

Chapter 600

Appraisal Ratio Studies

Introduction

Ratio studies in mass appraisal are regularly used to evaluate the quality and accuracy of appraisal. Ratio studies provide a set of statistics describing the distribution of the ratios (such as central tendency and spread), as well as summaries of uniformity (horizontal and vertical equity).

The basic measure in ratio studies is the ratio of the appraised value to the sale price. This procedure provides a variety of statistics to describe the set of ratios under evaluation, including the median, mean, weighted mean, interquartile range (IQR), coefficient of dispersion (COD), coefficient of variation (COV), coefficient of concentration (COC), price-related differential (PRD), and coefficient of price-related bias (PRB), among many others. Confidence intervals are available for the median, mean, weighted mean, and PRB. Normality assumption tests and plots are also included in this procedure.

Often there are natural groupings of the appraised values that are compared for equity in appraisal. Examples of such groupings are neighborhood (or market area), property type (or use), size, age, condition, quality rating, and style. Several tools are available in this procedure to evaluate horizontal equity among such groups. These tools include comparative box plots, comparative density plots (and several other distribution comparison plots), assumption tests, and horizontal equity tests such as the Kruskal-Wallis Rank test, the ANOVA F-test, and Welch's test.

Another area of interest in ratio studies is vertical equity (whether ratios are similar across the whole range of appraisal and sale price values). In addition to the PRD and PRB indices of vertical equity, this procedure provides several scatter plots and tests for evaluating ratios across sale price values and appraisal values. Both slope / Pearson correlation and nonparametric Spearman rank tests are available.

Two excellent sources for greater detail on the topic of appraisal ratio study analysis are Gloudemans and Almy (2011) and the Standard on Ratio Studies put forth by the International Association of Assessing Officers (available through iaao.org at the time of this writing).

Data Structure

In the Appraisal Ratio Studies procedure, the ratio data may be specified in any of three ways.

- One appraisal column, one sale price column
- Group of appraisal columns, group of sale price columns
- Ratio column containing the already computed (and adjusted, if applicable) ratios

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One appraisal column, one sale price column

The most common data structure for ratio studies is a single column containing the appraisal values, and a single column containing the sale price values. Ratios are computed as

$$\text{Ratio} = \text{Appraisal Value} / \text{Sale Price Value}$$

Appraisal	Sale_Price
62009	64000
21685	23000
41663	39500
52576	54000
32231	26500
90417	96000
.	.
.	.
.	.

Group of appraisal columns, group of sale price columns

In cases where the appraisal and/or the sale price is divided, multiple columns may be used to specify the components of the appraisal or sale. In these cases the ratios are computed as

$$\text{Ratio} = \text{Sum of Appraisal Values} / \text{Sum of Sale Price Values}$$

App_Land	App_Bldg	Price_Land	Price_Bldg
325000	350900	316700	317000
218800	321900	212300	327500
287200	231400	281400	242600
176200	219300	150200	214800
336400	275600	366700	274900
216400	247700	220400	259700
.	.	.	.
.	.	.	.
.	.	.	.

Ratio column containing the already computed (and adjusted, if applicable) ratios

For the case where the ratios have already been produced, perhaps in the NCSS spreadsheet or in another program, the ratio values may be contained in a single column. However, statistics and reports that require appraisal values and sale price values cannot be generated in this case.

Ratio
1.232
1.094
0.973
1.049
0.964
0.982
.
.
.

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Three optional column entries may also be used to enhance the appraisal study.

- Break columns for horizontal grouping
- Date column for filtering by date or for adjusting the sale price to a specific date
- Filter column for including a specific subset of the dataset

Break columns for horizontal grouping

In many mass appraisal ratio studies, investigators would like to compare ratios across horizontal groups (e.g., neighborhood or property type). In this procedure each set of groups is defined by a break column, a column with various group values. Any number of break columns may be used in this procedure. A separate analysis is performed for each break column entered.

Appraisal	Sale_Price	Neighborhood
62009	64000	Oak Hills
21685	23000	Green Point
41663	39500	Oak Hills
52576	54000	Oak Hills
32231	26500	Oak Hills
90417	96000	West Flats
20815	24400	Green Point
67843	62100	Green Point
28649	31000	Oak Hills
94812	95500	West Flats
55823	54000	West Flats
.	.	.
.	.	.
.	.	.

Date column for filtering by date or for adjusting the sale price to a specific date

In this procedure there are two potential uses for a column of sale dates. One available option is to remove the use of rows that fall outside a given date range. The second option compares the dates of this column to a basis date to make adjustments to the sale price.

Appraisal	Sale_Price	Neighborhood	Sale_Date
62009	64000	Oak Hills	4/18/2016
21685	23000	Green Point	3/2/2016
41663	39500	Oak Hills	1/23/2016
52576	54000	Oak Hills	4/14/2016
32231	26500	Oak Hills	7/3/2016
90417	96000	West Flats	2/19/2016
20815	24400	Green Point	5/12/2016
67843	62100	Green Point	4/25/2016
28649	31000	Oak Hills	1/20/2016
94812	95500	West Flats	9/6/2016
55823	54000	West Flats	3/13/2016
.	.	.	.
.	.	.	.
.	.	.	.

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Filter column for including a specific subset of the dataset

A column may be used to define a specific subset of the rows that are desired to be included in the ratio study. For example, in the dataset below the user may wish to filter for use of only residential properties in the ratio study. The filtering can be done through the filter system of the spreadsheet, or the column and desired value(s) may be used directly in the Appraisal Ratio Studies procedure.

Appraisal	Sale_Price	Neighborhood	Type
62009	64000	Oak Hills	Residential
21685	23000	Green Point	Residential
41663	39500	Oak Hills	Commercial
52576	54000	Oak Hills	Commercial
32231	26500	Oak Hills	Residential
90417	96000	West Flats	Residential
20815	24400	Green Point	Commercial
67843	62100	Green Point	Residential
28649	31000	Oak Hills	Commercial
94812	95500	West Flats	Residential
55823	54000	West Flats	Commercial
.	.	.	.
.	.	.	.
.	.	.	.

Technical Details

A considerable number of statistics, confidence intervals and statistical tests are available in this procedure. The technical details of many of these are given below. For those that are not given, the reader is pointed to another section of the documentation where the details can be viewed.

Definitions and Ratios

Let A indicate the appraised value, and let S indicate the sale price. Let R designate the ratio as

$$R = \frac{A}{S}$$

Since there are a number of ratios in a ratio study, let each ratio be defined as

$$R_i = \frac{A_i}{S_i}$$

where i is the row or sequence number. Thus, the series of ratios is R_1, R_2, R_3 , etc.

Median Ratio

The median ratio is one of several measures of the center of the ratios. It is less influence by outliers than the mean. The median ratio is the middle ratio of the ordered ratios if the number of ratios is odd, or it is the mean of the two middle ratios if the number of ratios is even. Symbolically, the median is \tilde{R} .

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Mean Ratio

The mean (or average) ratio of n ratios is

$$\bar{R} = \frac{\sum_{i=1}^n R_i}{n}$$

Weighted Mean

The weighted mean is the ratio of the total appraised values for the entire sample and the total sales prices of the entire sample

$$WM = \frac{\bar{A}}{\bar{S}} = \frac{\sum_{i=1}^n A_i}{\sum_{i=1}^n S_i}$$

The weighted mean weights each ratio by the sales price. Hence, high priced properties carry a larger weight than low priced properties.

Interquartile Range (IQR)

The interquartile range is the difference between the 75th percentile of the ratios and the 25th percentile. In other words, it is the range of middle 50% of the ratios. It is commonly used as a measure of spread when the median is used as the measure of the center.

Standard Deviation (SD)

The standard deviation is roughly the average distance of the ratios from the ratio mean. For more details, see the Descriptive Statistics chapter of the documentation.

Coefficient of Dispersion (COD)

The coefficient of dispersion is a widely used measure of appraisal uniformity. It expresses as a percentage the average deviation of the ratios from the median.

$$COD = \frac{100}{n} \frac{\sum_{i=1}^n |R_i - \tilde{R}|}{\tilde{R}}$$

Coefficient of Variation (COV)

The coefficient of variation is defined as follows:

$$COV = \frac{100 * SD}{\bar{R}}$$

Price-Related Differential (PRD)

The price-related differential measures the regressivity or progressivity of the assessments. Regressive appraisals occur when high-value properties are under-appraised relative to low-value properties. Progressive appraisals occur when the opposite pattern occurs.

$$PRD = \frac{\bar{R}}{\bar{A}/\bar{S}} = \frac{\bar{R}}{WM}$$

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Coefficient of Price-Related Bias (PRB)

The coefficient of price-related bias is used to analyze vertical equity (whether ratios are similar across the whole range of appraisal and sale price values). Further details of this index are given in the Vertical Equity section of this chapter.

Coefficient of Concentration (COC)

The coefficient of concentration is the percentage of ratios within a given percentage of the median. High COCs for low percentages indicate high performance.

Weighted Coefficient of Dispersion (Weighted COD)

The weighted COD can be compared to the COD to determine if high-value properties are being assessed with more or less variability than low-value properties.

$$COD_w = \frac{100}{\bar{R}} \times \left(\frac{\sum_{i=1}^n (S_i/\bar{S}) |R_i - \bar{R}|}{n} \right)$$

Weighted Coefficient of Variation (Weighted COV)

Similarly to the weighted COD, the weighted COV can be compared to the COV to determine if high-value properties are being assessed with more or less variability than low-value properties.

$$COV_w = \frac{100}{\bar{R}} \times \sqrt{\frac{\sum_{i=1}^n (S_i/\bar{S})(R_i - \bar{R})^2}{n}}$$

Median Absolute Deviation from the Median (MADM)

Comparable to the standard deviation, this value is the median of the absolute values of all the distances from the ratios to the median.

Median Absolute Percent Deviation from the Median (MAPDM)

This value is the median of the absolute values of all the percent differences of the ratios from the median.

Trimmed Mean

The trimmed mean is the mean of the ratios after removing a desired percentage of the extreme ratios of both sides of the mean.

Confidence Interval for the Median Ratio

The details of obtaining the confidence interval for the median are described in the Percentile section of Descriptive Statistics chapter.

Confidence Interval for the Mean Ratio

The lower and upper confidence limits of the mean ratio are given by

$$\bar{R} \pm t_{\alpha/2, n-1} \frac{SD}{\sqrt{n}}$$

where t is the appropriate value of the t distribution.

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Confidence Interval for the Weighted Mean Ratio

The confidence interval for the weighted mean ratio is calculated as described in Cochran (1971) and Gloudemans and Almy (2011). The lower and upper confidence limits of the weighted mean ratio are given by

$$\bar{A}/\bar{S} \pm t_{\frac{\alpha}{2}, n-1} \frac{\sqrt{\sum_{i=1}^n A_i^2 - 2\bar{A}/\bar{S} \sum_{i=1}^n A_i S_i + (\bar{A}/\bar{S})^2 \sum_{i=1}^n S_i^2}}{\bar{S} \sqrt{n(n-1)}}$$

where t is the appropriate value of the t distribution.

Skewness

The skewness measures the direction and degree of asymmetry. A value of zero indicates a symmetrical distribution. A positive value indicates skewness (long-tailedness) to the right while a negative value indicates skewness to the left. The method of calculation of skewness in this procedure is Fisher's g_1 . For more details, see the Descriptive Statistics chapter of the documentation.

Kurtosis

This statistic measures the heaviness of the tails of a distribution. The usual reference point in kurtosis is the Normal distribution. Unimodal distributions that have kurtosis greater than three have heavier or thicker tails than the Normal distribution. These same distributions also tend to have higher peaks in the center of the distribution (leptokurtic). Unimodal distributions whose tails are lighter than the Normal distribution tend to have a kurtosis that is less than three. In this case, the peak of the distribution tends to be broader than the Normal distribution (platykurtic). The method of calculation of kurtosis in this procedure is Fisher's g_2 . For more details, see the Descriptive Statistics chapter of the documentation.

Normality Tests

Seven normality tests are produced in this procedure. Each Normality test has its strengths. Typically, distributions of ratios that are strongly non-Normal will result in a rejection of Normality for most or all these tests. For technical details of these seven tests, go to the Normality Tests chapter of the documentation.

The seven Normality tests given in this procedure are

- Shapiro-Wilk Test
- Anderson-Darling Test
- Martinez-Iglewicz Test
- Kolmogorov-Smirnov (Lilliefors' adjusted) Test
- D'Agostino Skewness Test
- D'Agostino Kurtosis Test
- D'Agostino Omnibus (Skewness and Kurtosis) Test

Assumptions for Tests of Horizontal Equity – Equal Variance

There are four tests given in this procedure for testing equal variance among groups. For technical details of these four tests, go to the One-Way Analysis of Variance chapter of the documentation.

The four equal variance tests given in this procedure are

- Brown-Forsythe (Data - Medians)
- Levene (Data - Means)
- Conover (Ranks of Deviations)
- Bartlett (Likelihood Ratio)

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Tests of Horizontal Equity

The Tests of Horizontal Equity are used to determine whether the mean or median ratio differs across horizontal groups.

The three tests given in this procedure are

- Kruskal-Wallis Rank Test
- ANOVA F-Test
- Welch's Test

The ANOVA F-test is recommended when Normality and equal variance can be assumed. Welch's test is recommended when Normality can be assumed, but equal variance cannot. The Kruskal-Wallis test is recommended when the assumption of Normality is under question. For technical details of these three tests, go to the One-Way Analysis of Variance chapter of the documentation.

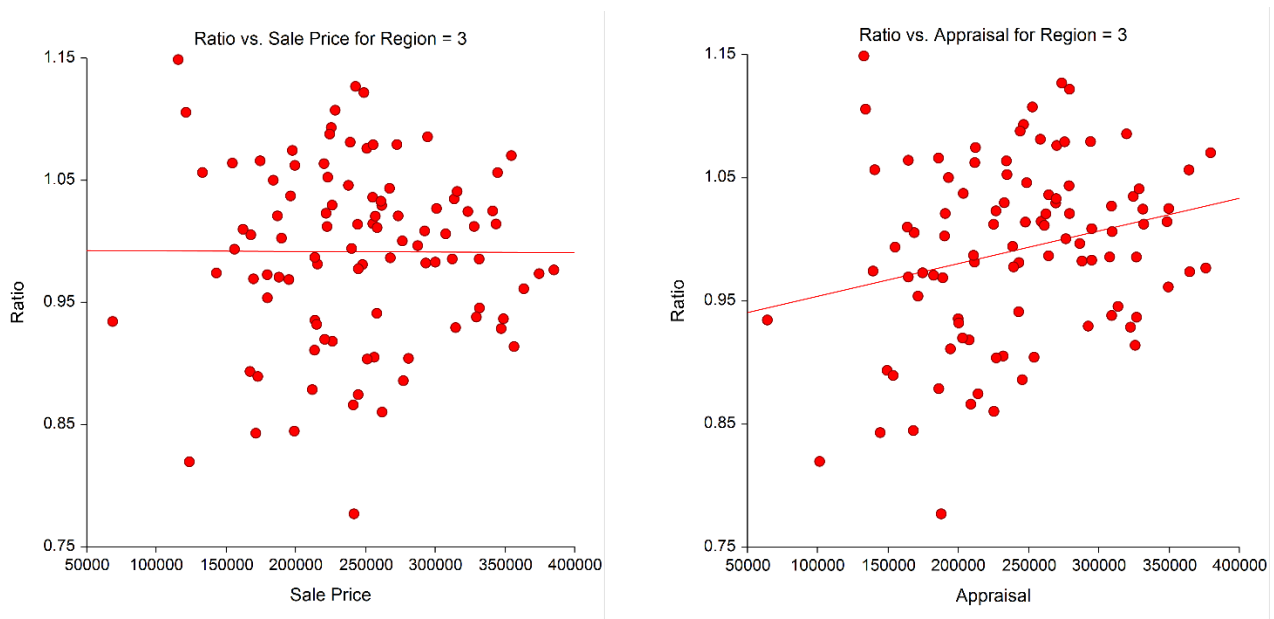
Vertical Equity

It is often of importance in ratio studies to determine whether ratios behave similarly for low-value, medium-value, and high-value properties. A variety of scatter plots, correlation values, and statistical tests may be used to assist in this determination.

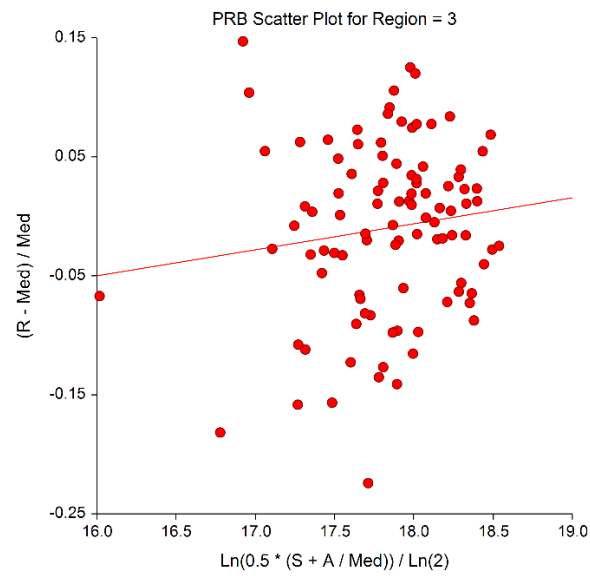
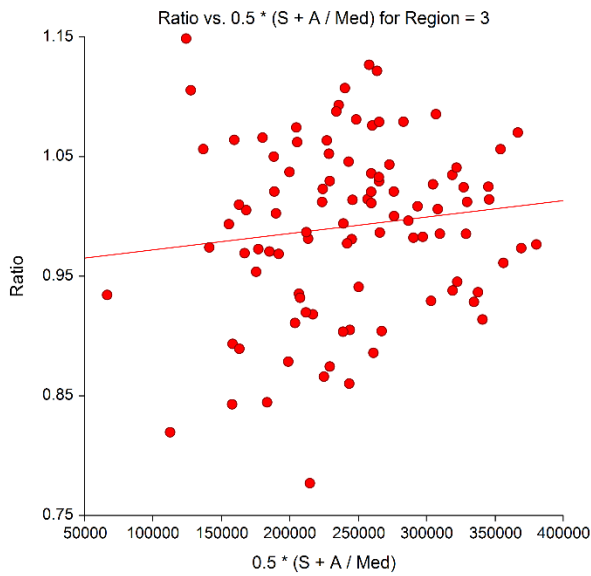
Four similar analyses can be used in the Appraisal Ratio Studies procedure to assess vertical equity. For three of these, the y values are the ratios and the x values are a measures of property value: sale price, appraisal, and adjusted average of sale price and appraisal. In the fourth case, the y values are based on the ratios and the x values are based on the sale price and appraisal values in such a way that the slope of the fitted line has a meaningful interpretation. The slope in this last case is called the coefficient of price-related bias (PRB). For the PRB method, the y and x values are computed as

$$y_i = \frac{R_i - \tilde{R}}{\tilde{R}}$$

$$x_i = \frac{\ln\left(0.5\left(S_i + \frac{A_i}{\tilde{R}}\right)\right)}{\ln(2)}$$



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For each of these scenarios, the correlation (slope) can be tested to determine whether there is evidence of inequity across property value. Details for the calculation of the correlation, slope, testing correlation (slope) significance, and confidence intervals are given the Linear Regression and Correlation chapter of the documentation. Details of the Spearman (rank) correlation and corresponding test are also given in that chapter. The Spearman rank correlation is sometimes used when the Y - X relationship appears to be curved rather than linear.

The slope in the PRB analysis has the following interpretation: when property values double, ratios rise (or fall) by $100 \times \text{PRB} \%$. For example, if PRB is 0.1418, ratios increase by 14.2% when sale prices go from \$100,000 to \$200,000. If PRB is -0.2358, ratios decrease by 23.6% when sale prices go from \$100,000 to \$200,000.

Adjustment to Sale Price based on Sale Date

In some cases it is of use to adjust sale prices of properties to a specific date, particularly when property values increase or decrease significantly during a short period of time. One simplistic approach to make these adjustments is available in this procedure. The date of interest (basis date) is compared to the date of each sale and the difference in months between the two dates is calculated. A monthly price adjustment factor (A) is used to produce sale prices adjusted to the basis date as follows

$$S_{i,adj} = (1 + A \times M)S_i$$

where $S_{i,adj}$ is the adjusted sale price, S_i is the original sale price, A is the adjustment factor, and M is the difference in months from the basis date. Positive A values indicate increasing sale prices, while negative A values indicate decreasing sale prices. Dates previous to the basis date result in positive M values, while dates after the basis date result in negative M values.

For example, suppose the basis date is January 1. Further suppose that a property is sold 4 months after the basis date for \$100,000. Also suppose that properties values in general increased about 6% during the year following the basis date. Increasing 6% per year, or 0.06, is comparable to 0.005 per month (if not compounding). The value for A in this example would be 0.005. The adjusted sale price used in the ratio analysis would be

$$S_{i,adj} = (1 + 0.005 \times (-4))\$100,000 = \$98,000$$

This is the estimated sale price for the property if the property had been sold 4 months earlier on January 1.

Procedure Options

This section describes the options available in this procedure.

Columns Tab

Ratio Values Specification

Ratio Specification

This selection determines how the ratios are produced for this procedure.

Appraisal Column, Sale Price Column

For this selection a single column containing the appraised values, and a single column containing the sale price values, are used. One ratio per row is computed, using appraisal / sale price.

Sum Appraisal Columns, Sum Sale Price Columns

For this choice, appraisal and/or sale price values are computed by summing values from multiple columns. One ratio per row is computed, using sum of appraisal values / sum of sale price values.

Ratio Column

For this selection, a single column containing the already computed ratio values will be used. With this selection, the various statistics and reports that require appraisal values and sale price values cannot be given.

Appraisal Column

Enter the name or number of the column containing the appraisal values. You may type the column name or number directly, or you may use the column selection tool by clicking the column selection button to the right.

Sale Price Column

Enter the name or number of the column containing the sale price values. You may type the column name or number directly, or you may use the column selection tool by clicking the column selection button to the right. If you wish to adjust the sale price values to a specific date, this may be done by specifying sale date column and desired date on the Sale Date tab.

Appraisal Column(s)

Enter the names or numbers of one or more columns containing the appraisal (sub)value(s). For each row, the values across these columns will be added together to produce the appraisal value. You may type the column names or numbers directly, or you may use the column selection tool by clicking the column selection button to the right.

Sales Price Column(s)

Enter the names or numbers of one or more columns containing the sale price (sub)value(s). For each row, the values across these columns will be added together to produce the sale price value. You may type the column names or numbers directly, or you may use the column selection tool by clicking the column selection button to the right. If you wish to adjust the (summed) sale price values to a specific date, this may be done by specifying sale date column and desired date on the Sale Date tab.

Ratio Column

Enter the name or number of the column containing the ratio values. You may type the column name or number directly, or you may use the column selection tool by clicking the column selection button to the right.

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Ratio Values Specification – Horizontal Grouping

Break Column(s)

The column(s) entered here specify horizontal groupings of the ratios. A variety of horizontal equity plots and tests can be specified on the 'Horizontal Equity' tab. The values in these columns should define categories, and should not be continuous. For example, if size in acres were to be used as a Break Column, a designation such as 'Small', 'Medium', and 'Large' should be used, than leaving the acreage values. Some common examples of break (horizontal grouping) columns are neighborhood (or market area), property type (or use), size, age, and style.

Ratio Values Specification – Removing Extreme Ratios

Remove Ratios less than

Check this box if you wish to remove all ratios from the analysis that are below the specified value. You can view a list of the removed ratios by checking the 'Removed Ratios Report' checkbox on the Summary Reports tab.

Minimum Ratio Kept Value

All ratios below this value will be removed from the analysis when the 'Remove Ratios less than' checkbox is checked. The scale for the minimum ratio entered here is the '1' scale (not the '100' scale).

Remove Ratios greater than

Check this box if you wish to remove all ratios from the analysis that are above the specified value. You can view a list of the removed ratios by checking the 'Removed Ratios Report' checkbox on the Summary Reports tab.

Maximum Ratio Kept Value

All ratios above this value will be removed from the analysis when the 'Remove Ratios greater than' checkbox is checked. The scale for the maximum ratio entered here is the '1' scale (not the '100' scale).

Remove Outliers using IQR Method

Check this box if you wish to have outliers removed from the analysis based on the IQR method. The IQR method removes ratios that are less than the first quartile minus the multiplier times the interquartile range, or greater than the third quartile plus the multiplier times the interquartile range. That is, the boundaries are $Q1 - M * IQR$ and $Q3 + M * IQR$. The first quartile, $Q1$, is the ratio such that 25% of the ratios are lower and 75% are higher. Similarly, the third quartile, $Q3$, is the ratio such that 75% of the ratios are lower and 25% are higher. You can view a list of the removed outliers by checking the 'Removed Ratios Report' checkbox on the Summary Reports tab.

IQR Multiplier

When using the IQR method for removing outliers, this value is the multiplier for the IQR. The IQR Multiplier is M for the boundaries $Q1 - M * IQR$ and $Q3 + M * IQR$.

Filtering

Include Column

This (optional) column can be used to select specific rows to be included in the analysis. It is an alternative to using the filter on the spreadsheet. If an 'Include Column' is entered here, only those rows with the 'Included Values' in this column will be used.

Included Values

The values entered here specify which rows will be used in the analysis. These values are compared to each row of the 'Include Column' column. If the value in the column matches one of these values, the ratio of this row is used. Otherwise, it is discarded from the analysis.

Summary Reports Tab

Sale Price Adjustment by Date

Sale Price Adjustment by Date Summary

Check this box to show a summary of the sale date adjustment to sale price as specified on the Sale Date tab.

Removed Ratios

Removed Ratios Report

Check this box to obtain a list of the ratios that were removed using the IQR method or the ratio boundaries. Ratios that are removed using the 'Include Column' or sale date boundaries are not summarized in this report.

Summary Statistics

Ratio Summary Statistics

Check this box to obtain the following ratio statistics:

- Count
- Median
- Mean
- Weighted Mean
- Interquartile Range (IQR)
- Standard Deviation
- Coefficient of Dispersion (COD)
- Coefficient of Variation (COV)
- Price-Related Differential (PRD)
- Coefficient of Price-Related Bias (PRB) *

* PRB is not available when ratios are input directly.

These statistics are shown for each group of the Break Column if a Break Column is specified.

Additional Summary Statistics

Check this box to obtain the following ratio statistics:

- Count
- Minimum
- Maximum
- Range
- Coefficient of Concentration (COC)
- Weighted Coefficient of Dispersion *
- Weighted Coefficient of Variation *
- Median Absolute Deviation from the Median
- Median Absolute Percent Deviation from the Median
- Trimmed Mean

* These statistics are not available when ratios are input directly.

These statistics are shown for each group of the Break Column if a Break Column is specified.

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COC %

This is the percentage associated with the coefficient of concentration. The coefficient of concentration is the percentage of ratios within 'COC %' of the median.

Trim %

For producing the trimmed mean, this percentage is the percent of the most extreme values trimmed from each side of the mean before the mean is calculated.

Appraisal and Sale Price Summary

Check this box to obtain the following statistics:

- Count
- Appraisal Mean
- Sale Price Mean
- Appraisal Median
- Sale Price Median

These statistics are shown for each group of the Break Column if a Break Column is specified.

Confidence Intervals

Check this box to obtain the following confidence interval statistics:

- Count
- Ratio Median and Confidence Limits
- Ratio Mean and Confidence Limits
- Ratio Weighted Mean and Confidence Limits

These statistics are shown for each group of the Break Column if a Break Column is specified.

Confidence Level

Enter the value of the desired confidence level for the confidence limits.

Normality Assumptions for each Horizontal Group

Normality Assumptions

Check this box to obtain the following ratio distribution statistics and Normality tests:

- Count
- Skewness
- Kurtosis
- Shapiro-Wilk Test
- Anderson-Darling Test
- Martinez-Iglewicz Test
- Kolmogorov-Smirnov (Lilliefors' adjusted) Test
- D'Agostino Skewness Test
- D'Agostino Kurtosis Test
- D'Agostino Omnibus (Skewness and Kurtosis) Test

These statistics and Normality tests are shown for each group of the Break Column if a Break Column is specified.

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Normality Test Alpha

This is the alpha level to which each Normality test p-value is compared to determine whether the assumption of Normality is rejected.

Histogram

This plot gives a visual representation of the distribution of the ratios. An individual histogram is shown for each group of the Break Column if a Break Column is specified.

Normal Probability Plot

A Normal Probability Plot allows the investigator to visually assess Normality by determining how well the points follow a straight line.

Horizontal Equity Tab

Plots of Horizontal Equity

Horizontal Equity Plot

The horizontal equity plots are used to compare the ratio distribution for each of the horizontal groups, as defined by the Break Column(s). Each plot gives a different perspective of the distribution of ratio values. The choice of the best plot(s) to be used depends on a variety of factors, including the number of ratios in each group, familiarity, and local requirements.

Tests of Horizontal Equity

Horizontal Equity Test Assumptions - Normality

This report can be helpful in determining which test of horizontal equity to use. When Normality is not a reasonable assumption, the Kruskal-Wallis Rank Test is a recommended nonparametric alternative to the ANOVA F-test or Welch's test. The three tests given in this report are

- D'Agostino Skewness Test
- D'Agostino Kurtosis Test
- D'Agostino Omnibus (Skewness and Kurtosis) Test

Each test uses the combined residuals from the group means.

Alpha

This is the alpha level to which each Normality test p-value is compared to determine whether the assumption of Normality is rejected.

Horizontal Equity Test Assumptions - Equal Variance

This report is used to determine whether the variances of the groups can be assumed to be approximately equal. Under approximate Normality, the equal variance assumption determines whether to use the ANOVA F-test or Welch's test for testing horizontal equity. The four equal variance tests given in this report are

- Brown-Forsythe (Data - Medians)
- Levene (Data - Means)
- Conover (Ranks of Deviations)
- Bartlett (Likelihood Ratio)

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Alpha

This is the alpha level to which each equal variance test p-value is compared to determine whether the assumption of equal variance is rejected.

Tests of Horizontal Equity

The Tests of Horizontal Equity are used to determine whether the mean or median ratio differs across horizontal groups. The groups are defined by the Break Column. The three tests given in this report are

- Kruskal-Wallis Rank Test
- ANOVA F-Test
- Welch's Test

The choice of which horizontal equity test should be used is determined by which assumptions are met. See the documentation for details.

Alpha

This is the alpha level to which each horizontal equity test p-value is compared to determine whether horizontal equity is rejected.

Vertical Equity Tab

Vertical Equity – Price Related Bias (PRB)

PRB Scatter Plots

Check this box to include the scatter plot corresponding to the coefficient of price-related bias (PRB). In this plot, the X and Y values are

$$X = \ln(0.5 * (\text{Sale Price} + \text{Appraisal} / \text{Median Ratio})) / \ln(2)$$

$$Y = (\text{Ratio} - \text{Median Ratio}) / \text{Median Ratio}$$

The slope of the regression line of this plot is the coefficient of price-related bias (PRB). A separate scatter plot is shown for each group of the Break Column if a Break Column is specified.

Price-Related Bias Details

Check this box to include details about PRB, the coefficient of price-related bias. PRB is used to evaluate vertical equity (whether ratios are similar across the whole range of appraisal and sale price values). The following statistics are available in this report

- Count
- Pearson Correlation
- Coefficient of Price-Related Bias (PRB)
- Standard Error of the PRB Estimate
- PRB Test T Value
- PRB Test P-Value
- PRB Confidence Interval Limits

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PRB is the slope given when regressing

$$Y = (\text{Ratio} - \text{Median Ratio}) / \text{Median Ratio}$$

on

$$X = \text{Ln}(0.5 * (\text{Sale Price} + \text{Appraisal} / \text{Median Ratio})) / \text{Ln}(2)$$

Alpha

This is the alpha level to which each vertical equity PRB test p-value is compared to determine whether vertical equity is rejected. It is also the alpha level used in the Spearman Rank test, if the Spearman Rank Details are to be shown.

Confidence Level

Enter the value of the desired confidence level for the confidence limits of PRB.

Include Spearman Rank Details

If the PRB Scatter Plot shows a relationship that is not flat or straight-lined, the nonparametric Spearman Rank Correlation may be a better assessment of vertical equity.

This report analyzes the relationship between

$$Y = (\text{Ratio} - \text{Median Ratio}) / \text{Median Ratio}$$

and

$$X = \text{Ln}(0.5 * (\text{Sale Price} + \text{Appraisal} / \text{Median Ratio})) / \text{Ln}(2)$$

using the Spearman Rank Correlation. The following statistics are available in this report

- Count
- Spearman Correlation
- Spearman Correlation Test P-Value

"

Vertical Equity – Ratio vs. Sale Price

R vs. S Scatter Plots

This plot visually represents the relationship between the ratios and sale prices. In this plot,

$$Y = \text{Ratio}$$

and

$$X = \text{Sale Price}$$

A separate scatter plot is shown for each group of the Break Column if a Break Column is specified.

Appraisal Ratio Studies

R vs. S Tests of Vertical Equity

This report shows how ratios trend across the range of sale price values.

It analyzes the relationship between

Y = Ratio

and

X = Sale Price

The following statistics are available in this report

- Count
- Pearson Correlation
- Pearson Correlation / Slope Test P-Value
- Spearman Correlation
- Spearman Correlation Test P-Value

Alpha

This is the alpha level to which each vertical equity test p-value is compared to determine whether vertical equity is rejected.

Vertical Equity – Ratio vs. Appraisal

R vs. A Scatter Plots

This plot visually represents the relationship between the ratios and appraisal values. In this plot,

Y = Ratio

and

X = Appraisal Value

A separate scatter plot is shown for each group of the Break Column if a Break Column is specified.

R vs. A Tests of Vertical Equity

This report shows how ratios trend across the range of appraisal values. It analyzes the relationship between

Y = Ratio

and

X = Appraisal Value

The following statistics are available in this report

- Count
- Pearson Correlation
- Pearson Correlation / Slope Test P-Value
- Spearman Correlation
- Spearman Correlation Test P-Value

Appraisal Ratio Studies

Alpha

This is the alpha level to which each vertical equity test p-value is compared to determine whether vertical equity is rejected.

Vertical Equity – Ratio vs. $0.5(\text{Sale Price} + \text{Appraisal} / \text{Median})$

R vs. $0.5(S + A')$ Scatter Plots

This plot visually represents the relationship between the ratios and the average of sale price and adjusted appraisal values. In this plot,

Y = Ratio

and

$X = 0.5 * (\text{Sale Price} + \text{Appraisal} / \text{Median Ratio})$

A separate scatter plot is shown for each group of the Break Column if a Break Column is specified.

R vs. $0.5(S + A')$ Tests of Vertical Equity

This report shows how ratios trend across the range of the average of sale price and adjusted appraisal values. It analyzes the relationship between

Y = Ratio

and

$X = 0.5 * (\text{Sale Price} + \text{Appraisal} / \text{Median Ratio})$

The following statistics are available in this report

- Count
- Pearson Correlation
- Pearson Correlation / Slope Test P-Value
- Spearman Correlation
- Spearman Correlation Test P-Value

Alpha

This is the alpha level to which each vertical equity test p-value is compared to determine whether vertical equity is rejected.

Report Options Tab

Report Options

Ratio Reporting Scale

Specify whether to report ratios and ratio statistics as ratios or as percents. Ratios are inherently on the ratio scale, that is, values near 1. However, some investigators may wish to view the ratio as the appraisal being a percent of the sale price. In this case, the typical values are near 100. The two available ratio reporting scales are

- Ratio (1)
- Percent (100)

Show Definitions

For several of the reports the column headings are shortened phrases or acronyms. Check this box to include report definitions at the end of those reports.

Column Names

Specify whether to use column names, column labels, or both to label output reports.

Names

Column names are the column headings that appear on the data table. They may be modified by clicking the Column Info button on the Data window or by clicking the right mouse button while the mouse is pointing to the column heading.

Labels

This refers to the optional labels that may be specified for each column. Clicking the Column Info button on the Data window allows you to enter them.

Both

Both the column names and labels are displayed.

Comments

1. Most reports are formatted to receive about 12 characters for variable names.
2. Column Names cannot contain blanks or math symbols (like + - * / . ,), but variable labels can.

Value Labels

Value Labels are used to make reports more legible by assigning meaningful labels to numbers and codes.

Data Values

All data are displayed in their original format, regardless of whether a value label has been set or not.

Value Labels

All values of columns that have a value label column designated are converted to their corresponding value label when they are output. This does not modify their value during computation.

Both

Both data value and value label are displayed.

Example

A variable named NBHD (used as a grouping variable) contains 1's, 2's, and 3's. By specifying a value label for NBHD, the report can display "Pine Acres" instead of 1, "Shadowcrest" instead of 2, and "Meadow Farms" instead of 3. This option specifies whether (and how) to use the value labels.

Appraisal Ratio Studies

Decimal Places

Decimal Places

This option allows the user to specify the number of decimal places directly or using an Auto function. If one of the Auto options is used, the ending zero digits are not shown. Your choice here will not affect calculations; it will only affect the format of the output.

Auto

If one of the 'Auto' options is selected, the ending zero digits are not shown. For example, if 'Auto (0 to 7)' is chosen,

0.0500 is displayed as 0.05

1.314583689 is displayed as 1.314584

The output formatting system is not designed to accommodate 'Auto (0 to 13)', and if chosen, this will likely lead to lines that run on to a second line. This option is included, however, for the rare case when a very large number of decimals is needed.

Sale Date Tab

Sale Date Specification

Sale Date Column

This (optional) column of dates can be used to either remove ratios by date or adjust sale prices to a date, or both. Only a column of date values can be used here. You may type the column name or number directly, or you may use the column selection tool by clicking the column selection button to the right.

Sale Date Specification – Removing Ratios by Date

Remove Ratios with Date before

Check this box to remove the use of all rows (and thus ratios) where the Sale Date Column has a date previous to the specified date for that row.

Minimum Date Kept

Specify the date such that the rows for all dates in the Sale Date Column previous to this date will be removed from the analysis.

Remove Ratios with Date after

Check this box to remove the use of all rows (and thus ratios) where the Sale Date Column has a date after the specified date for that row.

Maximum Date Kept

Specify the date such that the rows for all dates in the Sale Date Column after this date will be removed from the analysis.

Appraisal Ratio Studies

Sale Date Specification – Price Adjustment

Adjust Prices To This Date

Check this checkbox to (linearly) adjust each sale price value to a single date. This option uses the Basis Date and the Monthly Price Adjustment Factor in connection with the dates of the Sale Date Column to form the price adjustment.

Price Adjustment Formula:

$$\text{Adjusted Price} = (1 + A * M) * \text{Sales Price}$$

where

A is the Monthly Price Adjustment Factor

M is the number of months from the Basis Date to the sale date.

Dates previous to the Basis Date result in a positive M value, while dates after the Basis Date result in a negative M value.

For example, suppose the basis date is January 1. Further suppose that a property is sold 4 months after the basis date for \$100,000. Also suppose that properties values in general increased 6% during the year following the basis date. Increasing 6% per year, or 0.06, is comparable to 0.005 per month (if not compounding). The value for A in this example would be 0.005. The adjusted sale price used in the ratio analysis would be

$$(1 + 0.005 * (-4)) * \$100,000 = \$98,000$$

Sale prices for which there is no date in the Sale Date Column will be left as is. Adjusted sale price values can be stored to the dataset by entering the appropriate (usually empty) column on the Storage tab.

Basis Date

This date is compared to the date values of the Sale Date Column to determine the number of months the sale was made before or after the Basis Date.

Monthly Price Adjustment Factor

This value determine the amount per month that each sale price is adjusted. Positive values for the Monthly Price Adjustment Factor result in decreasing sale price values that are after the basis date. Positive Monthly Price Adjustment Factor values should be used when mean or median sale prices increased after the basis date. Negative values for the Monthly Price Adjustment Factor result in increasing sale price values that are after the basis date. Negative Monthly Price Adjustment Factor values should be used when mean or median sale prices decreased after the basis date. For example, suppose the basis date is January 1. Further suppose that a property is sold 4 months after the basis date for \$100,000. Also suppose that properties values in general increased 6% during the year following the basis date. Increasing 6% per year, or 0.06, is comparable to 0.005 per month (if not compounding). The value entered here in this example would be 0.005. The adjusted sale price used in the ratio analysis would be

$$(1 + 0.005 * (-4)) * \$100,000 = \$98,000$$

Storage Tab

Storage Columns

Store Ratios in Column

If you wish to store the computed ratios back to the dataset, enter the desired (usually empty) result column here. The ratios returned to the dataset will be only those used in the analysis. That is, removed ratios, whether by boundary, IQR method, filtering, or date, will not be stored. Also, these ratios will be those based on the adjusted sale price if the sale price adjustment option is used. The stored ratios will not be saved with the dataset until the dataset is saved. You may type the column name or number directly, or you may use the column selection tool by clicking the column selection button to the right.

Store Time-Adjusted Sale Price in Column

If you wish to store the time-adjusted sale prices back to the dataset, enter the desired (usually empty) result column here. These values will only be different from the input sale price column if the 'Adjust Prices To This Date:' checkbox on the Sale Date tab is checked. The stored sale prices will not be saved with the dataset until the dataset is saved. You may type the column name or number directly, or you may use the column selection tool by clicking the column selection button to the right.

Example 1 – Appraisal Ratio Study

This example presents a broad ratio study based on the Appraisals dataset. Only the first three columns will be used in this example. Later examples show the use of outlier removal, multiple (horizontal grouping) break columns, and sale date adjustment. Summary statistics, assumptions, plots, horizontal equity, and vertical equity will all be examined in this example.

Appraisals Dataset

Appraisal	Price	Region	Type	Date
310900	296000	Fox Knoll	Residential	10/26/2015
264800	256300	Fox Knoll	Commercial	10/18/2015
204600	199700	Fox Knoll	Residential	1/2/2015
290600	264000	Fox Knoll	Commercial	3/11/2015
287900	281100	Fox Knoll	Residential	2/17/2015
292600	267400	Fox Knoll	Residential	5/12/2015
244000	253200	Fox Knoll	Commercial	4/12/2015
270100	270000	Fox Knoll	Residential	8/24/2015
210500	191200	Fox Knoll	Residential	11/8/2015
304700	302000	Fox Knoll	Commercial	6/6/2015
248200	228400	Fox Knoll	Residential	6/16/2015
.
.
.

You may follow along here by making the appropriate entries or load the completed template **Example 1** by clicking on Open Example Template from the File menu of the Appraisal Ratio Studies window.

1 Open the Appraisals dataset.

- From the **File** menu of the NCSS Data window, select **Open Example Data**.
- Click on the file **Appraisals.NCSS**.
- Click **Open**.

2 Open the Appraisal Ratio Studies window.

- Using the Analysis menu or the Procedure Navigator, find and select the **Appraisal Ratio Studies** procedure.
- On the menus, select **File**, then **New Template (Reset)**. This will fill the procedure with the default template.

3 Specify the columns.

- On the Appraisal Ratio Studies window, select the **Columns tab**.
- With Ratio Specification set to 'Appraisal Column, Sale Price Column', set the **Appraisal Column** to **Appraisal**.
- Set the **Sale Price Column** to **Price**.
- Under Horizontal Grouping, set the **Break Column(s)** to **Region**.

4 Run the procedure.

- From the Run menu, select **Run Procedure**. Alternatively, just click the green Run button.

Appraisal Ratio Studies

Ratio Summary Statistics Section

Ratio Summary Statistics Section										
Region	Count	Median	Mean	Wtd. Mean	IQR	SD	COD	COV	PRD	PRB
Fox Knoll	117	1.0035	1.0025	1.0033	0.0672	0.0565	4.258	5.6357	0.9993	0.0415
Freedom Manor	184	0.9767	0.9697	0.9773	0.1075	0.0984	7.087	10.1452	0.9922	0.0812
Hastings Heights	98	1.0015	0.9918	0.9917	0.1038	0.0728	5.732	7.3422	1.0001	0.0218
Puente Real	206	1.0043	1.0049	1.0052	0.0553	0.0525	3.808	5.2233	0.9997	0.0103
Sussex Vale	143	1.0208	1.0209	1.0190	0.0450	0.0403	2.884	3.9449	1.0019	-0.0082
Combined	748	1.0035	0.9972	0.9994	0.0678	0.0703	4.901	7.0468	0.9979	0.0250

Wtd. Mean: Weighted Mean
IQR: Interquartile Range
COD: Coefficient of Dispersion
COV: Coefficient of Variation
PRD: Price-Related Differential
PRB: Coefficient of Price-Related Bias

This report gives a commonly used set of ratio summary statistics for each value of the break column (Region), as well as for all regions combined. The technical details for each statistic are given earlier in the chapter in the Technical Details section. This report gives a numeric snapshot of the ratios in this study. The definitions given at the end can be hidden by unchecking Show Definitions on the Report Options tab.

Additional Ratio Summary Statistics Section

Additional Ratio Summary Statistics Section										
Region	Count	Min	Max	Range	15% COC	Wtd. COD	Wtd. COV	MADM	MAPDM	5% Trim. Mean
Fox Knoll	117	0.8160	1.1731	0.3571	98.291	4.040	28.0740	0.0299	2.978	1.0032
Freedom Manor	184	0.4714	1.3214	0.8501	91.304	6.174	72.6110	0.0517	5.296	0.9744
Hastings Heights	98	0.7770	1.1488	0.3718	95.918	5.450	47.9581	0.0481	4.806	0.9936
Puente Real	206	0.7481	1.1631	0.4149	97.573	3.203	18.4352	0.0273	2.716	1.0047
Sussex Vale	143	0.9056	1.1584	0.2528	100.000	2.400	10.8974	0.0214	2.097	1.0201
Combined	748	0.4714	1.3214	0.8501	96.257	4.249	36.3042	0.0340	3.387	1.0000

15% COC: Coefficient of Concentration (Percentage of Ratios within 15% of the Median)
Wtd. COD: Weighted Coefficient of Dispersion
Wtd. COV: Weighted Coefficient of Variation
MADM: Median Absolute Deviation from the Median
MAPDM: Median Absolute Percent Deviation from the Median
5% Trim. Mean: 5% Trimmed Mean (Mean after the 5% most extreme Ratios of each side are removed)

This report gives (perhaps less commonly used) additional summary statistics for each region and the combined regions. The technical details for each statistic are given earlier in the chapter in the Technical Details section.

Appraisal Ratio Studies

Appraisal and Sales Price Summary Statistics Section

Appraisal and Sales Price Summary Statistics Section

Region	Count	Appraisal Mean	Sale Price Mean	Appraisal Median	Sale Price Median
Fox Knoll	117	251792.3	250976.1	254700.0	256300.0
Freedom Manor	184	273625.0	279974.5	263550.0	274500.0
Hastings Heights	98	243087.8	245115.3	243550.0	244500.0
Puente Real	206	321050.5	319381.1	321850.0	319200.0
Sussex Vale	143	298388.1	292823.8	292300.0	290800.0
Combined	748	284004.3	284180.6	275600.0	273900.0

Summary statistics of the original appraisal and sale price values are given here.

Confidence Interval Section

Confidence Interval Section

Region	Count	Ratio			Mean			Ratio		
		Median	Lower	Upper	Mean	Lower	Upper	Wtd. Mean	Lower	Upper
Fox Knoll	117	1.0035	0.9944	1.0136	1.0025	0.9922	1.0129	1.0033	0.9937	1.0128
Freedom Manor	184	0.9767	0.9629	0.9873	0.9697	0.9554	0.9840	0.9773	0.9662	0.9884
Hastings Heights	98	1.0015	0.9814	1.0143	0.9918	0.9772	1.0064	0.9917	0.9780	1.0054
Puente Real	206	1.0043	0.9954	1.0116	1.0049	0.9977	1.0121	1.0052	0.9997	1.0107
Sussex Vale	143	1.0208	1.0148	1.0260	1.0209	1.0143	1.0276	1.0190	1.0140	1.0240
Combined	748	1.0035	0.9969	1.0086	0.9972	0.9924	1.0021	0.9994	0.9954	1.0034

This report gives confidence intervals for the median, mean, and weighted mean. The technical details for each confidence interval are given earlier in the chapter in the Technical Details section.

Normality Assumption Section

Normality Assumption Section

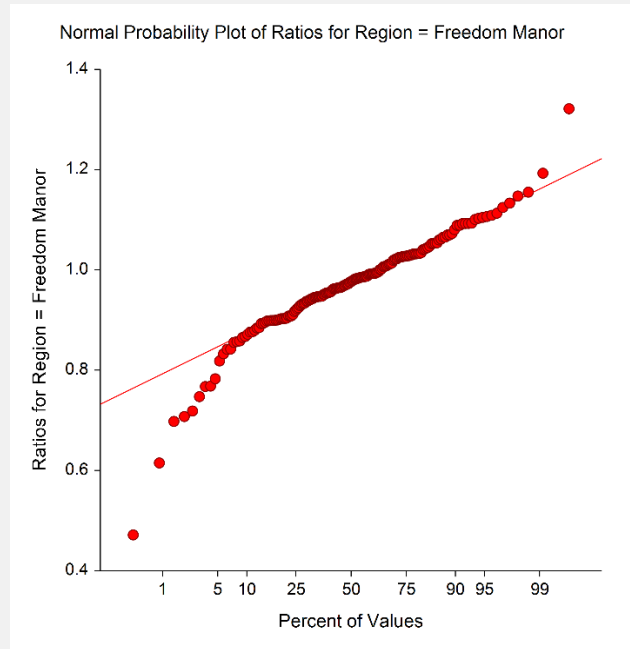
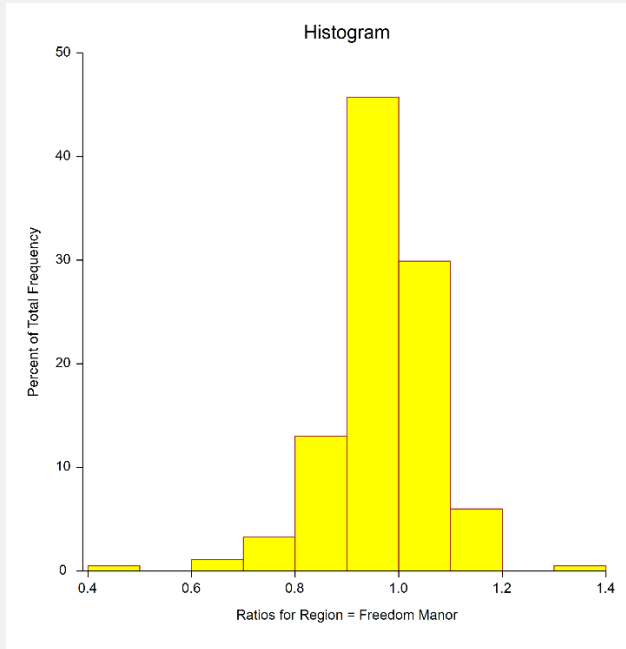
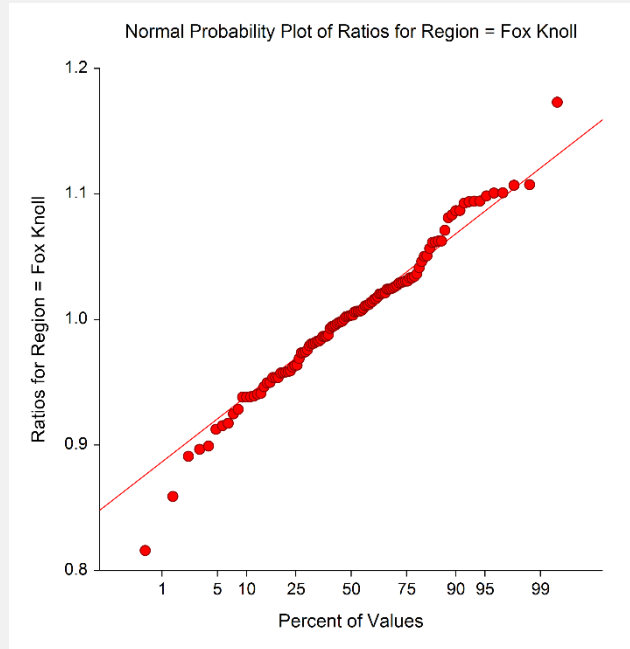
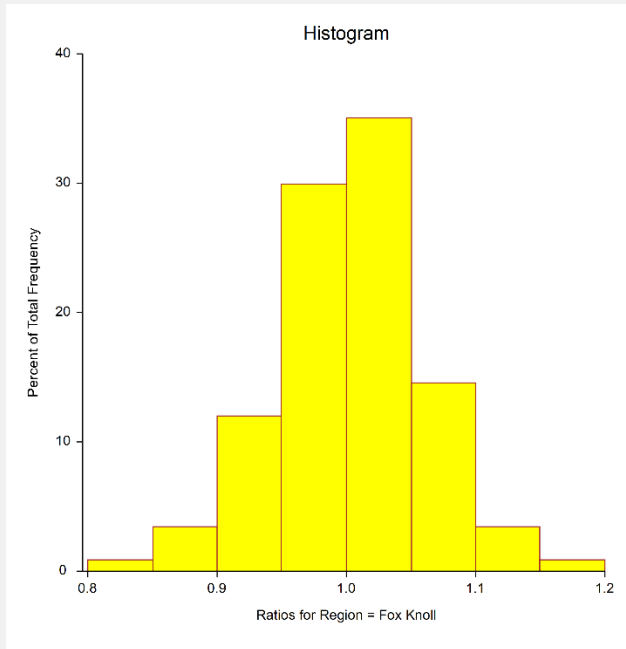
Region	Count	Skew	Kurt	P-Values (< 0.1 rejects Normality)						
				S-W	A-D	M-I	K-S	D-S	D-K	D-O
Fox Knoll	117	-0.0901	0.8100	0.3083	0.2056	< 0.1	> 0.1	0.6788	0.1037	0.2442
Freedom Manor	184	-0.9729	4.6534	0.0000	0.0000	< 0.1	< 0.1	0.0000	0.0000	0.0000
Hastings Heights	98	-0.4356	0.0173	0.3375	0.2245	> 0.1	> 0.1	0.0743	0.8216	0.1983
Puente Real	206	-0.2937	3.2823	0.0000	0.0006	< 0.1	< 0.1	0.0830	0.0000	0.0000
Sussex Vale	143	0.3837	1.6873	0.0036	0.0080	< 0.1	< 0.1	0.0594	0.0052	0.0034
Combined	748	-1.2057	6.9241	0.0000	0.0000	< 0.1	< 0.1	0.0000	0.0000	0.0000

Skew: Skewness (Negative values indicate left skewness; positive values indicate right skewness; values close to 0 indicate little skewness.)
 Kurt: Kurtosis (Negative values indicate fatter tails; positive values indicate a strong peak and lighter tails.)
 S-W: Shapiro-Wilk W Test
 A-D: Anderson-Darling Test
 M-I: Martinez-Iglewicz Test
 K-S: Kolmogorov-Smirnov (Lilliefors' adjusted) Test
 D-S: D'Agostino Skewness Test
 D-K: D'Agostino Kurtosis Test
 D-O: D'Agostino Omnibus (Skewness and Kurtosis) Test

Tests for which the Normality assumption is rejected are highlighted in red.

The skewness and kurtosis statistics, along with the seven Normality tests, give indication of whether the assumption of Normality is reasonable for each group. This report shows that Normality should not be assumed for the Freedom Manor, Puente Real, and Sussex Vale regions. For formulas and details, the user is pointed to the Descriptive Statistics and Normality Tests chapters in the documentation.

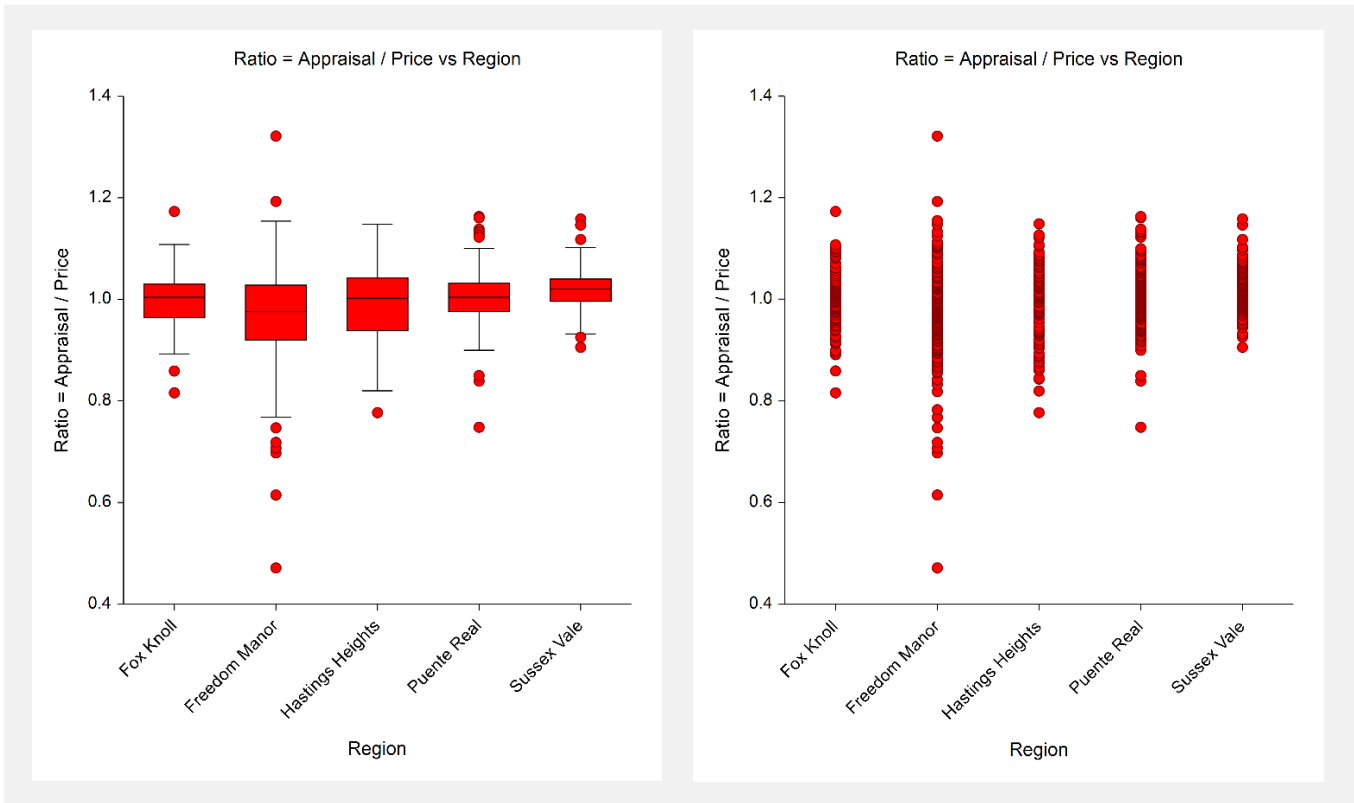
Normality Assumption Plots



Report continues with more Normality assumption plots...

The histograms and Normal probability plots give a visual representation of whether the assumption of Normality is reasonable. For the histograms, a bell-shaped appearance indicates Normality, while for the Normal probability plot, points that roughly follow the straight line indicate Normality. In this example the ratios for the Fox Knoll region appear to be Normally distributed, while those in the Freedom Manor region seem to exhibit a departure from Normality.

Comparative (Horizontal Equity) Plots



The comparative plots are used to compare the distribution of the values of the break column. In the box plot, the line across the near-middle of the box is the median line. The box itself shows the interquartile range (IQR), or the region containing the middle 50% of the ratios. The dot plot shows each ratio individually. Some quick takeaways from these plots are the lower median for the Freedom Manor region, and greater variability in the Freedom Manor and Hastings Heights ratios. These characteristics can be confirmed by examining the ratio summary statistics. For statistical comparisons of the means or medians, use the tests of horizontal equity.

Horizontal Equity Test Assumption: Normality

Horizontal Equity Test Assumption: Normality			
Normality Attributes	Test Value	P-Value	Reject Normality? ($\alpha = 0.1$)
Skewness Test	-8.1115	0.0000	Yes
Kurtosis Test	10.4382	0.0000	Yes
Skewness and Kurtosis (Omnibus)	174.7531	0.0000	Yes

These Normality tests, based in the residuals from the group means, are used to determine whether the Normality assumption is reasonable in the test of horizontal equity. The tests here show strong evidence that the residuals should not be assumed to be Normal, and therefore, a nonparametric test of horizontal equity should be used (Kruskal-Wallis rank test).

Appraisal Ratio Studies

Horizontal Equity Test Assumption: Equal Variance

Horizontal Equity Test Assumption: Equal Variance

Test Name	Test Value	P-Value	Reject Equal Variances? ($\alpha = 0.1$)
Brown-Forsythe (Data - Medians)	18.6639	0.0000	Yes
Levene (Data - Means)	19.2449	0.0000	Yes
Conover (Ranks of Deviations)	72.6317	0.0000	Yes
Bartlett (Likelihood Ratio)	152.2970	0.0000	Yes

These tests compare the spread of the ratios across the groups. When the ratios are Normally distributed, these tests can help determine whether the ANOVA F-test or Welch's test should be used for comparing the groups. These tests may also be used to evaluate whether the spread of ratios is significantly different across groups. In this example, the spread of the five groups is clearly not equal.

Tests of Horizontal Equity - Region

Tests of Horizontal Equity - Region

Test Name	Test Statistic	P-Value	Conclusion ($\alpha = 0.05$)
Kruskal-Wallis Rank Test*	45.8080	0.0000	Reject Equality of Ratio Medians
ANOVA F-Test**	12.8003	0.0000	Reject Equality of Ratio Means
Welch's Test***	12.1189	0.0000	Reject Equality of Ratio Means

* The Kruskal-Wallis Test is recommended when the Normality assumption is not met. The test statistic is the Z-Value.

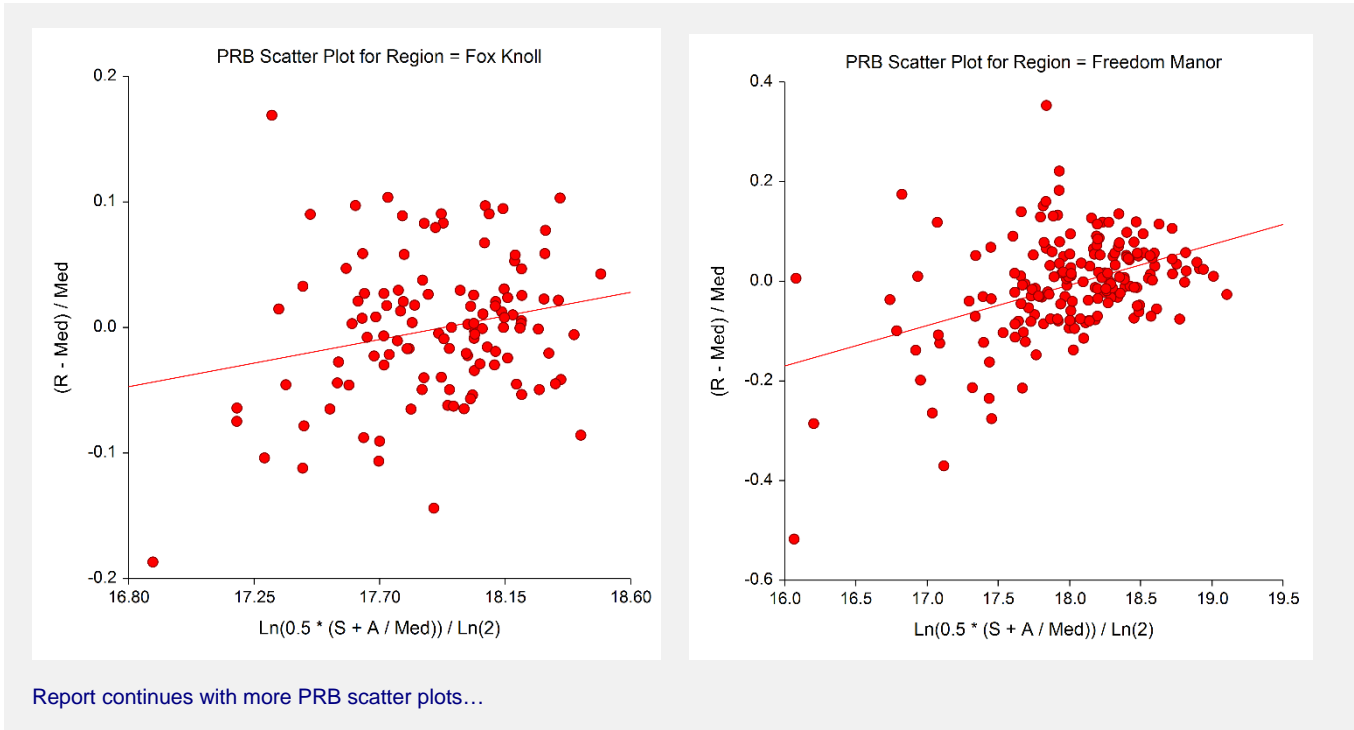
** The ANOVA F-Test assumes the variances are similar across groups, and that the distribution of residuals is approximately Normal. The test statistic is the F-Value.

*** Welch's Test is recommended when the data can be assumed to be approximately Normally distributed, but the variances are unequal. The test statistic is the F-Value.

These tests are used to compare the center (medians or means) of the groups. Since the assumption of Normality is not reasonable (based on previous Normality tests), the Kruskal-Wallis test should be used here. However, all three tests show strong evidence that there is a difference among the centers of the five groups. If the user wishes to find out specifically which groups are different from the others, the ratios could be output to the spreadsheet and multiple comparison tests could be conducted using the One-Way Analysis of Variance procedure (use the Tukey-Kramer Test for Normally distributed data, or the Kruskal-Wallis Z Test (Dunn's Test) when the Normality assumption is not met).

Appraisal Ratio Studies

Price-Related Bias (PRB) Scatter Plots



These plots give a visual representation of the vertical bias. The greater the slope, the greater the indication of vertical bias.

Price-Related Bias (PRB) Details Section

Price-Related Bias (PRB) Details
 $Y = (\text{Ratio} - \text{Median Ratio}) / \text{Median Ratio}$
 $X = \text{Ln}(0.5 * (\text{Sale Price} + \text{Appraisal} / \text{Median Ratio})) / \text{Ln}(2)$

Region	Count	Pearson Correlation	PRB (Slope)	Std Error	T Value	PRB (Slope) Test P-Value	Reject Vertical Equity? ($\alpha = 0.05$)	95% C.I. of PRB (Slope)	
								Lower	Upper
Fox Knoll	117	0.2207	0.0415	0.0171	2.4262	0.0168	Yes	0.0076	0.0753
Freedom Manor	184	0.4203	0.0812	0.0130	6.2487	0.0000	Yes	0.0556	0.1068
Hastings Heights	98	0.1283	0.0218	0.0172	1.2679	0.2079	No	-0.0123	0.0558
Puente Real	206	0.1191	0.0103	0.0060	1.7137	0.0881	No	-0.0016	0.0222
Sussex Vale	143	-0.1337	-0.0082	0.0051	-1.6025	0.1113	No	-0.0184	0.0019
Combined	748	0.1943	0.0250	0.0046	5.4103	0.0000	Yes	0.0160	0.0341

Pearson Correlation: the common measure of the linear relationship of two variables.
 PRB (Slope): Coefficient of Price-Related Bias. This is the slope of the regression line (for example, a PRB of 0.032 indicates a 3.2% increase in assessment ratio for every 100% increase in value).
 Std Error: The standard error of the estimate of PRB.
 T Value: The test statistic for testing whether PRB is statistically different from 0.
 PRB (Slope) Test P-Value: This P-Value indicates the likelihood that the slope is flat (PRB = 0) given the sale price and appraisal values in question.
 Lower and Upper: Lower and upper confidence limits for the value of PRB.

The PRB report gives tests of whether the slope (PRB) is significantly different from zero. For slopes significantly different from zero there is a meaningful interpretation of the slope. For example, for the Fox Knoll region, PRB is 0.0415 and the test indicates the slope is significantly different from zero. The interpretation may be made that for a doubling in property value, there is approximately a 4.15% increase in ratio. Higher valued properties are valued relatively higher than lower valued properties by this amount. Some technical details are

Appraisal Ratio Studies

given in the Vertical Equity sub-section of the Technical Details of this chapter. For the technical details of regression analysis, the reader is referred to the Linear Regression and Correlation chapter.

Example 2 – Removing Outliers with the IQR Method

In this example, each row is examined to determine whether it should be considered an outlier based on the IQR method. An IQR multiplier of 3 (and later 1.5) will be used in this example. In the IQR method, IQR is the range of the middle 50% of the ratios. A ratio is deemed an outlier if it is greater than 3 times the (group) IQR from the (group) first and third quartiles. Only the first three columns will be used in this example.

Appraisals Dataset

Appraisal	Price	Region	Type	Date
310900	296000	Fox Knoll	Residential	10/26/2015
264800	256300	Fox Knoll	Commercial	10/18/2015
204600	199700	Fox Knoll	Residential	1/2/2015
290600	264000	Fox Knoll	Commercial	3/11/2015
287900	281100	Fox Knoll	Residential	2/17/2015
292600	267400	Fox Knoll	Residential	5/12/2015
244000	253200	Fox Knoll	Commercial	4/12/2015
270100	270000	Fox Knoll	Residential	8/24/2015
210500	191200	Fox Knoll	Residential	11/8/2015
304700	302000	Fox Knoll	Commercial	6/6/2015
248200	228400	Fox Knoll	Residential	6/16/2015
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You may follow along here by making the appropriate entries or load the completed template **Example 2** by clicking on Open Example Template from the File menu of the Appraisal Ratio Studies window.

1 Open the Appraisals dataset.

- From the **File** menu of the NCSS Data window, select **Open Example Data**.
- Click on the file **Appraisals.NCSS**.
- Click **Open**.

2 Open the Appraisal Ratio Studies window.

- Using the Analysis menu or the Procedure Navigator, find and select the **Appraisal Ratio Studies** procedure.
- On the menus, select **File**, then **New Template (Reset)**. This will fill the procedure with the default template.

3 Specify the columns.

- On the Appraisal Ratio Studies window, select the **Columns tab**.
- Set the **Appraisal Column** to **Appraisal**.
- Set the **Sale Price Column** to **Price**.
- Under Horizontal Grouping, set the **Break Column(s)** to **Region**.

4 Specify the IQR Outlier Removal method.

- Check the box next to **Remove Outliers using IQR Method**. The **IQR Multiplier** should be set to **3.0**.

Appraisal Ratio Studies

5 Specify to obtain only the Removed Ratios Report and the summary statistics.

- On the Appraisal Ratio Studies window, select the **Summary Reports** tab.
- Check the box next to **Removed Ratios Report**.
- Uncheck all other reports and plots except Ratio Summary Statistics.

6 Run the procedure.

- From the Run menu, select **Run Procedure**. Alternatively, just click the green Run button.

Removed Ratios Section

Removed Ratios Section

Row removal options used in this analysis:

Remove rows with ratios $< (Q1 - 3 * IQR)$ or $> (Q3 + 3 * IQR)$

The following ratios were removed from the analysis for the reason(s) given.

Row #	Region	Removed Ratio	Reason
239	Freedom Manor	0.4714	$< 0.5974 (Q1 - 3 * IQR)$
547	Puente Real	0.7481	$< 0.8108 (Q1 - 3 * IQR)$

This report shows the ratios that were removed based on the IQR method. The IQR and quartiles (Q1 and Q3) are different for each region, so the boundary value for each region is different.

Ratio Summary Statistics Section

Ratio Summary Statistics Section

Region	Count	Median	Mean	Wtd. Mean	IQR	SD	COD	COV	PRD	PRB
Fox Knoll	117	1.0035	1.0025	1.0033	0.0672	0.0565	4.258	5.6357	0.9993	0.0415
Freedom Manor	183	0.9776	0.9724	0.9782	0.1050	0.0914	6.836	9.4023	0.9941	0.0661
Hastings Heights	98	1.0015	0.9918	0.9917	0.1038	0.0728	5.732	7.3422	1.0001	0.0218
Puente Real	205	1.0049	1.0062	1.0055	0.0544	0.0494	3.700	4.9133	1.0006	0.0034
Sussex Vale	143	1.0208	1.0209	1.0190	0.0450	0.0403	2.884	3.9449	1.0019	-0.0082
Combined	746	1.0035	0.9983	0.9997	0.0676	0.0649	4.809	6.4982	0.9986	0.0188

Some of the summary statistics are slightly different from Example 1 for the Freedom Manor, Puente Real, and Combined regions.

Appraisal Ratio Studies

Removed Ratios Section using the IQR Multiplier 1.5

Generally an IQR Multiplier of around 3 is recommended. If the IQR Multiplier is set to 1.5, considerably more ratios will be deemed outliers.

Removed Ratios Section

Row removal options used in this analysis:

Remove rows with ratios $< (Q1 - 1.5 * IQR)$ or $> (Q3 + 1.5 * IQR)$

The following ratios were removed from the analysis for the reason(s) given.

Row #	Region	Removed Ratio	Reason
45	Fox Knoll	1.1731	$> 1.1314 (Q3 + 1.5 * IQR)$
91	Fox Knoll	0.8160	$< 0.8628 (Q1 - 1.5 * IQR)$
96	Fox Knoll	0.8590	$< 0.8628 (Q1 - 1.5 * IQR)$
128	Freedom Manor	0.6149	$< 0.7587 (Q1 - 1.5 * IQR)$
149	Freedom Manor	1.1928	$> 1.1888 (Q3 + 1.5 * IQR)$
199	Freedom Manor	0.6977	$< 0.7587 (Q1 - 1.5 * IQR)$
234	Freedom Manor	0.7185	$< 0.7587 (Q1 - 1.5 * IQR)$
239	Freedom Manor	0.4714	$< 0.7587 (Q1 - 1.5 * IQR)$
259	Freedom Manor	1.3214	$> 1.1888 (Q3 + 1.5 * IQR)$
286	Freedom Manor	0.7074	$< 0.7587 (Q1 - 1.5 * IQR)$
300	Freedom Manor	0.7471	$< 0.7587 (Q1 - 1.5 * IQR)$
303	Hastings Heights	0.7770	$< 0.7821 (Q1 - 1.5 * IQR)$
433	Puente Real	1.1270	$> 1.1148 (Q3 + 1.5 * IQR)$
447	Puente Real	1.1631	$> 1.1148 (Q3 + 1.5 * IQR)$
487	Puente Real	1.1605	$> 1.1148 (Q3 + 1.5 * IQR)$
489	Puente Real	0.8500	$< 0.8937 (Q1 - 1.5 * IQR)$
506	Puente Real	1.1319	$> 1.1148 (Q3 + 1.5 * IQR)$
527	Puente Real	0.8392	$< 0.8937 (Q1 - 1.5 * IQR)$
547	Puente Real	0.7481	$< 0.8937 (Q1 - 1.5 * IQR)$
549	Puente Real	1.1333	$> 1.1148 (Q3 + 1.5 * IQR)$
571	Puente Real	1.1382	$> 1.1148 (Q3 + 1.5 * IQR)$
596	Puente Real	1.1225	$> 1.1148 (Q3 + 1.5 * IQR)$
643	Sussex Vale	1.1179	$> 1.1083 (Q3 + 1.5 * IQR)$
654	Sussex Vale	0.9056	$< 0.9285 (Q1 - 1.5 * IQR)$
723	Sussex Vale	1.1584	$> 1.1083 (Q3 + 1.5 * IQR)$
725	Sussex Vale	1.1466	$> 1.1083 (Q3 + 1.5 * IQR)$
727	Sussex Vale	1.1464	$> 1.1083 (Q3 + 1.5 * IQR)$
734	Sussex Vale	0.9256	$< 0.9285 (Q1 - 1.5 * IQR)$

These ratios are the same as the ratios outside the whiskers of the box plot of Example 1, since the end of the whisker of the box plot is defined the same way as the IQR method with 1.5 as the IQR multiplier.

Ratio Summary Statistics Section using the IQR Multiplier 1.5

Ratio Summary Statistics Section

Region	Count	Median	Mean	Wtd. Mean	IQR	SD	COD	COV	PRD	PRB
Fox Knoll	114	1.0035	1.0039	1.0045	0.0630	0.0503	3.931	5.0076	0.9994	0.0377
Freedom Manor	176	0.9795	0.9770	0.9805	0.0968	0.0740	5.956	7.5751	0.9965	0.0428
Hastings Heights	97	1.0026	0.9940	0.9939	0.1025	0.0698	5.553	7.0223	1.0001	0.0202
Puente Real	196	1.0034	1.0031	1.0032	0.0504	0.0404	3.231	4.0286	0.9999	0.0050
Sussex Vale	137	1.0202	1.0189	1.0186	0.0441	0.0329	2.514	3.2326	1.0003	-0.0008
Combined	720	1.0035	0.9986	0.9999	0.0652	0.0550	4.323	5.5112	0.9988	0.0157

While the median is relatively unaffected by the removal of so many outliers, several of the other statistics are affected considerably.

Example 3 – More than One Break Column

Using the same dataset as Examples 1 and 2, this example show the use of more than one break column (region as well as property type). When more than one break column is specified, a separate analysis is given for each column. In this example, only the ratio summary statistics and box plots will be produced.

Appraisals Dataset

Appraisal	Price	Region	Type	Date
310900	296000	Fox Knoll	Residential	10/26/2015
264800	256300	Fox Knoll	Commercial	10/18/2015
204600	199700	Fox Knoll	Residential	1/2/2015
290600	264000	Fox Knoll	Commercial	3/11/2015
287900	281100	Fox Knoll	Residential	2/17/2015
292600	267400	Fox Knoll	Residential	5/12/2015
244000	253200	Fox Knoll	Commercial	4/12/2015
270100	270000	Fox Knoll	Residential	8/24/2015
210500	191200	Fox Knoll	Residential	11/8/2015
304700	302000	Fox Knoll	Commercial	6/6/2015
248200	228400	Fox Knoll	Residential	6/16/2015
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You may follow along here by making the appropriate entries or load the completed template **Example 3** by clicking on Open Example Template from the File menu of the Appraisal Ratio Studies window.

1 Open the Appraisals dataset.

- From the **File** menu of the NCSS Data window, select **Open Example Data**.
- Click on the file **Appraisals.NCSS**.
- Click **Open**.

2 Open the Appraisal Ratio Studies window.

- Using the Analysis menu or the Procedure Navigator, find and select the **Appraisal Ratio Studies** procedure.
- On the menus, select **File**, then **New Template (Reset)**. This will fill the procedure with the default template.

3 Specify the columns.

- On the Appraisal Ratio Studies window, select the **Columns tab**.
- With Ratio Specification set to 'Appraisal Column, Sale Price Column', set the **Appraisal Column** to **Appraisal**.
- Set the **Sale Price Column** to **Price**.
- Under Horizontal Grouping, set the **Break Column(s)** to **Region Type**.

4 Specify to obtain only the summary statistics.

- Uncheck all reports and plots except **Ratio Summary Statistics** and **Box Plot**.

5 Run the procedure.

- From the Run menu, select **Run Procedure**. Alternatively, just click the green Run button.

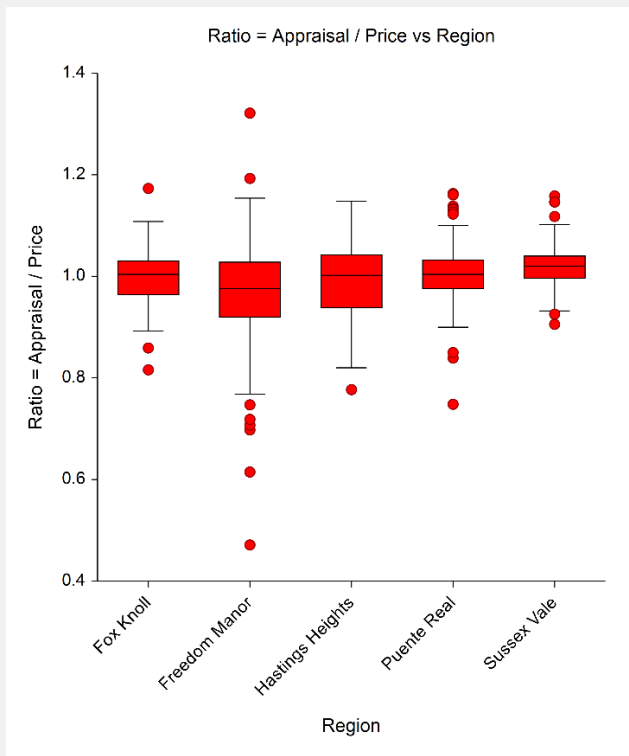
Appraisal Ratio Studies

Ratio Summary Statistics and Box Plot – Region

Ratio Summary Statistics Section

Region	Count	Median	Mean	Wtd. Mean	IQR	SD	COD	COV	PRD	PRB
Fox Knoll	117	1.0035	1.0025	1.0033	0.0672	0.0565	4.258	5.6357	0.9993	0.0415
Freedom Manor	184	0.9767	0.9697	0.9773	0.1075	0.0984	7.087	10.1452	0.9922	0.0812
Hastings Heights	98	1.0015	0.9918	0.9917	0.1038	0.0728	5.732	7.3422	1.0001	0.0218
Puente Real	206	1.0043	1.0049	1.0052	0.0553	0.0525	3.808	5.2233	0.9997	0.0103
Sussex Vale	143	1.0208	1.0209	1.0190	0.0450	0.0403	2.884	3.9449	1.0019	-0.0082
Combined	748	1.0035	0.9972	0.9994	0.0678	0.0703	4.901	7.0468	0.9979	0.0250

Wtd. Mean: Weighted Mean
 IQR: Interquartile Range
 COD: Coefficient of Dispersion
 COV: Coefficient of Variation
 PRD: Price-Related Differential
 PRB: Coefficient of Price-Related Bias



These results are the same as those of Example 1.

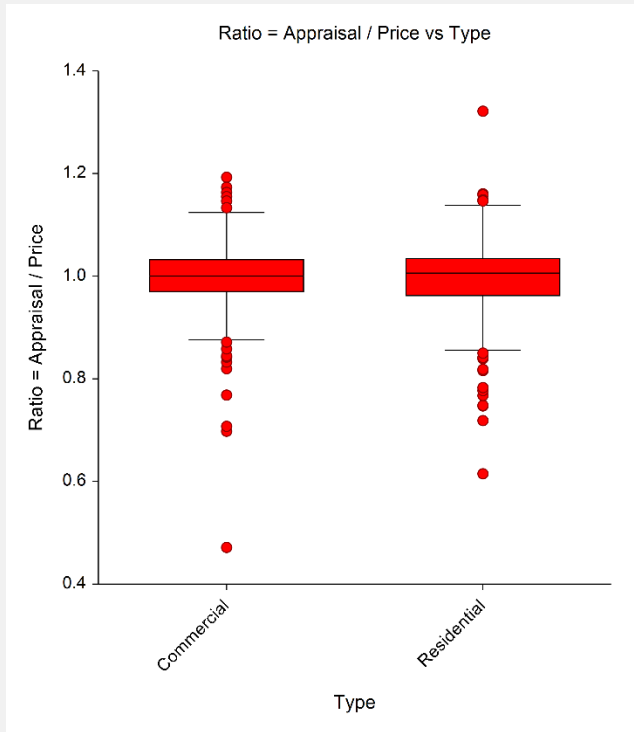
Appraisal Ratio Studies

Ratio Summary Statistics and Box Plot – Type

Ratio Summary Statistics Section

Region	Count	Median	Mean	Wtd. Mean	IQR	SD	COD	COV	PRD	PRB
Commercial	305	1.0008	0.9976	1.0004	0.0632	0.0697	4.644	6.9843	0.9973	0.0330
Residential	443	1.0060	0.9970	0.9987	0.0722	0.0708	5.068	7.0972	0.9983	0.0201
Combined	748	1.0035	0.9972	0.9994	0.0678	0.0703	4.901	7.0468	0.9979	0.0250

Wtd. Mean: Weighted Mean
 IQR: Interquartile Range
 COD: Coefficient of Dispersion
 COV: Coefficient of Variation
 PRD: Price-Related Differential
 PRB: Coefficient of Price-Related Bias



This report shows the results for comparing commercial to residential properties. Additional tests could be examined, but there appears to be very little difference in the statistics of these two groups.

Example 4 – Adjusting Sale Price based on Date of Sale

This example presents a similar analysis to that of Example 1, except that each sale price will be adjusted to the date January 1, 2015.

Suppose a mass appraisal process has produced all the appraised values based on the date January 1, 2015. Suppose further that in the area under examination, the overall increase in property values was about 12% over the course of the year 2015. The analyst would like to perform a ratio analysis based on sale prices that are adjusted back to an estimated sale price for January 1, 2015. (In some cases, the sale date may be taken into account as part of the appraisal process, in which case a sale price adjustment should likely not be used.)

Appraisals Dataset

Appraisal	Price	Region	Type	Date
310900	296000	Fox Knoll	Residential	10/26/2015
264800	256300	Fox Knoll	Commercial	10/18/2015
204600	199700	Fox Knoll	Residential	1/2/2015
290600	264000	Fox Knoll	Commercial	3/11/2015
287900	281100	Fox Knoll	Residential	2/17/2015
292600	267400	Fox Knoll	Residential	5/12/2015
244000	253200	Fox Knoll	Commercial	4/12/2015
270100	270000	Fox Knoll	Residential	8/24/2015
210500	191200	Fox Knoll	Residential	11/8/2015
304700	302000	Fox Knoll	Commercial	6/6/2015
248200	228400	Fox Knoll	Residential	6/16/2015
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You may follow along here by making the appropriate entries or load the completed template **Example 4** by clicking on Open Example Template from the File menu of the Appraisal Ratio Studies window.

1 Open the Appraisals dataset.

- From the **File** menu of the NCSS Data window, select **Open Example Data**.
- Click on the file **Appraisals.NCSS**.
- Click **Open**.

2 Open the Appraisal Ratio Studies window.

- Using the Analysis menu or the Procedure Navigator, find and select the **Appraisal Ratio Studies** procedure.
- On the menus, select **File**, then **New Template (Reset)**. This will fill the procedure with the default template.

3 Specify the columns.

- On the Appraisal Ratio Studies window, select the **Columns** tab.
- With Ratio Specification set to 'Appraisal Column, Sale Price Column', set the **Appraisal Column** to **Appraisal**.
- Set the **Sale Price Column** to **Price**.
- Under Horizontal Grouping, set the **Break Column(s)** to **Region**.

4 Specify the dates.

- On the Appraisal Ratio Studies window, select the **Sale Date** tab.
- Set the **Sale Date Column** to **Date**.
- Check the box next to **Adjust Prices To This Date**.

Appraisal Ratio Studies

- Set the basis date to **January 1, 2015**.
- Set the **Monthly Price Adjustment Factor** to **0.01** (based on 12% / 12 months = 1% per month).

5 Specify the reports.

- On the **Summary Reports** tab, check the box **Sale Price Adjustment by Date Summary**.
- Uncheck all other boxes and plots except **Ratio Summary Statistics**.

6 Specify the column for saving adjusted prices to the spreadsheet.

- On the Appraisal Ratio Studies window, select the **Storage** tab.
- To the right of **Store Time-Adjusted Sale Price in Column:**, enter **C7** (assuming there is nothing currently in that column on the spreadsheet).

7 Run the procedure.

- From the Run menu, select **Run Procedure**. Alternatively, just click the green Run button.

Sale Price Adjustment by Date Section

Sale Price Adjustment by Date Section

Sale Date Column: Date
 Ratios removed if before the date: None Specified or no Sale Date Column
 Ratios removed if after the date: None Specified or no Sale Date Column

Basis Date: January 1, 2015 (Prices are adjusted to be as if they were sold on this date.)
 Monthly Price Adjustment Factor: 0.01
 Price Adjustment Formula: Adjusted Price = (1 + 0.01 * M) * Sale Price
 M is the number of months from the Basis Date to the sale date. Dates previous to the Basis Date result in a positive M value, while dates after the Basis Date result in a negative M value.

Time-adjusted sale prices were stored in the C7 column.

The first portion of this report shows that no date boundaries were set. The second portion shows the summary of how each sale price is adjusted to the basis date. Since each date is after the basis date, the *M* values used are negative (the sale prices are adjusted down). The final line indicates that the time-adjusted sale prices were stored in the C7 column. You can go to the spreadsheet to see the values listed there.

	Appraisal	Price	Region	Type	Date	C6	C7	C8
1	310900	296000	Fox Knoll	Residential	10/26/2015		267020	
2	264800	256300	Fox Knoll	Commercial	10/18/2015		231880.5	
3	204600	199700	Fox Knoll	Residential	1/2/2015		199634.4	
4	290600	264000	Fox Knoll	Commercial	3/11/2015		258015.3	
5	287900	281100	Fox Knoll	Residential	2/17/2015		276759.4	
6	292600	267400	Fox Knoll	Residential	5/12/2015		255891.4	
7	244000	253200	Fox Knoll	Commercial	4/12/2015		244798.1	
8	270100	270000	Fox Knoll	Residential	8/24/2015		249154	
9	210500	191200	Fox Knoll	Residential	11/8/2015		171663.8	
10	304700	302000	Fox Knoll	Commercial	6/6/2015		286521.7	
11	248200	228400	Fox Knoll	Residential	6/16/2015		215943.5	
12	144900	154300	Fox Knoll	Commercial	3/1/2015		151309.1	
13	328100	341100	Fox Knoll	Residential	3/13/2015		333143.3	
14	232100	230400	Fox Knoll	Residential	7/7/2015		216244.8	
15	215100	215800	Fox Knoll	Commercial	12/4/2015		191906.9	
16	265600	274100	Fox Knoll	Residential	8/18/2015		253477.8	

The sale prices shown in column C7 are those used in all statistics, plots, and tests of the report.

Appraisal Ratio Studies

Ratio Summary Statistics Section

Ratio Summary Statistics Section

Region	Count	Median	Mean	Wtd. Mean	IQR	SD	COD	COV	PRD	PRB
Fox Knoll	117	1.0687	1.0704	1.0684	0.1080	0.0747	5.672	6.9743	1.0018	0.0140
Freedom Manor	184	1.0307	1.0264	1.0324	0.1175	0.1113	7.753	10.8417	0.9941	0.0736
Hastings Heights	98	1.0635	1.0537	1.0539	0.1001	0.0827	5.879	7.8466	0.9998	0.0306
Puente Real	206	1.0694	1.0713	1.0687	0.0894	0.0718	5.149	6.6989	1.0024	0.0026
Sussex Vale	143	1.0775	1.0869	1.0802	0.0900	0.0624	4.624	5.7421	1.0061	-0.0203
Combined	748	1.0629	1.0608	1.0604	0.1028	0.0833	6.009	7.8535	1.0003	0.0170

Wtd. Mean: Weighted Mean

IQR: Interquartile Range

COD: Coefficient of Dispersion

COV: Coefficient of Variation

PRD: Price-Related Differential

PRB: Coefficient of Price-Related Bias

Because the sale prices are all adjusted down in this scenario, the ratios are all increased, and this is reflected in the summary statistics (compare to the results of Example 1).