

Chapter 249

Exponentially Weighted Moving Average (EWMA) Charts

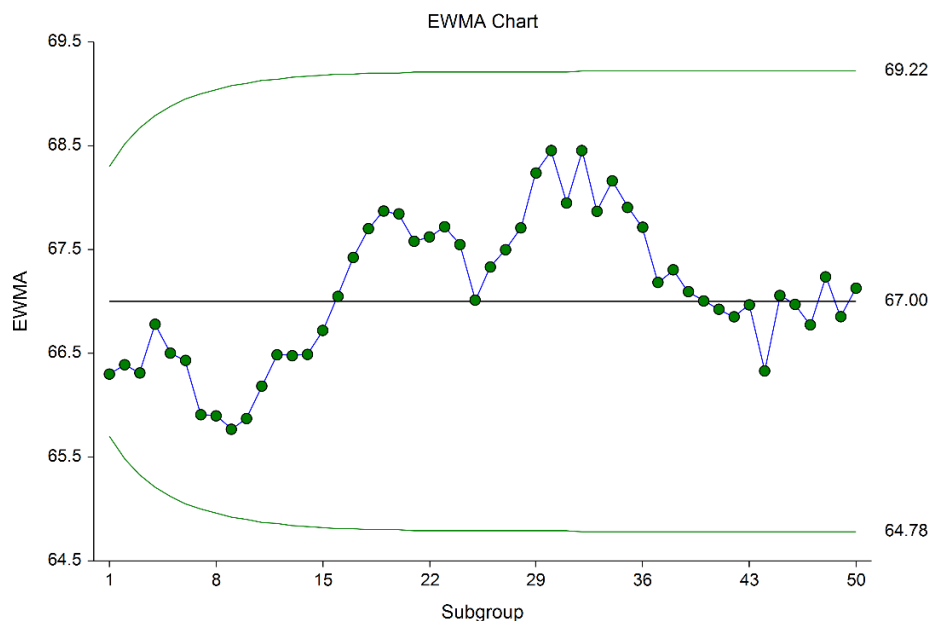
Introduction

This procedure generates exponentially weighted moving average (EWMA) control charts for variables. Charts for the mean and for the variability can be produced. The format of the control charts is fully customizable. The data for the subgroups can be in a single column or in multiple columns. This procedure permits the defining of stages. The target value can be entered directly or estimated from the data, or a sub-set of the data. Sigma may be estimated from the data or a standard sigma value may be entered. Means and ranges/standard deviations can be stored to the spreadsheet.

Exponentially Weighted Moving Average Control Charts

Similarly to the CUSUM chart, the EWMA chart is useful in detecting small shifts in the process mean. These charts are used to monitor the mean of a process based on samples taken from the process at given times (hours, shifts, days, weeks, months, etc.). The measurements of the samples at a given time constitute a subgroup.

The EWMA chart relies on the specification of a target value and a known or reliable estimate of the standard deviation. For this reason, the moving average chart is better used after process control has been established.



Other Control Charts for the Mean and Variation of a Process

Common control charts for detecting large process changes, or for establishing initial control of the process are the \bar{X} -bar and R charts (or \bar{X} -bar and s charts).

Once a reliable estimate of the mean and standard deviation is available, the EWMA and CUSUM charts are useful in detecting smaller shifts in the process mean.

The EWMA chart may also be used when only a single response is available at each time point. Another option for single responses are the individuals and moving range (I-MR) control charts.

Control Chart Formulas

Suppose we have k subgroups, each of size n . Let x_{ij} represent the measurement in the j^{th} sample of the i^{th} subgroup.

Formulas for the Points on the Chart

The i^{th} subgroup mean is calculated using

$$\bar{x}_i = \frac{\sum_{j=1}^n x_{ij}}{n}$$

The points of the chart are obtained from the \bar{x}_i 's by the following exponential smoothing operation

$$z_i = \lambda \bar{x}_i + (1 - \lambda)z_{i-1}$$

The value of z_0 is set to the target mean. The value of λ is specified by the user.

Values for the points of the EWMA variation chart are given by substituting \bar{x}_i with the range or standard deviation.

Estimating the EWMA Chart Center Line (Grand Mean)

In the EWMA Charts procedure, the target mean may be input directly, or it may be estimated from a series of subgroups. If it is estimated from the subgroups the formula for the grand average is

$$\bar{\bar{x}} = \frac{\sum_{i=1}^k \sum_{j=1}^{n_i} x_{ij}}{\sum_{i=1}^k n_i}$$

If the subgroups are of equal size, the above equation for the grand mean reduces to

$$\bar{\bar{x}} = \frac{\sum_{i=1}^k \bar{x}_i}{k} = \frac{\bar{x}_1 + \bar{x}_2 + \cdots + \bar{x}_k}{k}$$

Exponentially Weighted Moving Average (EWMA) Charts

Estimating Sigma – Sample Ranges

Either the range or the standard deviation of the subgroups may be used to estimate sigma, or a known (standard) sigma value may be entered directly. If the standard deviation (sigma) is to be estimated from the ranges, it is estimated as

$$\hat{\sigma} = \frac{\bar{R}}{d_2}$$

where

$$\bar{R} = \frac{\sum_{i=1}^k R_i}{k}$$

$$d_2 = \frac{E(R)}{\sigma} = \frac{\mu_R}{\sigma}$$

The calculation of $E(R)$ requires the knowledge of the underlying distribution of the x_{ij} 's. Making the assumption that the x_{ij} 's follow the normal distribution with constant mean and variance, the values for d_2 are derived through the use of numerical integration. It is important to note that the normality assumption is used and that the accuracy of this estimate requires that this assumption be valid.

When n is one, we cannot calculate R_i since it requires at least two measurements. The procedure in this case is to use the ranges of successive pairs of observations. Hence, the range of the first and second observation is computed, the range of the second and third is computed, and so on. The average of these approximate ranges is used to estimate σ .

Estimating Sigma – Sample Standard Deviations

If the standard deviation (sigma) is to be estimated from the standard deviations, it is estimated as

$$\hat{\sigma} = \frac{\bar{s}}{c_4}$$

where

$$\bar{s} = \frac{\sum_{i=1}^k s_i}{k}$$

$$c_4 = \frac{E(s)}{\sigma} = \frac{\mu_s}{\sigma}$$

The calculation of $E(s)$ requires the knowledge of the underlying distribution of the x_{ij} 's. Making the assumption that the x_{ij} 's follow the normal distribution with constant mean and variance, the values for c_4 are obtained from

$$c_4 = \sqrt{\frac{2}{n-1}} \frac{\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)}$$

Exponentially Weighted Moving Average (EWMA) Charts

Estimating Sigma – Weighted Approach

When the sample size is variable across subgroups, a weighted approach is recommended for estimating sigma (Montgomery, 2013):

$$\hat{\sigma} = \bar{s} = \left[\frac{\sum_{i=1}^k (n_i - 1) s_i^2}{\sum_{i=1}^k n_i - k} \right]^{1/2}$$

EWMA Chart Limits

The lower and upper control limits for the EWMA chart are calculated using the formulas

$$LCL_i = \mu_0 - m \left(\frac{\hat{\sigma}}{\sqrt{n}} \right) \sqrt{\frac{\lambda}{2 - \lambda} [1 - (1 - \lambda)^{2i}]}$$

$$UCL_i = \mu_0 + m \left(\frac{\hat{\sigma}}{\sqrt{n}} \right) \sqrt{\frac{\lambda}{2 - \lambda} [1 - (1 - \lambda)^{2i}]}$$

where m is a multiplier (often set to 2.7 or 3), and λ is the exponential smoothing constant.

Note that the values of limits change with each successive subgroup, but tend to level off at around the tenth subgroup.

Values for the limits of the EWMA variation chart are given by substituting μ_0 with the range or standard deviation center line value (such as R-bar or s-bar).

Exponentially Weighted Moving Average (EWMA) Charts

Data Structure

In this procedure, the data may be in either of two formats. The first data structure option is to have the data in several columns, with one subgroup per row.

Example dataset

S1	S2	S3	S4	S5
2	6	3	8	5
8	8	7	7	9
6	2	2	4	3
5	6	7	6	10
48	2	6	5	0
.
.
.

The second data structure option uses one column for the response data, and either a subgroup size or a second column defining the subgroups.

Alternative example dataset

Response	Subgroup
2	1
6	1
3	1
8	1
5	1
8	2
8	2
7	2
7	2
9	2
6	3
2	3
.	.
.	.
.	.

In the alternative example dataset, the Subgroup column is not needed if every subgroup is of size 5 and the user specifies 5 as the subgroup size. If there are missing values, the Subgroup column should be used, or the structure of the first example dataset.

Procedure Options

This section describes the options available in this procedure. To find out more about using a procedure, go to the Procedures chapter.

Variables Tab

This panel specifies the variables that will be used in the analysis.

Input Type

Specify whether the data is in a single response column or in multiple columns with one subgroup per row.

- **Response Column and Subgroup Column or Subgroup Size**

Response	Subgroup
5	1
6	1
4	1
3	2
7	2
6	2
4	3
5	3
8	3
.	.
.	.
.	.

- **Multiple Columns with One Subgroup Per Row**

X1	X2	X3
5	6	4
3	7	6
4	5	8
.	.	.
.	.	.
.	.	.

Variables – Response Column

Response Variable

Specify the column with the data values. The data values are separated into subgroups below using the Subgroup Specification options.

Subgroup Specification

Specify whether subgroups are defined by a Subgroup ID variable, or by a subgroup size. If the subgroup size is 3, then subgroups are formed by going down the response column in groups of 3. The first subgroup would be 5, 6, 4; the second would be 3, 7, 6; and so on.

Exponentially Weighted Moving Average (EWMA) Charts**Subgroup ID Variable**

Specify the column containing the subgroup identifiers.

Response	ID Variable
5	1
6	1
4	1
3	2
7	2
6	2
4	3
5	3
8	3
.	.
.	.
.	.

A new subgroup is created for each change in the Subgroup ID Variable, going down.

Subgroup Size

Specify the number of individuals in each subgroup.

Response
5
6
4
3
7
6
4
5
8
.
.
.

If the subgroup size is 3, then subgroups are formed by going down the response column in groups of 3. The first subgroup would be 5, 6, 4; the second would be 3, 7, 6; and so on.

Variables – Multiple Columns**Data Variables**

Specify the columns containing the sample responses. Each row represents a subgroup.

X1	X2	X3
5	6	4
3	7	6
4	5	8
.	.	.
.	.	.
.	.	.

Exponentially Weighted Moving Average (EWMA) Charts

If only one variable is specified, NCSS automatically generates an individuals chart with a moving-range of size 2.

Stages

Number of Stages

Specify whether the analyses and charts are to be produced based on a single set of subgroups, or a series of sets of subgroups. Typically a single stage is used unless you wish to have separate estimation for various segments of the subgroups. When using multiple stages, the software requires that there be at least one subgroup in every stage specified.

Stage Specification

Specify whether the various stages will be defined using a variable (column) with a unique value for each stage, or by entering a range of rows for each stage.

- **Stage Variable**

X1	X2	X3	Stage
5	6	4	1
3	7	6	1
4	5	7	1
6	5	4	1
3	7	6	1
5	8	3	2
2	6	4	2
3	6	5	2
4	5	8	2
.	.	.	.
.	.	.	.
.	.	.	.

- **Enter a range for each stage**

1-50, 51-100, 101-150

This would produce three stages. The first stage would be made up of rows 1 to 50, the second stage would be rows 51 to 100, and the third stage would be rows 101 to 150.

Stage Variable

Specify the variable (column) that contains the identifiers for each stage.

X1	X2	X3	Stage Variable
5	6	4	1
3	7	6	1
4	5	7	1
6	5	4	1
3	7	6	1
5	8	3	2
2	6	4	2
3	6	5	2
4	5	8	2
.	.	.	.
.	.	.	.
.	.	.	.

A new stage is created for each change in the Stage Variable, going down.

Exponentially Weighted Moving Average (EWMA) Charts

Stage Ranges

Enter a range for each stage using a dash. Separate each stage with a comma.

Example: 1-50, 51-100, 101-150

This would produce three stages. The first stage would be made up of rows 1 to 50, the second stage would be rows 51 to 100, and the third stage would be rows 101 to 150.

Specify Rows in Calculations

Specification Method

Select which method will be used to specify the rows of the data to be used to form subgroups.

- **All Rows**
All rows in the response column(s) will be used.
- **Enter First Row and Last Row**
Specify the first row and the last row of the data for use in calculations.
- **First N Rows (Enter N)**
The data beginning at Row 1 and ending at Row N will be used in calculations.
- **Last N Rows (Enter N)**
Subgroups will be formed from the last N rows of the dataset.
- **Keep Rows Variable**
Specify a variable and a value in that variable column that will be used to determine which rows are used to form the subgroups.
- **Remove Rows Variable**
Specify a variable and a value in that variable column that will be used to determine which rows will not be used to form the subgroups.

First Row

Specify the beginning row to be used for the first subgroup.

Last Row

Specify the last row to be used for the last subgroup.

N

Enter the number of rows to be used in forming subgroups.

Keep Rows Variable

This variable (column) is used to specify which rows of the data will be used to form the subgroups for the calculations.

Keep Rows Value

This value determines which rows of the Keep Rows Variable will be used in the calculation portion of the analysis.

Exponentially Weighted Moving Average (EWMA) Charts

Remove Rows Variable

This variable (column) is used to specify which rows of the data will not be used to form the subgroups for the calculations.

Remove Rows Value

This value determines which rows of the Remove Rows Variable will not be used in the calculation portion of the analysis.

Specify Rows in Charts

Specification Method

Select which method will be used to specify the rows of the data to be used to form subgroups for the charts.

- **All Rows**
All rows in the response column(s) will be used.
- **Enter First Row and Last Row**
Specify the first row and the last row of the data for use in the plots.
- **First N Rows (Enter N)**
The data beginning at Row 1 and ending at Row N will be used in the plots.
- **Last N Rows (Enter N)**
Subgroups will be formed from the last N rows of the dataset.
- **Keep Rows Variable**
Specify a variable and a value in that variable column that will be used to determine which rows are used to form the subgroups.
- **Remove Rows Variable**
Specify a variable and a value in that variable column that will be used to determine which rows will not be used to form the subgroups.

First Row

Specify the beginning row to be used for the first subgroup.

Last Row

Specify the last row to be used for the last subgroup.

N

Enter the number of rows to be used in forming subgroups.

Keep Rows Variable

This variable (column) is used to specify which rows of the data will be used to form the subgroups for the plots.

Keep Rows Value

This value determines which rows of the Keep Rows Variable will be used in the plots.

Remove Rows Variable

This variable (column) is used to specify which rows of the data will not be used to form the subgroups for the plots.

Exponentially Weighted Moving Average (EWMA) Charts

Remove Rows Value

This value determines which rows of the Remove Rows Variable will not be used in the plots.

Labels (Optional)

Subgroup Label Variable

Specify a variable (column) that contains the desired x axis (subgroup) labels for plots. If left blank, the plot will use the subgroup number. The format of the labels is controlled on the x axis tab of the plot format window.

Point Label Variable

Specify a variable (column) that contains the desired individual point labels for plots. If left blank, no point labels are shown. The format of the labels is controlled on the main chart tab of the plot format window.

Target & Sigma Tab

The options on the Target & Sigma tab are used to specify the target value and sigma.

Target Value Options

Target Value Specification

Specify whether the target value is estimated from the data, or whether it will be specified directly.

- **From Rows in Calculations Data**
Estimate the target value from the subgroups specified for calculations.
- **Enter Target Value(s)**
Specify the target value directly. If multiple stages are used, separate the target value for each stage by spaces.
- **Use a Variable with Target Value(s)**
Specify a column containing the target value in row 1. If multiple stages are used, a target value should be entered in a separate cell for each stage, beginning with row 1 for the first stage.

Target Value(s)

Enter the target value to be used. If multiple stages are used, separate the target values for each stage by spaces.

Target Value(s) Variable

Specify a column containing the target value in row 1. If multiple stages are used, a target value should be entered in a separate cell for each stage, beginning with row 1 for the first stage.

Limit Multipliers

Primary Limit Multiplier

This option specifies the multiplier of sigma for the primary control limits. For the well-known 3-sigma limits, the multiplier is set to 3.

Specification Limits

Lower Limit

Enter an optional lower specification limit for display on the chart. These limits are not the control limits.

Exponentially Weighted Moving Average (EWMA) Charts

Upper Limit

Enter an optional upper specification limit for display on the chart. These limits are not the control limits.

Spec Value

Enter an optional specification value for display on the chart.

Sigma Estimation Options

Range or SD Estimation

Specify whether the estimation of sigma will be based on the average range or the average standard deviation.

- **Range**
The average range will be used to estimate sigma.
- **SD**
The average standard deviation will be used to estimate sigma.

Sigma Specification

Specify the method by which Sigma will be estimated for use in the charts.

- **From Data – R-bar or s-bar Estimate**
Estimate sigma based on the average of the ranges or standard deviations (whichever is specified under Range or SD Estimation). Only the subgroups specified for use in calculations will be used.
- **From Data – Weighted Approach Estimate**
This method estimates s-bar using a special formula that is recommended when the subgroup size varies across subgroups. This option should only be used when the Range or SD Estimation is set to SD. Only the subgroups specified for use in calculations will be used.
- **Enter Standard Sigma Value(s)**
In this case the sigma value is entered directly. The corresponding R-bar or s-bar value is back-calculated from this value. If multiple stages are used, separate the sigma values for each stage by spaces.
- **Use a Variable with Standard Sigma Value(s)**
Specify a column containing the standard sigma value in row 1. The corresponding R-bar or s-bar value is back-calculated from this value. If multiple stages are used, a sigma value should be entered in a separate cell for each stage, beginning with row 1 for the first stage.

Sigma Value(s)

Enter the value to be used for the standard sigma. If multiple stages are used, separate the sigma values for each stage by spaces.

Sigma Variable

Specify a column containing the standard sigma value in row 1. The corresponding R-bar or s-bar value for variation plots is back-calculated from this value. If multiple stages are used, a sigma value should be entered in a separate cell for each stage, beginning with row 1 for the first stage.

Reports Tab

The following options control the format of the reports.

Specify Reports

Target Value and Sigma Summary Section

This report gives the numeric values of the target value and sigma, as well as the sigma estimation.

Report Options

Precision

Specify the precision of numbers in the report. A single-precision number will show seven-place accuracy, while a double-precision number will show thirteen-place accuracy. Note that the reports are formatted for single precision. If you select double precision, some numbers may run into others. Also note that all calculations are performed in double precision regardless of which option you select here. This is for reporting purposes only.

Variable Names

This option lets you select whether to display variable names, variable labels, or both.

Page Title

This option specifies a title to appear at the top of each page.

Plot Subtitle

This option specifies a subtitle to appear at the top of each plot.

EWMA Charts Tab

This panel sets the options used to define the appearance of the two EWMA charts.

Select Plots

EWMA Chart, EWMA Variability Chart

These charts are controlled by three form objects:

1. A checkbox to indicate whether the chart is displayed.
2. A format button used to call up the plot format window (see Quality Control Chart Format Options below for more formatting details).
3. A second checkbox used to indicate whether the chart can be edited during the run.

EWMA Parameter

This specifies the value of the smoothing parameter (λ) in the EWMA charts. The range for this parameter is 0 to 1 (not including 0). Typically, a value between 0.05 and 0.25 works well in practice. Popular choices include 0.05, 0.10, and 0.20.

Exponentially Weighted Moving Average (EWMA) Charts

Storage Tab

The options on this panel control the automatic storage of the means on the current dataset.

Storage Columns

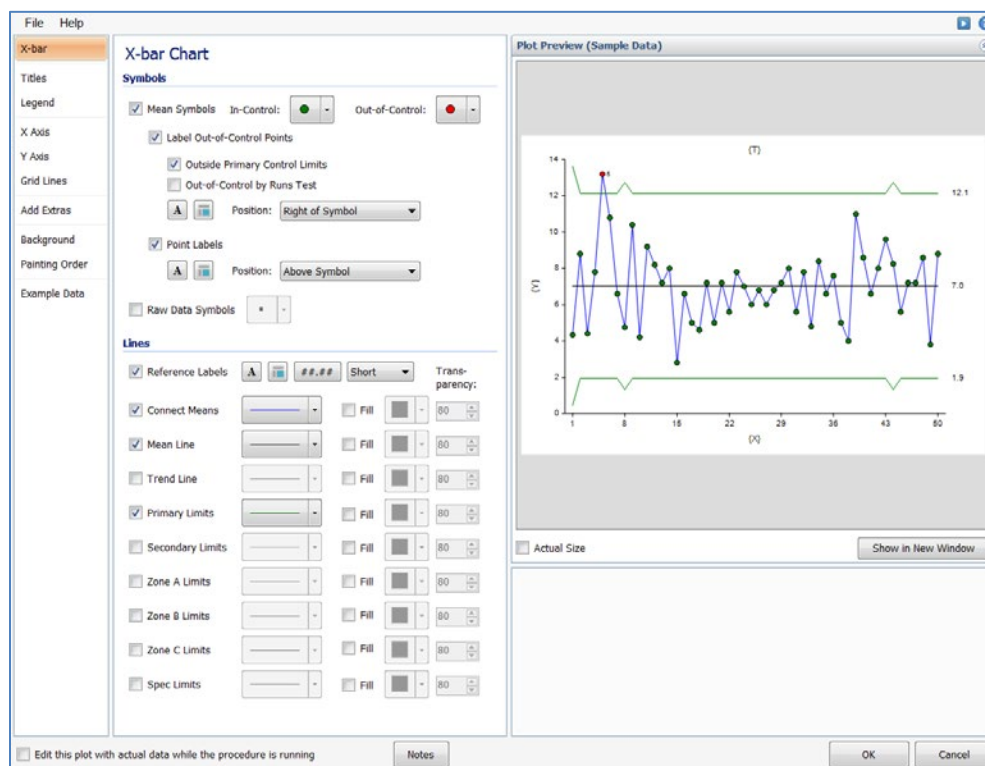
Store Means or Ranges/SDs in Column

You can automatically store the means, ranges, or standard deviations of each subgroup into the column specified here. Whether the range or standard deviation is stored depends on the choice of Range or SD Estimation on the Target & Sigma tab.

Warning: Any data already in this column is replaced. Be careful not to specify columns that contain important data.

Quality Control Chart Format Window Options

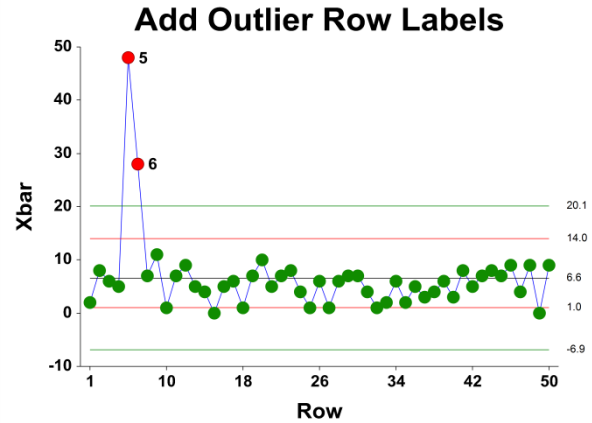
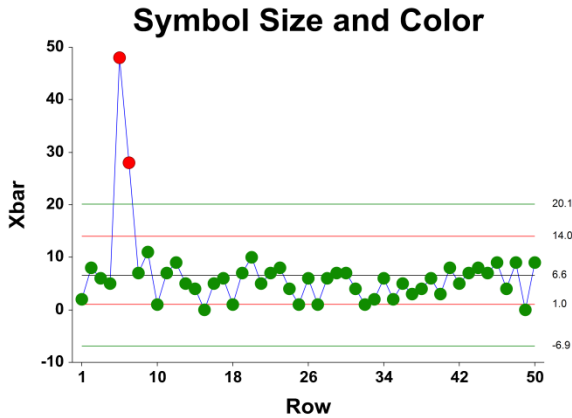
This section describes a few of the specific options available on the first tab of the control chart format window, which is displayed when a quality control chart format button is pressed. Common options, such as axes, labels, legends, and titles are documented in the Graphics Components chapter.



[Xbar] / [Range] Chart Tab

Symbols Section

You can modify the attributes of the symbols using the options in this section.

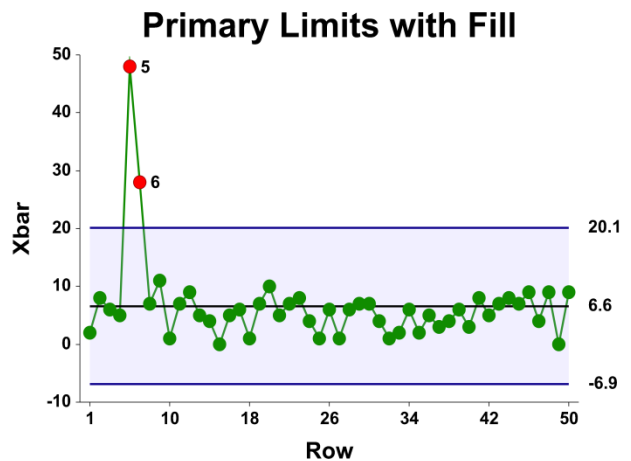
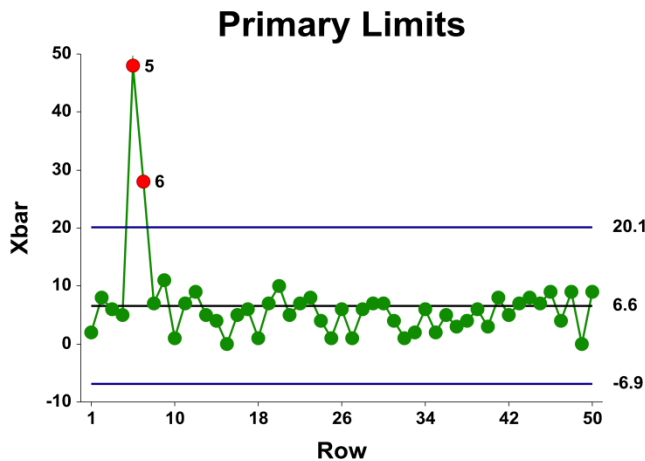


A wide variety of sizes, shapes, and colors are available for the symbols. The symbols for in-control and out-of-control points are specified independently. There are additional options to label out-of-control points. The label for points outside the primary control limits is the subgroup number. The label for points that are out-of-control based on the runs test is the number of the first runs test that is signaled by this point.

The user may also specify a column of point labels on the procedure variables tab, to be used to label all or some of the points of the chart. The raw data may also be shown, based on customizable raw data symbols.

Lines Section

You can specify the format of the various lines using the options in this section. Note that when shading is desired, the fill will be to the bottom for single lines (such as the mean line), and between the lines for pairs of lines (such as primary limits).



Lines for the zones, secondary limits, and specification limits are also specified here.

Titles, Legend, Numeric Axis, Group Axis, Grid Lines, and Background Tabs

Details on setting the options in these tabs are given in the Graphics Components chapter. The legend does not show by default, but can easily be included by going to the Legend tab and clicking the Show Legend checkbox.

Example 1 – EWMA Chart

This section presents an example of how to produce an EWMA chart. The data represent 50 subgroups of size 5. The data used are in the QC dataset. We will analyze the variables D1 through D5 of this dataset.

You may follow along here by making the appropriate entries or load the completed template **Example 1** by clicking on Open Example Template from the File menu of the EWMA Charts window.

1 Open the QC dataset.

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Click on the file **QC.NCSS**.
- Click **Open**.

2 Open the EWMA Charts window.

- Using the Analysis or Graphics menu or the Procedure Navigator, find and select the **EWMA Charts** procedure.
- On the menus, select **File**, then **New Template**. This will fill the procedure with the default template.

3 Specify the variables.

- On the EWMA Charts window, select the **Variables tab**.
- Double-click in the **Data Variables** text box. This will bring up the variable selection window.
- Select **D1** through **D5** from the list of variables and then click **Ok**. “D1-D5” will appear in the Data Variables box.

4 Specify the target value and sigma.

- On the Moving Average Charts window, select the **Target & Sigma tab**.
- Set **Target Value Specification** to **Enter Target Value(s)**. Set the **Target Value** to **67**.
- Set **Sigma Specification** to **Enter Sigma Value(s)**. Set the **Sigma Value** to **8**.

5 Run the procedure.

- From the Run menu, select **Run Procedure**. Alternatively, just click the green Run button.

Target Value Section

Target Value Section for Subgroups 1 to 50
Number of Subgroups 50

Target Value Specification	Value
User-Specified Target Value	67

This section displays the target value that is used in the EWMA chart.

Target Value Specification

This is the method by which the target value is obtained.

Exponentially Weighted Moving Average (EWMA) Charts

Target Value

This is value of the target value used to create the moving average chart.

Sigma Specification Section

Sigma Specification Section for Subgroups 1 to 50

Sigma Specification User-Specified	User-Specified Sigma Value 8
--	--

This section shows the specification of the standard deviation used in the EWMA chart.

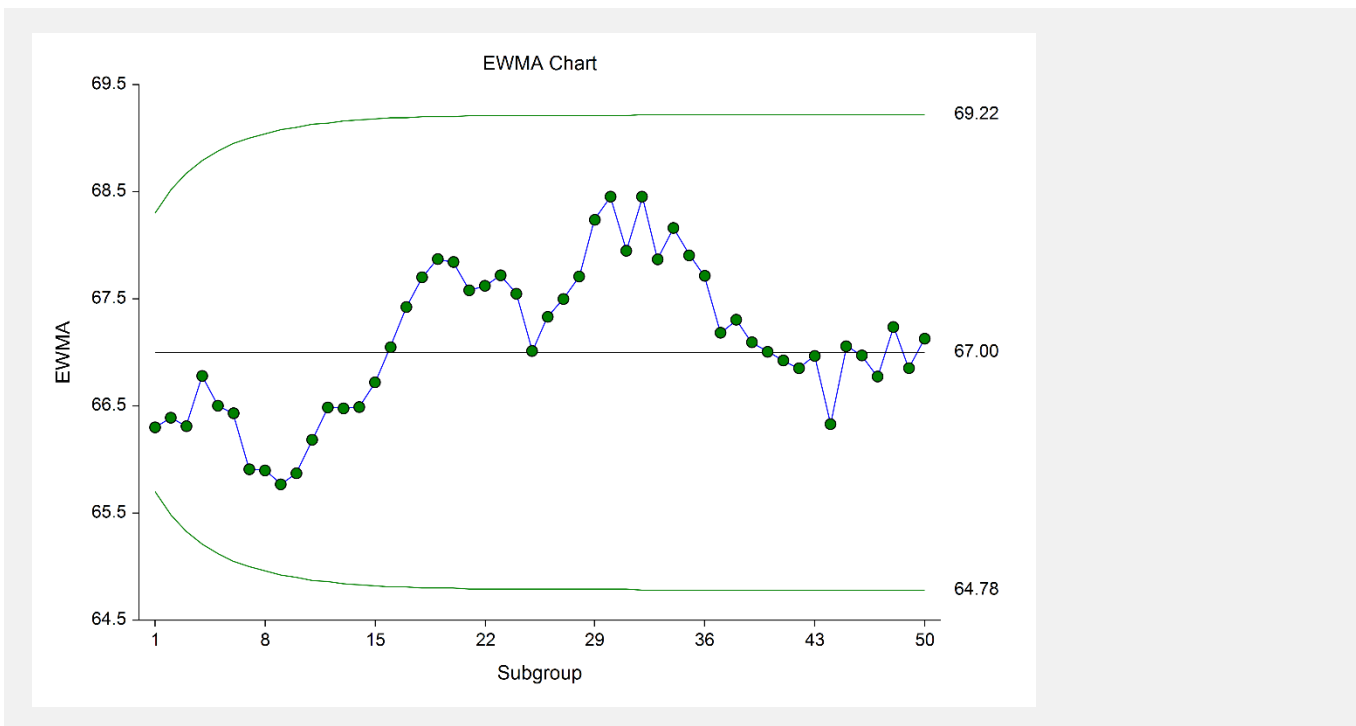
Sigma Specification

This is the method by which the sigma value is obtained.

User-Specified Sigma Value

This is value of the sigma entered by the user, and is the value used to create the moving average chart.

EWMA Chart



This EWMA plot shows the progress of the weighted moving averages across the 50 subgroups. There does not appear to be an indication of a change in the process mean.

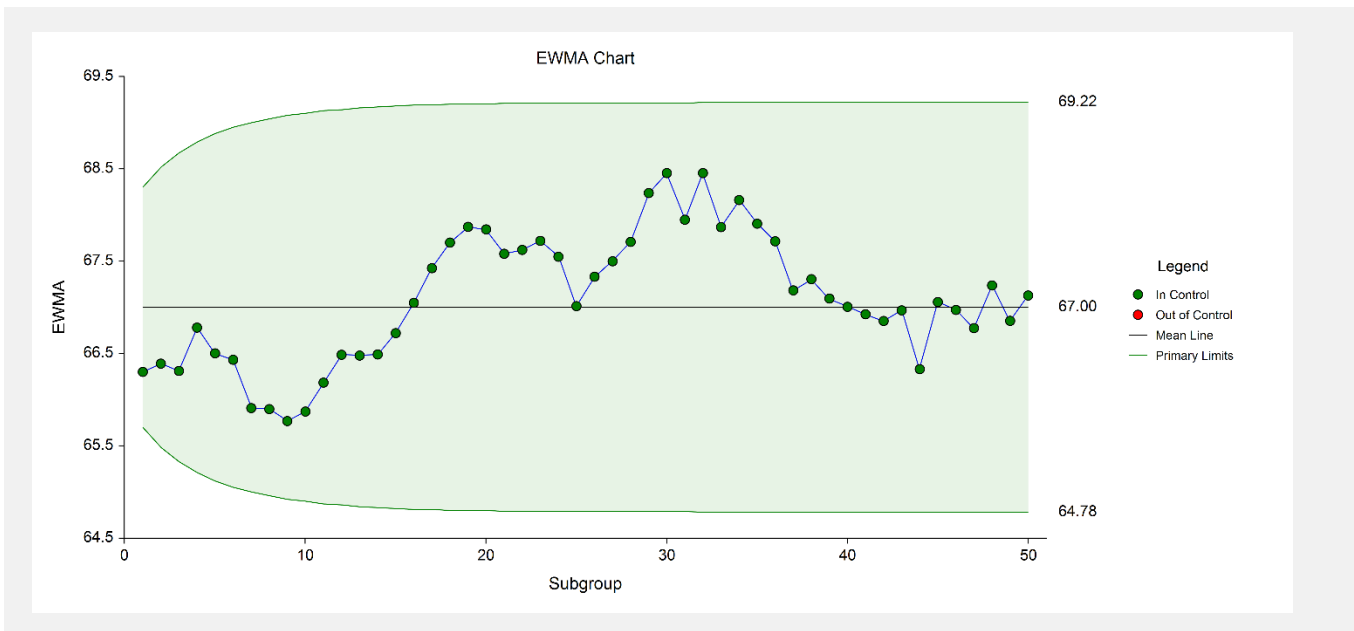
Exponentially Weighted Moving Average (EWMA) Charts

Example 2 – EWMA Chart with Additional Formatting

This example uses the same setup as Example 1, except that a variety of improvements are made in the plot format. These improvements are made by clicking the EWMA Chart format button on the EWMA Chart tab.

You can load the completed template **Example 2** by clicking on Open Example Template from the File menu of the EWMA Charts window.

EWMA Chart



As shown here, a variety of enhancements can be made to the formatting of the control charts to make the chart as easy to read as possible.