

Chapter 466

Exponential Smoothing – Trend

Introduction

This module forecasts series with upward or downward trends. Three techniques are available: least squares trend, double smoothing, and Holt's linear trend algorithm.

Least Squares Trend

Least squares trend computes a straight-line trend equation through the data using standard least squares techniques in which the dependent variable is the time series and the independent variable is the row (sequence) number. The forecasting equation is

$$F_t = a + bt$$

where F_t is the forecast at time period t , a is the y-intercept, and b is the slope of the trend. The slope indicates how much is added (or subtracted if b is negative) from each time period to the next.

This method is useful for series that show a stable, long-term trend. It places the largest weights in estimation on the two ends of the series, while the rows near the middle with an insignificant impact on the estimates.

Double Exponential Smoothing

Double exponential smoothing computes a trend equation through the data using a special weighting function that places the greatest emphasis on the most recent time periods. The forecasting equation changes from period to period.

The forecasting algorithm makes use of the following formulas:

$$F_t = a_t + b_t$$

$$a_t = X_t + (1 - \alpha)^2 e_t$$

$$b_t = b_{t-1} + \alpha^2 e_t$$

$$e_t = F_t - X_t$$

The smoothing constant, α , dictates the amount of smoothing that takes place. It ranges from zero to one.

The forecast at time period T for the value at time period $T+k$ is $a_T + b_T k$. Double smoothing is discussed in detail in Thomopoulos (1980).

This method is included more for its historical significance, since Holt's algorithm is usually preferred to it.

Holt's Linear Trend

Holt's Linear Trend computes an evolving trend equation through the data using a special weighting function that places the greatest emphasis on the most recent time periods. Instead of the global trend equation of the least squares trend algorithm, this technique uses a local trend equation. The trend equation is modified from period to period. The forecasting equation changes from period to period.

The forecasting algorithm makes use of the following formulas:

$$a_t = \alpha X_t + (1 - \alpha)(a_{t-1} + b_{t-1})$$

$$b_t = \beta(a_t - a_{t-1}) + (1 - \beta)b_{t-1}$$

Here α and β are smoothing constants which are each between zero and one. Again, a_t gives the y-intercept (or level) at time t , while b_t is the slope at time t .

The forecast at time T for the value at time $T+k$ is $a_T + b_T k$.

Smoothing Constants

Notice that in both double smoothing and Holt's linear trend, the *smoothing constant(s)* determines how fast the weights of the series decays. The values may be chosen either subjectively or objectively. Values of a smoothing constant near one put almost all weight on the most recent observations. Values of a smoothing constant near zero allow the distant past observations to have a large influence.

When selecting the smoothing constant *subjectively*, you use your own experience with this, and similar, series. Also, specifying the smoothing constant yourself lets you tune the forecast to your own beliefs about the future of the series. If you believe that the mechanism generating the series has recently gone through some fundamental changes, use a smoothing constant value of 0.9 which will cause distant observations to be ignored. If, however, you think the series is fairly stable and only going through random fluctuations, use a value of 0.1.

To select the value of the smoothing constant(s) *objectively*, you search for values that are best in some sense. Our program searches for that values that minimizes the size of the combined forecast errors of the currently available series. Three methods of summarizing the amount of error in the forecasts are available: the mean square error (MSE), the mean absolute error (MAE), and the mean absolute percent error (MAPE). The forecast error is the difference between the forecast of the current period made at the last period and the value of the series at the current period. This is written as

$$e_t = X_t - F_{t-1}$$

Using this formulation, we can define the three error-size criterion as follows:

$$MSE = \frac{1}{n} \sum e_t^2$$

$$MAE = \frac{1}{n} \sum |e_t|$$

$$MAPE = \frac{100}{n} \sum \left| \frac{e_t}{X_t} \right|$$

To find the value of the smoothing constants objectively, we select one of these criterion and search for those values of α and β that minimize this function. The program conducts a search for the appropriate values using an efficient grid-searching algorithm.

Initial Values

Both double smoothing and Holt's linear trend require initialization since the forecast for period one requires the forecast at period zero, which we do not, by definition, have. Several methods have been proposed for generating starting values. We have adopted the backcasting method which is currently considered to be one of the best methods. Backcasting is simply reversing the series so that we forecast into the past instead of into the future. This produces the required starting values for the slope and intercept. Once we have done this, we can then switch the series back and apply the algorithm in the regular manor.

Relationship to ARIMA Method

It can be shown that both double exponential smoothing and Holt's linear trend technique are equivalent to the ARIMA(0,2,2) model (see Kendall and Ord (1990) page 133). This is why backcasting is recommended for initial values.

Assumptions and Limitations

These algorithms are useful for forecasting non-seasonal time series with (local or global) trend.

Data Structure

The data are entered in a single variable.

Missing Values

When missing values are found in the series, they are either replaced or omitted. The replacement value is the average of the nearest observation in the future and in the past or the nearest non-missing value in the past.

If you do not feel that this is a valid estimate of the missing value, you should manually enter a more reasonable estimate before using the algorithm. These missing value replacement methods are particularly poor for seasonal data. We recommend that you replace missing values manually before using the algorithm.

Procedure Options

This section describes the options available in this procedure.

Variables Tab

Specify the variable(s) on which to run the analysis.

Time Series Variables

Time Series Variable(s)

Specify the variable(s) on which to run the analysis. A separate analysis will be conducted for each variable listed.

Use Logarithms

Specifies that the log (base 10) transformation should be applied to the values of the variable. The forecasts are converted back to there original metric before display.

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Missing Values

Choose how missing (blank) values are processed.

The algorithm used in this procedure cannot tolerate missing values since each row is assumed to represent the next point in a time sequence. Hence, when missing values are found, they must be removed either by imputation (filling in with a reasonable value) or by skipping the row and pretending it does not exist.

Whenever possible, we recommend that you replace missing values manually.

Here are the available options.

Average the Adjacent Values

Replace the missing value with the average of the nearest values in the future (below) and in the past (above).

Carry the Previous Value Forward

Replace the missing value with the first non-missing value immediately above (previous) this value.

Omit Row from Calculations

Ignore the row in all calculations. Analyze the data as if the row was not on the database.

Forecasting Options

Forecast Method

Select LS (least squares) linear trend, double exponential smoothing, or Holt's linear trend.

Number of Forecasts

This option specifies the number of forecasts to be generated.

Smoothing Constant Search Options

Search Method

This option specifies whether a search is conducted for the best values of the smoothing constants and what the criterion for the search will be.

- **Specified Value**
No search is conducted. The values of the smoothing constants that are given in the next options are used.
- **Search on MSE**
A search is conducted to find the values of the smoothing constants that minimize MSE.
- **Search on MAE**
A search is conducted to find the values of the smoothing constants that minimize MAE.
- **Search on MAPE**
A search is conducted to find the values of the smoothing constants that minimize MAPE.

Smoothing Constant Search Options – Pre-Specified Smoothing Constants

Alpha Smoothing Constant

When the Search Method is set to Specified Value, this option specifies the value of alpha used in double exponential smoothing and Holt's linear trend. The limits of this value are zero and one. Usually, a value between 0.1 and 0.3 are used. As the value gets closer to one, more and more weight is given to recent observations.

Beta Smoothing Constant

When the Search Method is set to Specified Value, this option specifies the value of beta used in Holt's linear trend. The limits of this value are zero and one. Usually, a value between 0.1 and 0.3 are used. As the value gets closer to one, more and more weight is given to recent observations.

Reports Tab

The following options control which reports are displayed.

Select Reports

Summary Report

This option specifies whether the indicated report is displayed.

Forecast Report

This option specifies which parts of the series are listed on the numeric reports: the original data and forecasts, just the forecasts, or neither.

Report Options

Precision

Specify the precision of numbers in the report. Single precision will display seven-place accuracy, while the double precision will display thirteen-place accuracy. Note that all reports are formatted for single precision only.

Variable Names

Specify whether to use variable names or (the longer) variable labels in report headings.

Page Title

Specify a title to be shown at the top of the reports.

Plots Tab

A plot of the data and forecast over time may be displayed, as well as the residual plot.

Select Plots

Forecast Plot - Residual Plot

Each of these options specifies whether the indicated plot is displayed. Click the plot format button to change the plot settings.

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Horizontal Axis Variable if there are Missing or Filtered Values

Horizontal Variable

This option controls the spacing on the horizontal axis when missing or filtered values occur.

Your choices are

Actual Row Number

Use the actual row number of each row from the dataset along the horizontal axis.

Sequence Number

Use the sequence (relative row) number formed by ignoring any missing or filtered values.

Storage Tab

The forecasts and residuals may be stored on the current dataset for further analysis. These options let you designate which statistics (if any) should be stored by designating which columns should receive the statistics.

Note that existing data is replaced. Be careful that you do not specify columns that contain important data.

Data Storage Columns

Forecasts

The forecasts are stored in this column.

Residuals

The residuals are stored in this column.

Example 1 – Trend Exponential Smoothing

This section presents an example of how to generate a forecast of a series using Holt's linear trend. The data in the Intel dataset gives price and volume data for Intel stock during August, 1995. We will forecast values for daily volumes. These values are contained in the variable Intel_Volume.

You may follow along here by making the appropriate entries or load the completed template **Example 1** by clicking on Open Example Template from the File menu of the Exponential Smoothing – Trend window.

1 Open the Intel dataset.

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Click on the file **Intel.NCSS**.
- Click **Open**.

2 Open the Exponential Smoothing – Trend window.

- Using the Analysis menu or the Procedure Navigator, find and select the **Exponential Smoothing – Trend** procedure.
- On the menus, select **File**, then **New Template**. This will fill the procedure with the default template.

3 Specify the variables.

- On the Exponential Smoothing – Trend window, select the **Variables tab**.
- Double-click in the **Time Series Variable** box. This will bring up the variable selection window.
- Select **Intel_Volume** from the list of variables and then click **Ok**.
- Select **Holt's Linear Trend** in the **Forecast Method** list box.

4 Specify the reports.

- On the Exponential Smoothing – Trend window, select the **Reports tab**.
- Select **Data and Forecasts** in the **Forecast Report** list box.

5 Run the procedure.

- From the Run menu, select **Run Procedure**. Alternatively, just click the green Run button.

Forecast Summary Section

Forecast Summary Section

Variable	Intel_Volume
Number of Rows	20
Mean	10974.54
Missing Values	None
Pseudo R-Squared	0.000000
Mean Square Error	1.80109E+07
Mean Error	3229.513
Mean Percent Error	28.64767
Forecast Method	Holt's Linear Trend
Search Iterations	59
Search Criterion	Mean Square Error
Alpha	0.4157034
Beta	0.1182474
Intercept (A)	9277.523
Slope (B)	210.8949

This report summarizes the forecast equation.

Variable

The name of the variable for which the forecasts are generated.

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Number of Rows

The number of rows that were in the series. This is provided to allow you to double-check that the correct series was used.

Missing Values

If missing values were found, this option lists the method used to estimate them.

Mean

The mean of the variable across all time periods.

Pseudo R-Squared

This value generates a statistic that acts like the R-Squared value in multiple regression. A value near zero indicates a poorly fitting model, while a value near one indicates a well fitting model. The statistic is calculated as follows:

$$R^2 = 100 \left(1 - \frac{SSE}{SST} \right)$$

where *SSE* is the sum of square residuals and *SST* is the total sum of squares after correcting for the mean.

Mean Square Error

The average squared residual (MSE) is a measure of how closely the forecasts track the actual data. The statistic is popular because it shows up in analysis of variance tables. However, because of the squaring, it tends to exaggerate the influence of outliers (points that do not follow the regular pattern).

Mean |Error|

The average absolute residual (MAE) is a measure of how closely the forecasts track the actual data without the squaring.

Mean |Percent Error|

The average percent absolute residual (MAPE) is a measure of how closely the forecasts track the actual data put on a percentage basis.

Forecast Method

This line shows which of the three possible trend forecasting algorithms was selected.

Search Iterations

This line shows how many iterations were needed to find the best value(s) for the smoothing constant(s).

Search Criterion

If a search was made to find the best values of the smoothing constants, this row gives the criterion used during the search.

Alpha

The value of the smoothing constant alpha that was used to generate the forecasts.

Beta

The value of the smoothing constant beta that was used to generate the forecasts.

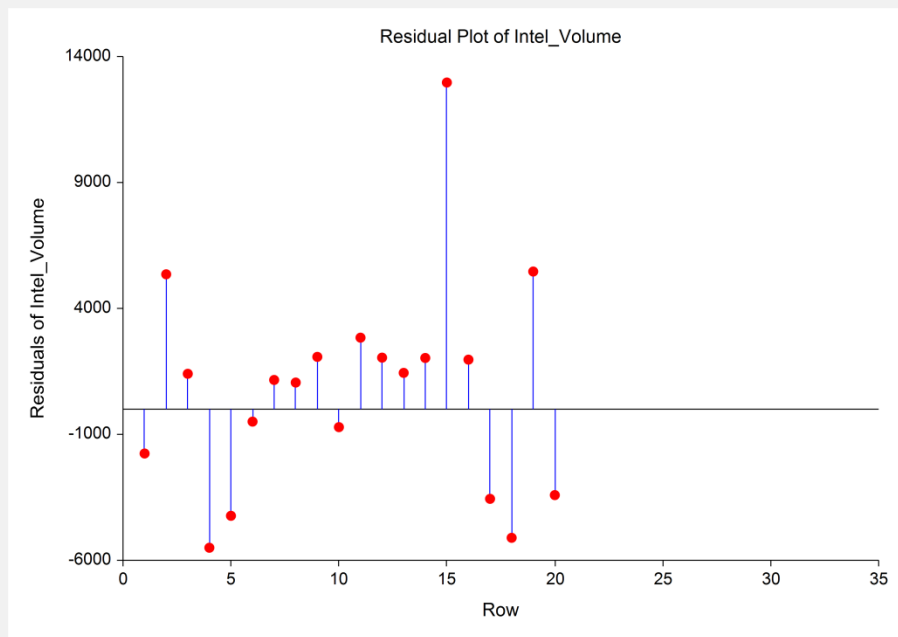
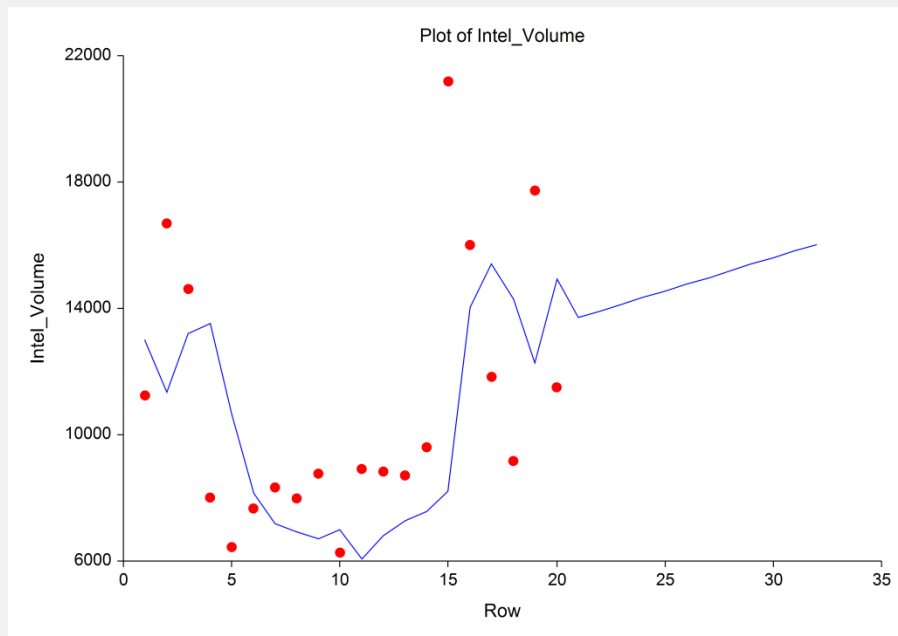
Intercept (A)

The value of the y-intercept for time period one! Hence, to forecast for time period 21 (the next period after the current period) we would use $9277.523 + 21(210.8949) = 13706.32$.

Slope (B)

The value of the slope.

Forecast and Residuals Plots



Forecast Plot

The forecast plot lets you analyze how closely the forecasts track the data. The plot also shows the forecasts at the end of the data series.

Residual Plot

This plot lets you analyze the residuals themselves. You are looking for patterns, outliers, or any other information that may help you improve the forecasting model. The first thing to compare is the scale of the Residual Plot versus the scale of the Forecast Plot. If your forecasting is working well, the vertical scale of the Residual Plot will be much less than the scale of the Forecast Plot.

Forecasts Section

Row No.	Forecast Intel_Volume	Actual Intel_Volume	Residuals
1	13003.77	11242.2	-1761.574
2	11334.51	16689.9	5355.391
3	13210.71	14613.3	1402.594
4	13512.66	8009	-5503.658
5	10673.12	6441.8	-4231.32
6	8154.504	7664.5	-490.0037
7	7167.079	8330.3	1163.221
8	6924.084	7983	1058.916
9	6689.781	8767.1	2077.319
10	6980.944	6266.4	-714.5444
11	6076.396	8915.3	2838.904
12	6788.578	8833	2044.422
13	7270.985	8709.7	1438.714
14	7572.32	9603	2030.68
15	8219.556	21185.2	12965.64
16	14049.83	16006.5	1956.668
17	15399.82	11832.4	-3567.42
18	14278.07	9168.1	-5109.966
19	12263.89	17729.3	5465.414
20	14914.58	11500.7	-3413.885
21	13706.32		
22	13917.21		
23	14128.11		
24	14339		
25	14549.9		
26	14760.79		
27	14971.68		
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This section shows the values of the forecasts, the actual values, and the residuals.