Chapter 521

Cochran's Q Test

Introduction

This procedure computes the non-parametric Cochran's Q test for related categories where the response is binary. Cochran's Q is used for testing $k = 2$ or more matched sets, where a binary response (e.g. 0 or 1) is recorded from each category within each subject. Cochran's Q tests the null hypothesis that the proportion of "successes" is the same in all groups versus the alternative that the proportion is different in at least one of the groups.

Cochran's Q test is an extension of the McNemar test to a situation where there are more than two matched samples. When Cochran's Q test is computed with only $k = 2$ groups, the results are equivalent to those obtained from the McNemar test (without continuity correction). Cochran's Q is also considered to be a special case of the non-parametric Friedman test, which is similar to repeated measures ANOVA and is used to detect differences in multiple matched sets with numeric responses. When the responses are binary, the Friedman test becomes Cochran's Q test.

This procedure also computes two-sided, pairwise multiple comparison tests that allow you to determine which of the individual groups are different if the null hypothesis in Cochran's Q test is rejected. The individual alpha level is adjusted using the Bonferroni method to control the overall experiment-wise error rate.

This procedure is based on the results and formulas given in chapter 26 of Sheskin (2011). We refer you there for additional information about Cochran's Q test.

Experimental Design

A typical design for this scenario involves N individuals where a binary measurement (e.g. 0 or 1) is made on each individual for each of k categories, where $k \geq 2$. Typical data might appear as

Subject	Condition 1	Condition 2	Condition 3
1	0	1	0
2	1	1	0
3	1	1	1
4	0	0	0
5	1	0	0
6	0	1	1
7	0	0	0
8	1	1	0
9	0	1	0
10	1	1	1
11	0	1	0
12	1	1	0

where, in this case, each subject responds to 3 different conditions with either a Yes (1) or No (0). In **NCSS**, the responses may be coded as either text values (e.g. Yes, No) or numeric values (e.g. 0, 1).

Technical Details

Suppose we have k binary measurements on each of N subjects (where the "subject" may be a set of matched individuals). Let $Y_{i,j}$ be the binary response from subject i in category j (i = 1 to N, j = 1 to k), with success = 1 and failure = 0. Let the proportions, $\pi_1, \pi_2, \ldots, \pi_k$, represent the proportion of "successes" in each of the k groups.

Cochran's Q is used to test the null hypothesis

$$H_0: \pi_1 = \pi_2 = \ldots = \pi_k$$

versus the alternative

$H_A: \pi_{\mathrm{a}} \neq \pi_{\mathrm{b}}$ for at least one pair $\pi_{\mathrm{a}}, \pi_{\mathrm{b}}$, with a ≠ b and 1 ≤ a, b ≤ k.

In **NCSS**, these proportions, π_{a} and π_{b}, are displayed as percentages.

Assumptions

The Cochran's Q test and associated multiple comparisons require the following assumptions:

1. Responses are binary and from k matched samples.
2. The subjects are independent of one another and were selected at random from a larger population.
3. The sample size is sufficiently "large". (As a rule of thumb, the number of subjects for which the responses are not all 0's or 1's, n, should be ≥ 4 and nk should be ≥ 24. This assumption is not required for the exact binomial McNemar test.)

Cochran's Q Test Statistic

For binary responses, $Y_{i,j}$, in k matched groups from N subjects, the Cochran's Q test statistic is computed as

$$Q = \frac{(k-1)[kC - T^2]}{kT - R}$$

where

$$C = \sum_{j=1}^{k}\left(\sum_{i=1}^{N} Y_{i,j}\right)^2$$

$$T = \sum_{i=1}^{N}\left(\sum_{j=1}^{k} Y_{i,j}\right)$$

$$R = \sum_{i=1}^{N}\left(\sum_{j=1}^{k} Y_{i,j}\right)^2$$

For "large" samples, the test statistic, Q, is distributed as chi-square with $k - 1$ degrees of freedom. As in the McNemar test, only subjects who do not have the same response in all categories contribute to the overall Q statistic.

The p-value for the test is computed as

$$\text{P-Value} = \Pr\left(Q > \chi^2_{1-\alpha,k-1}\right)$$

where $\chi^2_{1-\alpha,k-1}$ is the value of the $(1-\alpha)$ quantile of the chi-square distribution with $k-1$ degrees of freedom.

Multiple Comparisons

When the null hypothesis of success proportion equality is rejected by Cochran's Q test, you can proceed to determine which of the groups are different by computing multiple pairwise comparisons.

Pairwise tests between groups "a" and "b" test the null hypothesis

$$H_0: \pi_{\mathrm{a}} = \pi_{\mathrm{b}}$$

versus the alternative

$$H_A: \pi_{\mathrm{a}} \neq \pi_{\mathrm{b}}$$

In **NCSS**, these proportions, π_{a} and π_{b}, are displayed as percentages.

NCSS provides two methods as described in chapter 26 of Sheskin (2011). The first method identifies the minimum required difference, *MRD*, needed to declare a pair of experimental conditions as significantly different. The second method simply employs pairwise McNemar tests among groups to find significant differences.

Both multiple comparison methods use the Bonferroni alpha adjustment to control the overall experiment-wise error of the tests. The adjustment simply divides the overall required alpha, α, by the number of pairwise tests, c, where

$$c = \frac{k(k-1)}{2}.$$

The alpha-level for each individual test, α_{adj}, is

$$\alpha_{adj} = \frac{a}{c}.$$

For example, if the desired overall alpha were 0.05 and three groups were being compared, then the individual alpha level for each test would be 0.05/[3(2)/2] = 0.05/3 = 0.0167.

Minimum Required Difference

For sufficiently large sample sizes (i.e. $n \geq 4$ and $nk \geq 24$, where n is the number of subjects for which the responses are not all 0's or 1's), the minimum required difference in proportions for any pair of k experimental groups to be declared different is

$$MRD = z_{adj}\sqrt{2\left[\frac{kT-R}{N^2k(k-1)}\right]}$$

where N, T, and R are defined as in Cochran's Q statistic, with

$$T = \sum_{i=1}^{N}\left(\sum_{j=1}^{k} Y_{i,j}\right)$$

$$R = \sum_{i=1}^{N}\left(\sum_{j=1}^{k} Y_{i,j}\right)^2$$

and

z_{adj} is the value of the $\left(1-\alpha_{adj}/2\right)$ quantile from the standard normal distribution.

Two groups are declared to be significantly different with protected overall alpha, α, if their absolute difference in proportions is greater than *MRD*, that is if

$$|\pi_{\mathrm{a}}-\pi_{\mathrm{b}}|>MRD$$

McNemar Tests

The McNemar test statistic for each pair of groups is computed as

$$M=\frac{(n_1-n_2)^2}{n_1+n_2}$$

where

n_1 = number of subjects where group "a" response = 0 and group "b" response = 1.

n_2 = number of subjects where group "a" response = 1 and group "b" response = 0.

Large-Sample (Asymptotic)

For sufficiently large sample sizes (i.e. $n \geq 4$ and $nk \geq 24$, where n is the number of subjects for which the responses are not all 0's or 1's), the test statistic, *M*, is asymptotically distributed as chi-square with 1 degree of freedom. The p-value for the individual test with protected overall alpha, α, is computed as

$$\text{P-Value} = \Pr\left(M>\chi^2_{1-\alpha_{adj},1}\right)$$

where $\chi^2_{1-\alpha_{adj},1}$ is the value of the $\left(1-\alpha_{adj}\right)$ quantile of the chi-square distribution with 1 degree of freedom.

Exact Test

Exact p-values for the McNemar test can also be computed by enumerating and summing individual binomial probabilities of results more extreme than the observed. The exact test results are more accurate than the asymptotic test results because there is no approximation.

A Note on the Power of the Two Multiple Comparison Tests

The Minimum Required Difference multiple comparison method uses all of the available information in the data in its calculations, but the multiple McNemar tests comparison method uses just the values from subjects who have different responses for the two categories, not all of the data. For this reason some argue that the Minimum Required Difference method is more powerful for finding differences than using multiple McNemar tests (see Note #9 on page 1135 of Sheskin (2011)).

Data Structure

The data may be entered in two formats, as shown in the examples below. The examples give the binary responses of 12 subjects to each of 3 experimental conditions.

The first format, shown in the first table below, puts the responses for each group in separate columns; that is, each column contains all responses for each condition. Each row corresponds to a single subject. This format allows for the use of an additional optional frequency variable for summarized data.

The second format, shown in the second table below, arranges the data so that all responses are entered in a single column. A grouping variable contains an index that gives the group (Condition 1, 2, or 3) to which each row of data belongs. The subject variable specifies the individual to which each response belongs. This second format allows you to specify multiple response variables; a separate analysis is carried out for each response variable.

Multiple Response Variables

Subject	Condition 1	Condition 2	Condition 3
1	0	1	0
2	1	1	0
3	1	1	1
4	0	0	0
5	1	0	0
6	0	1	1
7	0	0	0
8	1	1	0
9	0	1	0
10	0	1	0
11	0	1	0
12	0	1	0

Response Variable, Grouping Variable, and a Subject Variable

Subject	Condition	Response
1	1	0
1	2	1
1	3	0
2	1	1
2	2	1
2	3	0
3	1	1
3	2	1
3	3	1
4	1	0
4	2	0
4	3	0
5	1	1
5	2	0
5	3	0
6	1	0
6	2	1
6	3	1
7	1	0
7	2	0
7	3	0
8	1	1
8	2	1
8	3	0
9	1	0
9	2	1
9	3	0
10	1	0
10	2	1
10	3	0
11	1	0
11	2	1
11	3	0
12	1	0
12	2	1
12	3	0

Procedure Options

This section describes the options available in this procedure.

Variables Tab

This panel specifies the variables used in the analysis.

Input Type

In this procedure, there are two ways to format the data for analysis. Specify which method you want to use.

- **Multiple Response Variables, One Variable per Group Category**

  The dataset includes two or more binary response variables. A single analysis is performed for all variables at once. Each category is represented by a separate column. All information about a specific subject is entered on a single row.

  Example Dataset

R1	R2	R3
0	1	1
1	0	1
1	0	0

- **Response Variable(s), a Grouping Variable, and a Subject Variable**

  The dataset includes a subject variable, a grouping variable, and one or more binary response variables. A separate analysis is performed for each response variable.

  Example Dataset

Subj	Grp	Resp
1	A	1
1	B	1
1	C	0
2	A	0
2	B	1
2	C	1

Variables

Response Variables (Displayed when Input Type = Multiple Response Variables)

Enter two or more binary response variables. Values may be text or numeric.

You can enter the column names or numbers directly, or double-click in the box to display a Column Selection window that will let you select the variables from a list.

The dataset must include two or more binary response variables. Each category is represented by a separate column. All information about a specific subject is entered on a single row. A single analysis is performed for all of the variables at once.

Example Dataset

R1	R2	R3
0	1	1
1	0	1
1	0	0

Frequency Variable (Displayed when Input Type = Multiple Response Variables)

Specify an optional frequency (count) variable. This data column contains integers that represent the number of observations (frequency) associated with each row of the dataset.

If this option is left blank, each dataset row has a frequency of one. This variable lets you modify that frequency. This may be useful when your data are tabulated and you want to enter counts.

Response Variable(s) (Displayed when Input Type = Response Variable(s), a Grouping Variable, and a Subject Variable)

Enter one or more binary response variables. Values may be text or numeric. A separate analysis is performed for each response variable.

You can enter the column names or numbers directly, or double-click in the box to display a Column Selection window that will let you select the variables from a list.

The dataset includes a subject variable, a grouping variable, and one or more binary response variables. In the example data below "Resp" is the response variable.

Example Dataset

Subj	Grp	Resp
1	A	1
1	B	1
1	C	0
2	A	0
2	B	1
2	C	1

Grouping Variable (Displayed when Input Type = Response Variable(s), a Grouping Variable, and a Subject Variable)

Enter a single categorical grouping variable. The values of this variable indicate which category each response belongs in. Values may be text or numeric.

You can enter the column names or numbers directly, or double-click in the box to display a Column Selection window that will let you select the variables from a list.

The dataset includes a subject variable, a grouping variable, and one or more binary response variables. In the example data below "Grp" is the grouping variable.

Example Dataset

Subj	Grp	Resp
1	A	1
1	B	1
1	C	0
2	A	0
2	B	1
2	C	1

Subject Variable (Displayed when Input Type = Response Variable(s), a Grouping Variable, and a Subject Variable)

Enter a single categorical subject variable. The values of this variable indicate which subject each response came from. Values may be text or numeric.

You can enter the column names or numbers directly, or double-click in the box to display a Column Selection window that will let you select the variables from a list.

The dataset includes a subject variable, a grouping variable, and one or more binary response variables. In the example data below "Subj" is the subject variable.

Example Dataset

Subj	Grp	Resp
1	A	1
1	B	1
1	C	0
2	A	0
2	B	1
2	C	1

Reports Tab

This tab controls which statistical reports and tables are displayed in the output.

Data Summary

Data Summary Report

Check this option to display a summary of the data used in the analysis with key statistics and helpful information.

Count and Percentage Tables

Show Combined Table

Check this option to display a single table containing the selected statistics. After activating this option, you must specify which items you would like to display in the table.

The items to choose from are:

- **Counts**
- **Table Percentages**
- **Row Percentages**
- **Column Percentages**

Show Individual Tables

Check this option to display a separate table for each statistic. After activating this option, you must specify which tables you would like to display.

The tables to choose from are:

- **Counts**
- **Table Percentages**
- **Row Percentages**
- **Column Percentages**

Tests

Cochran's Q Test

Check this option to output Cochran's Q Test. This is the primary test for this procedure.

Multiple Comparisons using Minimum Required Absolute Difference

Check this option to output a report with pairwise comparisons between all groups, made using the Minimum Required Absolute Difference. This multiple comparison procedure uses the Bonferroni alpha-level adjustment so that the overall experiment-wise alpha level is achieved.

A Note on Power

This multiple comparison method uses all of the data, not just the values from subjects who have different responses for the two categories, so some argue that this method is more powerful than using multiple McNemar tests (see Note #9 on page 1135 of Sheskin (2011)).

Multiple Comparisons using the McNemar Test

Check this option to output a report with pairwise comparisons between all groups, made using the McNemar Test. The Bonferroni adjustment is applied and the alpha level is adjusted based the number of pairwise comparisons so that the overall experiment-wise alpha level is achieved.

A Note on Power

This multiple comparison method uses just the values from subjects who have different responses for the two categories, not all of the data. For this reason some argue that this method is less powerful than using the Minimum Required Absolute Difference (see Note #9 on page 1135 of Sheskin (2011)).

Alpha for Test and Multiple Comparisons

Alpha

Enter the value of alpha to be used for all hypothesis tests in this procedure. The probability level (*p*-value) is compared to alpha to determine whether to reject the null hypothesis.

Report Options Tab

The following options control the format of the reports.

Report Options

Variable Names

Specify whether to use variable names, variable labels, or both to label output reports. In this discussion, the variables are the columns of the data table.

- **Names**

  Variable names are the column headings that appear on the data table. They may be modified by clicking the Column Info button on the Data window or by clicking the right mouse button while the mouse is pointing to the column heading.

- **Labels**

  This refers to the optional labels that may be specified for each column. Clicking the Column Info button on the Data window allows you to enter them.

- **Both**

  Both the variable names and labels are displayed.

Comments

1. Most reports are formatted to receive about 12 characters for variable names.
2. Variable Names cannot contain blanks or math symbols (like + - * / . ,), but variable labels can.

Value Labels

Value Labels are used to make reports more legible by assigning meaningful labels to numbers and codes.

The options are

- **Data Values**
  All data are displayed in their original format, regardless of whether a value label has been set or not.

- **Value Labels**
  All values of variables that have a value label variable designated are converted to their corresponding value label when they are output. This does not modify their value during computation.

- **Both**
  Both data value and value label are displayed.

Example

A variable named GENDER (used as a grouping variable) contains 1's and 2's. By specifying a value label for GENDER, the report can display "Male" instead of 1 and "Female" instead of 2. This option specifies whether (and how) to use the value labels.

Table Formatting

Column Justification

Specify whether data columns in the contingency tables will be left or right justified.

Column Widths

Specify how the widths of columns in the contingency tables will be determined.

The options are

- **Autosize to Minimum Widths**
  Each data column is individually resized to the smallest width required to display the data in the column. This usually results in columns with different widths. This option produces the most compact table possible, displaying the most data per page.

- **Autosize to Equal Minimum Width**
  The smallest width of each data column is calculated and then all columns are resized to the width of the widest column. This results in the most compact table possible where all data columns have the same with. This is the default setting.

- **Custom (User-Specified)**
  Specify the widths (in inches) of the columns directly instead of having the software calculate them for you.

Custom Widths (Single Value or List)

Enter one or more values for the widths (in inches) of columns in the contingency tables. This option is only displayed if Column Widths is set to “Custom (User-Specified)”.

- **Single Value**

  If you enter a single value, that value will be used as the width for all data columns in the table.

- **List of Values**

  Enter a list of values separated by spaces corresponding to the widths of each column. The first value is used for the width of the first data column, the second for the width of the second data column, and so forth. Extra values will be ignored. If you enter fewer values than the number of columns, the last value in your list will be used for the remaining columns.

  Type the word “Autosize” for any column to cause the program to calculate it's width for you. For example, enter “1 Autosize 0.7” to make column 1 be 1 inch wide, column 2 be sized by the program, and column 3 be 0.7 inches wide.

Wrap Column Headings onto Two Lines

Check this option to make column headings wrap onto two lines. Use this option to condense your table when your data are spaced too far apart because of long column headings.

Decimal Places

Item Decimal Places

These decimal options allow the user to specify the number of decimal places for items in the output. Your choice here will not affect calculations; it will only affect the format of the output.

- **Auto**

  If one of the “Auto” options is selected, the ending zero digits are not shown. For example, if “Auto (0 to 7)” is chosen,

  0.0500 is displayed as 0.05
  1.314583689 is displayed as 1.314584

  The output formatting system is not designed to accommodate “Auto (0 to 13)”, and if chosen, this will likely lead to lines that run on to a second line. This option is included, however, for the rare case when a very large number of decimals is needed.

Omit Percent Sign after Percentages

The program normally adds a percent sign, %, after each percentage. Checking this option will cause this percent sign to be omitted.

Example 1 – Multiple Response Variables

This section presents an example of how to run an analysis on hypothetical data where the responses are stored in separate columns, one for each category. In this example, a series of physical exams were given to 12 subjects. If the subject passed the exam, a "1" was recorded, otherwise a "0" was recorded for failure. They wish to determine if there is a difference among the three exams.

You may follow along here by making the appropriate entries or load the completed template **Example 1** by clicking on Open Example Template from the File menu of the procedure window.

1 Open the PhysExam1 example dataset.

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Click on the file **PhysExam1.NCSS**.
- Click **Open**.

2 Open the Cochran's Q Test procedure window.

- Using the Analysis menu or the Procedure Navigator, find and select the **Cochran's Q Test** procedure.
- On the menus, select **File**, then **New Template**. This will fill the procedure with the default template.

3 Specify the variables.

- Select the **Variables** tab.
- Leave **Input Type** as **Multiple Response Variables, One Variable per Group Category**.
- Double-click in the **Response Variables** text box. This will bring up the variable selection window.
- Select **Exam1**, **Exam2**, and **Exam3** from the list of variables and then click **OK**. **"Exam1-Exam3"** will appear in the **Response Variables** box.
- Leave the **Frequency Variable** box empty.

4 Specify the reports and plots.

- Leave all report and plot options at their default settings.

5 Run the procedure.

- From the Run menu, select **Run Procedure**. Alternatively, just click the green Run button.

Data Summary Section

Data Summary

Rows Processed:	12
Rows with Missing Values:	0
Rows Used in the Analysis:	12
Responses:	2 (0, 1)
Groups (k):	3 (Exam1, Exam2, Exam3)
Subjects or Blocks (N):	12
• Number with responses that are not all equal (n):	11 (nk = 33)*

* Large-sample (asymptotic) test results should be used only if n ≥ 4 and nk ≥ 24.
Status: Conditions are met.

The Data Summary report gives a summary description of the data used in the analysis. The summary indicates that the large-sample conditions ($n \geq 4$ and $nk \geq 24$) are met. One subject passed every exam, so his/her responses will not contribute to the Cochran's Q test statistic.

Combined Table Section

Combined Table

Variable		Response 0	1	Total
Exam1	Count	6	6	12
	% within Group	50.00	50.00	100.00
Exam2	Count	2	10	12
	% within Group	16.67	83.33	100.00
Exam3	Count	9	3	12
	% within Group	75.00	25.00	100.00
Total	Count	17	19	36
	% within Group	47.22	52.78	100.00

This report give the counts and percentages of each response within each category. Exam 2 has the highest pass rate with 83.33%, while Exam 3 has the lowest pass rate with only 25%.

Cochran's Q Test Section

Cochran's Q Test
(Exam1 by Exam2 by Exam3)
H0: The proportions of [Response = "1"] in all groups are equal.
H1: The proportion of [Response = "1"] in at least one group is different.

Test	Type	Chi-Square Statistic Q	DF	Asymptotic Prob Level	Reject H0 at α = 0.05?
Cochran's Q	2-Sided	6.7273	2	0.03461	Yes

Cochran's Q test has a p-value of 0.03461, indicating that the success rate for at least one group is different from the others. Because this result is significant, we can proceed to consider the multiple comparison tests.

Multiple Comparisons using Minimum Required Absolute Difference Section

Multiple Comparisons using Minimum Required Absolute Difference
H0: $\pi_i = \pi_j$ (The proportions of [Response = "1"] in the two groups are equal.)
H1: $\pi_i \neq \pi_j$ (The proportions of [Response = "1"] in the two groups are not equal.)
Number of Comparisons (c): 3

Comparison*	π_i (%)	π_j (%)	Absolute Difference $\lvert\pi_i - \pi_j\rvert$	Minimum Required Absolute Difference	Reject H0 with Overall α = 0.05?†
Exam1 vs. Exam2	50.00	83.33	33.33	54.02	No
Exam1 vs. Exam3	50.00	25.00	25.00	54.02	No
Exam2 vs. Exam3	83.33	25.00	58.33	54.02	Yes

* These tests should only be considered if the null hypothesis of equality was rejected by Cochran's Q Test.
† Individual Comparison Alpha = (Overall Alpha)/c = 0.05/3 = 0.01667.

These multiple comparison results indicate that exams 2 and 3 are significantly different from each other with an absolute different of 58.33%.

Multiple Comparisons using the McNemar Test Section

Multiple Comparisons using the McNemar Test
H0: $\pi_i = \pi_j$ (The proportions of [Response = "1"] in the two groups are equal.)
H1: $\pi_i \neq \pi_j$ (The proportions of [Response = "1"] in the two groups are not equal.)
Number of Comparisons (c): 3

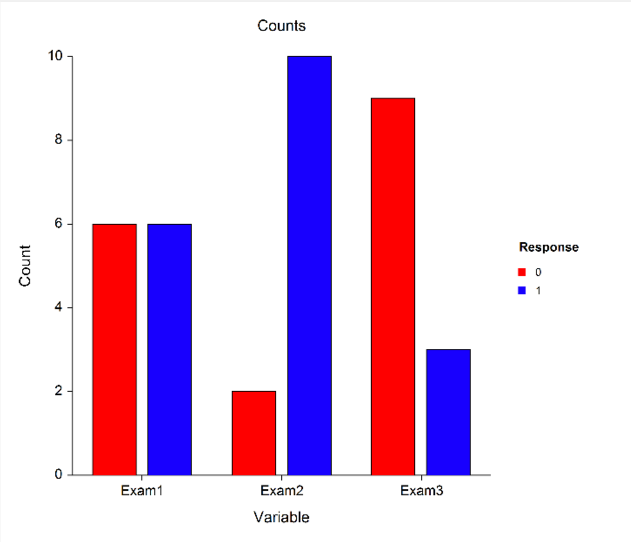
Comparison*	π_i (%)	π_j (%)	Test	Chi-Square Value	DF	Prob Level	Reject H0 with Overall α = 0.05?†
Exam1 vs. Exam2	50.00	83.33	Asymptotic	2.0000	1	0.15730	No
			Binomial Exact			0.28906	No
Exam1 vs. Exam3	50.00	25.00	Asymptotic	1.8000	1	0.17971	No
			Binomial Exact			0.37500	No
Exam2 vs. Exam3	83.33	25.00	Asymptotic	5.4444	1	0.01963	No
			Binomial Exact			0.03906	No

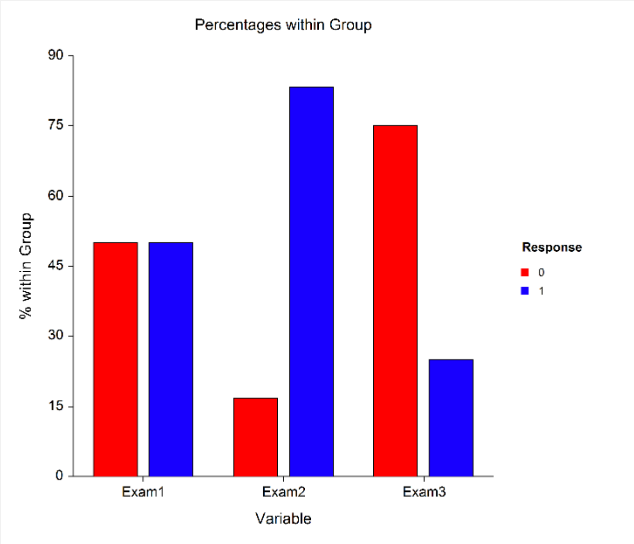
* These tests should only be considered if the null hypothesis of equality was rejected by Cochran's Q Test.
† Individual Comparison Alpha = (Overall Alpha)/c = 0.05/3 = 0.01667.

These multiple comparison results indicate that there are no pairs significantly different from one another. It's interesting to note here that no pairs were found to be different even though the overall Cochran's Q test found a significant difference and the multiple comparisons test using minimum required absolute difference found a difference between exams 2 and 3. This is likely due to the fact that McNemar test has lower overall power because only discordant pairs are used in the computation of the test statistic. The other multiple comparison procedure uses all of the data and, thus, has more power.

Plots Section

Plots Section





This section provides a graphical representation of the counts and percentages for each response within each category.

Example 2 – Multiple Response Variables with a Frequency Variable

Continuing from Example 1, we'll now show you how to analyze data that has been tabulated (summarized) using a frequency variable. The data for this example is exactly the same as that from Example 1 except that it has been summarized. The variable "Count" indicates how many subjects correspond to each series of responses.

You may follow along here by making the appropriate entries or load the completed template **Example 2** by clicking on Open Example Template from the File menu of the procedure window.

1 Open the PhysExam2 example dataset.

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Click on the file **PhysExam2.NCSS**.
- Click **Open**.

2 Open the Cochran's Q Test procedure window.

- Using the Analysis menu or the Procedure Navigator, find and select the **Cochran's Q Test** procedure.
- On the menus, select **File**, then **New Template**. This will fill the procedure with the default template.

3 Specify the variables.

- Select the **Variables** tab.
- Leave **Input Type** as **Multiple Response Variables, One Variable per Group Category**.
- Double-click in the **Response Variables** text box. This will bring up the variable selection window.
- Select **Exam1**, **Exam2**, and **Exam3** from the list of variables and then click **OK**. **"Exam1-Exam3"** will appear in the **Response Variables** box.
- For **Frequency Variable** enter **Count**.

4 Specify the reports and plots.

- Leave all report and plot options at their default settings.

5 Run the procedure.

- From the Run menu, select **Run Procedure**. Alternatively, just click the green Run button.

Output

Data Summary

Rows Processed:	8
Rows with Missing Values:	0
Rows Used in the Analysis:	8
Responses:	2 (0, 1)
Groups (k):	3 (Exam1, Exam2, Exam3)
Subjects or Blocks (N):	12
• Number with responses that are not all equal (n):	11 (nk = 33)*

* Large-sample (asymptotic) test results should be used only if n ≥ 4 and nk ≥ 24.
Status: Conditions are met.

Cochran's Q Test
(Exam1 by Exam2 by Exam3)
H0: The proportions of [Response = "1"] in all groups are equal.
H1: The proportion of [Response = "1"] in at least one group is different.

Test	Type	Chi-Square Statistic Q	DF	Asymptotic Prob Level	Reject H0 at α = 0.05?
Cochran's Q	2-Sided	6.7273	2	0.03461	Yes

The Data Summary report indicates that only 8 rows were processed this time (12 rows were processed in Example 1), but the number of subjects, 12, is the same as in Example 1 because some of the rows represent more than one individual. The Cochran's Q test results are exactly the same as in Example 1.

Example 3 – Responses All in a Single Column

This section presents an example of how to run an analysis on hypothetical data where the responses are stored in a single column with a separate subject and grouping variable. In this example, responses from 20 individuals are recorded for each of 4 pain relief drugs (A, B, C, and D). One hour after the administration of each drug, each subject was asked whether the medication was effective for them in controlling pain. Responses were recorded as either "Yes" or "No". Drugs were administered in random order, each after a washout period.

You may follow along here by making the appropriate entries or load the completed template **Example 3** by clicking on Open Example Template from the File menu of the procedure window.

1 Open the PainDrug example dataset.

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Click on the file **PainDrug.NCSS**.
- Click **Open**.

2 Open the Cochran's Q Test procedure window.

- Using the Analysis menu or the Procedure Navigator, find and select the **Cochran's Q Test** procedure.
- On the menus, select **File**, then **New Template**. This will fill the procedure with the default template.

3 Specify the variables.

- Select the **Variables** tab.
- Change **Input Type** to **Response Variable(s), a Grouping Variable, and a Subject Variable**.
- Double-click in the **Response Variable(s)** text box. This will bring up the variable selection window.
- Select **Response** from the list of variables and then click **OK**.
- Double-click in the **Grouping Variable** text box. This will bring up the variable selection window.
- Select **Drug** from the list of variables and then click **OK**.
- Double-click in the **Subject Variable** text box. This will bring up the variable selection window.
- Select **Subject** from the list of variables and then click **OK**.

4 Specify the reports.

- Click on the **Reports** tab.
- Deselect all reports except the **Combined Table** (with **Counts** and **Percentages within Groups**) and **Cochran's Q Test**.
- Click on the **Report Options** tab.
- For **Variable Names** select **Labels**.
- Click on the **Plots** tab.
- Uncheck **Show Plots**.

5 Run the procedure.

- From the Run menu, select **Run Procedure**. Alternatively, just click the green Run button.

Output

Combined Table

Drug		Was the Drug Effective? No	Yes	Total
A	Count	7	13	20
	% within Group	35.00	65.00	100.00
B	Count	11	9	20
	% within Group	55.00	45.00	100.00
C	Count	7	13	20
	% within Group	35.00	65.00	100.00
D	Count	14	6	20
	% within Group	70.00	30.00	100.00
Total	Count	39	41	80
	% within Group	48.75	51.25	100.00

Cochran's Q Test
(A by B by C by D)
H0: The proportions of [Response = "Yes"] in all groups are equal.
H1: The proportion of [Response = "Yes"] in at least one group is different.

Test	Type	Chi-Square Statistic Q	DF	Asymptotic Prob Level	Reject H0 at α = 0.05?
Cochran's Q	2-Sided	6.2239	3	0.10121	No

Cochran's Q test indicates that there is not a significant difference among the 4 medications. There is no reason to look at the multiple comparison tests at this point.

Example 4 – Validation of Cochran's Q and Multiple Comparison Tests using Sheskin (2011)

Sheskin (2011) presents an example of computing Cochran's Q test and the associated multiple comparisons in chapter 26, starting on page 1120. The data for the example consists of responses from 12 female subjects about whether or not they would purchase an automobile manufactured by three different companies: Chenesco, Howasaki, and Gemini. The data for this validation example are contained in the dataset called "Sheskin".

Sheskin (2011) computes the group proportions for Response = "1" in Chenesco, Howasaki, and Gemini as 0.25 (25%), 0.75 (75%), and 0.25 (25%), respectively. They compute a Cochran's Q value of 8.0. They do not compute the p-value directly, but state that it is between 0.01 and 0.05. For the multiple comparison test using the minimum required absolute difference with an overall alpha level of 0.05, they find the minimum required difference to be 0.49 (or 49%). They conclude that Howasaki is different from both Chenesco and Gemini, each with differences of 50%, which are both greater than 49%. Sheskin further computes the binomial exact test p-values for the multiple McNemar test comparisons as 0.0312 for Chenesco vs. Howasaki and 0.0704 for Howasaki vs. Gemini. Both are greater than the Bonferroni-adjusted alpha level of 0.0167 so both tests fail to reject the null hypothesis.

The results from **NCSS** match all of these results, with slight difference due to rounding.

You may follow along here by making the appropriate entries or load the completed template **Example 4** by clicking on Open Example Template from the File menu of the procedure window.

1 Open the Sheskin example dataset.

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Click on the file **Sheskin.NCSS**.
- Click **Open**.

2 Open the Cochran's Q Test procedure window.

- Using the Analysis menu or the Procedure Navigator, find and select the **Cochran's Q Test** procedure.
- On the menus, select **File**, then **New Template**. This will fill the procedure with the default template.

3 Specify the variables.

- Select the **Variables** tab.
- Leave **Input Type** as **Multiple Response Variables, One Variable per Group Category**.
- Double-click in the **Response Variables** text box. This will bring up the variable selection window.
- Select **Chenesco**, **Howasaki**, and **Gemini** from the list of variables and then click **OK**. **"Chenesco-Gemini"** will appear in the **Response Variables** box.
- Leave the **Frequency Variable** box empty.

4 Specify the reports and plots.

- Leave all report and plot options at their default settings.

5 Run the procedure.

- From the Run menu, select **Run Procedure**. Alternatively, just click the green Run button.

Output

Combined Table

Variable		Response 0	1	Total
Chenesco	Count	9	3	12
	% within Group	75.00	**25.00**	100.00
Gemini	Count	9	3	12
	% within Group	75.00	**25.00**	100.00
Howasaki	Count	3	9	12
	% within Group	25.00	**75.00**	100.00
Total	Count	21	15	36
	% within Group	58.33	41.67	100.00

Cochran's Q Test
(Chenesco by Gemini by Howasaki)
H0: The proportions of [Response = "1"] in all groups are equal.
H1: The proportion of [Response = "1"] in at least one group is different.

Test	Type	Chi-Square Statistic Q	DF	Asymptotic Prob Level	Reject H0 at α = 0.05?
Cochran's Q	**2-Sided**	**8.0000**	**2**	**0.01832**	**Yes**

Multiple Comparisons using Minimum Required Absolute Difference
H0: $\pi_i = \pi_j$ (The proportions of [Response = "1"] in the two groups are equal.)
H1: $\pi_i \neq \pi_j$ (The proportions of [Response = "1"] in the two groups are not equal.)
Number of Comparisons (c): 3

Comparison*	π_i (%)	π_j (%)	Absolute Difference $\lvert\pi_i - \pi_j\rvert$	Minimum Required Absolute Difference	Reject H0 with Overall α = 0.05?†
Chenesco vs. Gemini	**25.00**	**25.00**	**0.00**	**48.87**	**No**
Chenesco vs. Howasaki	**25.00**	**75.00**	**50.00**	**48.87**	**Yes**
Gemini vs. Howasaki	**25.00**	**75.00**	**50.00**	**48.87**	**Yes**

* These tests should only be considered if the null hypothesis of equality was rejected by Cochran's Q Test.
† Individual Comparison Alpha = (Overall Alpha)/c = 0.05/3 = 0.01667.

Multiple Comparisons using the McNemar Test
H0: $\pi_i = \pi_j$ (The proportions of [Response = "1"] in the two groups are equal.)
H1: $\pi_i \neq \pi_j$ (The proportions of [Response = "1"] in the two groups are not equal.)
Number of Comparisons (c): 3

Comparison*	π_i (%)	π_j (%)	Test	Chi-Square Value	DF	Prob Level	Reject H0 with Overall α = 0.05?†
Chenesco vs. Gemini	25.00	25.00	Asymptotic	0.0000	1	1.00000	No
			Binomial Exact			1.00000	No
Chenesco vs. Howasaki	25.00	75.00	Asymptotic	6.0000	1	0.01431	Yes
			Binomial Exact			**0.03125**	**No**
Gemini vs. Howasaki	25.00	75.00	Asymptotic	4.5000	1	0.03389	No
			Binomial Exact			**0.07031**	**No**

* These tests should only be considered if the null hypothesis of equality was rejected by Cochran's Q Test.
† Individual Comparison Alpha = (Overall Alpha)/c = 0.05/3 = **0.01667**.

The results from **NCSS** match Sheskin (2011), with slight differences due to rounding. Key matched items are highlighted in purple.