

Chapter 420

Factor Analysis

Introduction

Factor Analysis (FA) is an exploratory technique applied to a set of observed variables that seeks to find underlying factors (subsets of variables) from which the observed variables were generated. For example, an individual's response to the questions on a college entrance test is influenced by underlying variables such as intelligence, years in school, age, emotional state on the day of the test, amount of practice taking tests, and so on. The answers to the questions are the observed variables. The underlying, influential variables are the factors.

Factor analysis is carried out on the correlation matrix of the observed variables. A factor is a weighted average of the original variables. The factor analyst hopes to find a few factors from which the original correlation matrix may be generated.

Usually the goal of factor analysis is to aid data interpretation. The factor analyst hopes to identify each factor as representing a specific theoretical factor. Therefore, many of the reports from factor analysis are designed to aid in the interpretation of the factors.

Another goal of factor analysis is to reduce the number of variables. The analyst hopes to reduce the interpretation of a 200-question test to the study of 4 or 5 factors. One of the most subtle tasks in factor analysis is determining the appropriate number of factors.

Factor analysis has an infinite number of solutions. If a solution contains two factors, these may be rotated to form a new solution that does just as good a job at reproducing the correlation matrix. Hence, one of the biggest complaints of factor analysis is that the solution is not unique. Two researchers can find two different sets of factors that are interpreted quite differently yet fit the original data equally well.

NCSS provides the *principal axis method* of factor analysis. The results may be rotated using varimax or quartimax rotation. The factor scores may be stored for further analysis.

Many books are devoted to factor analysis. We suggest you obtain a book on the subject from an author in your own field. An excellent introduction to the subject is provided by Tabachnick (1989).

Technical Details

Mathematical Development

This section will document the basic formulas used by NCSS in performing a factor analysis. The following table lists many of the matrices that are used in the discussion to follow.

Label	Matrix Name	Size	Description
R	Correlation	p×p	Matrix of correlations between each pair of variables.
X	Data	N×p	Observed data matrix with N rows (observations) and p columns (variables).
Z	Standardized data	N×p	Matrix of standardized data. The standardization of each variable is made by subtracting its mean and dividing by its standard deviation.
A	Factor loading	p×m	Matrix of correlations between the original variables and the factors. Also represents the contribution of each factor in estimating the original variables.
L	Eigenvalue	m×m	Diagonal matrix of eigenvalues. Only the first m eigenvalues are considered.
V	Eigenvector	p×m	Matrix of eigenvectors. Only the first m columns of this matrix are used.
B	Factor-score coefficients	p×m	Matrix of regression weights used to construct the factor scores from the original variables.
U	Uniqueness	p×p	Matrix of uniqueness values.
F	Factor score	N×m	Matrix of factor scores. For each observation in the original data, the values of each of the retained factors are estimated. These are the factor scores.

The principal-axis method is used by NCSS to solve the factor analysis problem. Factor analysis assumes the following partition of the correlation matrix, R :

$$R = AA' + U$$

The principal-axis method proceeds according to the following steps:

1. Estimate U from the communalities as discussed below.
2. Find L and V , the eigenvalues and eigenvectors of $R-U$ using standard eigenvalue analysis.
3. Calculate the loading matrix as follows:

$$A = VL^{\frac{1}{2}}$$

4. Calculate the score matrix as follows:

$$B = VL^{-\frac{1}{2}}$$

5. Calculate the factor scores as follows:

$$F = ZB$$

Steps 1-3 may be iterated since a new U matrix may be estimated from the current loading matrix.

Initial Communality Estimation

We close this section with a discussion of obtaining an initial value of U . NCSS uses the initial estimation of Cureton (1983), which will be outlined here. The initial communality estimates, c_{ii} , are calculated from the correlation and inverse correlation matrices as follows:

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$$c_{ii} = \left(I - \frac{I}{R^{ii}} \right) \frac{\sum_{k=1}^p \max_{\text{over } j \neq k} (|r_{jk}|)}{\sum_{k=1}^p \left(I - \frac{I}{R^{kk}} \right)}$$

where R_{ii} is the i^{th} diagonal element of $R-I$ and r_{jk} is an element of R . The value of U is then estimated by $I-c_{ii}$.

Missing Values and Robust Estimation

Missing values and robust estimation are done the same way as in principal components analysis. Refer to that chapter for details.

How Many Factors

Several methods have been proposed for determining the number of factors that should be kept for further analysis. Several of these methods will now be discussed. However, remember that important information about possible outliers and linear dependencies may be determined from the factors associated with the relatively small eigenvalues, so these should be investigated as well.

Kaiser (1960) proposed dropping factors whose eigenvalues are less than one since these provide less information than is provided by a single variable. Jolliffe (1972) feels that Kaiser's criterion is too large. He suggests using a cutoff on the eigenvalues of 0.7 when correlation matrices are analyzed. Other authors note that if the largest eigenvalue is close to one, then holding to a cutoff of one may cause useful factors to be dropped. However, if the largest factors are several times larger than one, then those near one may be reasonably dropped.

Cattell (1966) documented the *scree graph*, which will be described later in this chapter. Studying this chart is probably the most popular method for determining the number of factors, but it is subjective, causing different people to analyze the same data with different results.

Another criterion is to preset a certain percentage of the variation that must be accounted for and then keep enough factors so that this variation is achieved. Usually, however, this cutoff percentage is used as a lower limit. That is, if the designated number of factors do not account for at least 50% of the variance, then the whole analysis is aborted.

Varimax and Quartimax Rotation

Factor analysis finds a set of dimensions (or coordinates) in a subspace of the space defined by the set of variables. These coordinates are represented as axes. They are orthogonal (perpendicular) to one another. For example, suppose you analyze three variables that are represented in three-dimensional space. Each variable becomes one axis. Now suppose that the data lie near a two-dimensional plane within the three dimensions. A factor analysis of this data should uncover two factors that would account for the two dimensions. You may rotate the axes of this two-dimensional plane while keeping the 90-degree angle between them, just as the blades of a helicopter propeller rotate yet maintain the same angles among themselves. The hope is that rotating the axes will improve your ability to interpret the "meaning" of each factor.

Many different types of rotation have been suggested. Most of them were developed for use in factor analysis. NCSS provides two orthogonal rotation options: varimax and quartimax.

Varimax Rotation

Varimax rotation is the most popular orthogonal rotation technique. In this technique, the axes are rotated to maximize the sum of the variances of the squared loadings within each column of the loadings matrix. Maximizing according to this criterion forces the loadings to be either large or small. The hope is that by rotating the factors, you will obtain new factors that are each highly correlated with only a few of the original variables.

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This simplifies the interpretation of the factor to a consideration of these two or three variables. Another way of stating the goal of varimax rotation is that it clusters the variables into groups; each “group” is actually a new factor.

Since varimax seeks to maximize a specific criterion, it produces a unique solution (except for differences in sign). This has added to its popularity. Let the matrix $G = \{g_{ij}\}$ represent the rotated factors. The goal of varimax rotation is to maximize the quantity:

$$Q_I = \sum_{j=1}^k \left(\frac{p \sum_{i=1}^p g_{ij}^4 - \sum_{i=1}^p g_{ij}^2}{p} \right)$$

This equation gives the raw varimax rotation. This rotation has the disadvantage of not spreading the variance very evenly among the new factors. Instead, it tends to form one large factor followed by many small ones. To correct this, NCSS uses the normalized-varimax rotation. The quantity maximized in this case is:

$$Q_N = \sum_{j=1}^k \left[\frac{p \sum_{i=1}^p \left(\frac{g_{ij}}{c_i} \right)^4 - \sum_{i=1}^p \left(\frac{g_{ij}}{c_i} \right)^2}{p^2} \right]$$

where c_i is the square root of the communality of variable i .

Quartimax Rotation

Quartimax rotation is similar to varimax rotation except that the rows of G are maximized rather than the columns of G . This rotation is more likely to produce a “general” factor than will varimax. Often, the results are quite similar. The quantity maximized for the quartimax is:

$$Q_N = \sum_{j=1}^k \left[\frac{p \sum_{i=1}^p \left(\frac{g_{ij}}{c_i} \right)^4}{p} \right]$$

Miscellaneous Topics

Using Correlation Matrices Directly

Occasionally, you will be provided with only the correlation matrix from a previous analysis. This happens frequently when you want to analyze data that is presented in a book or a report. You can perform a factor analysis on a correlation matrix using NCSS.

NCSS can store the correlation matrix on the current database. If it takes a great deal of computer time to build the correlation matrix, you might want to save it so you can use it while you determine the number of factors. You could then return to the original data to analyze the factor scores.

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Principal Component Analysis versus Factor Analysis

Both principal component analysis (PCA) and factor analysis (FA) seek to reduce the dimensionality of a data set. The most obvious difference is that while PCA is concerned with the total variation as expressed in the correlation matrix, R , FA is concerned with a correlation in a partition of the total variation called the common portion. That is, FA separates R into two matrices R_c (common factor portion) and R_u (unique factor portion). FA models the R_c portion of the correlation matrix. Hence, FA requires the discovery of R_c as well as a model for it. The goals of FA are more concerned with finding and interpreting the underlying, common factors. The goals of PCA are concerned with a direct reduction in the dimensionality.

Put another way, PCA is directed towards reducing the diagonal elements of R . Factor analysis is directed more towards reducing the off-diagonal elements of R . Since reducing the diagonal elements reduces the off-diagonal elements and vice versa, both methods achieve much the same thing.

Data Structure

The data for a factor analysis consists of two or more columns. We have created an artificial data set in which each of the six variables (X1 - X6) were created using weighted averages of two original variables (V1 and V2) plus a small random error. For example, $X1 = .33 V1 + .65 V2 + \text{error}$. Each variable had a different set of weights (.33 and .65 are the weights) in the weighted average.

Rows two and three of the data set were modified to be outliers so that their influence on the analysis could be observed. Note that even though these two rows are outliers, their values on each of the individual variables are not outliers. This shows one of the challenges of multivariate analysis: multivariate outliers are not necessarily univariate outliers. In other words, a point may be an outlier in a multivariate space and yet you cannot detect it by scanning the data one variable at a time.

This data is contained in the dataset PCA2. The data given below are the first few rows of this dataset.

PCA2 dataset (subset)

X1	X2	X3	X4	X5	X6
50	102	103	70	75	102
4	2	5	11	11	5
81	98	94	5	85	97
31	81	86	46	50	74
65	50	51	60	57	53
22	30	39	17	15	17
36	33	39	29	27	25
31	91	96	50	56	85

Procedure Options

This section describes the options available in this procedure.

Variables Tab

This panel specifies the variables used in the analysis.

Input Variables

Variables

Designates the variables to be analyzed. If matrix input is selected, indicate the variables containing the matrix. Note that for matrix input, the number of rows used is equal to the number of variables specified. Other rows will be ignored.

Data Input Format

Indicates whether raw data is to be analyzed or if a previously summarized correlation or covariance matrix is to be used.

- **Regular Data**

The data is to be input in its raw format.

- **Lower-Triangular**

The data is in a correlation or covariance matrix in lower-triangular format. This matrix could have been created by a previous run of an NCSS program or from direct keyboard input.

- **Upper-Triangular**

The data is in a correlation or covariance matrix in upper triangular format. The number of rows used is equal to the number of variables specified. This matrix could have been created by a previous run of an NCSS program or from direct keyboard input.

Covariance Estimation Options

Robust Covariance Matrix Estimation

This option indicates whether robust estimation is to be used. A full discussion of robust estimation is provided in the PCA chapter. If checked, robust estimates of the means, variances, and covariances are formed.

Robust Weight

This option specifies the value of v_1 for robust estimation. This parameter controls the weighting function in robust estimation of the covariance matrix. Jackson (1991) recommends the value 4.

Missing Value Estimation

This option indicates the type of missing value imputation method that you want to use. (Note that if the number of iterations is zero, this option is ignored.)

- **None**

No missing value imputation. Rows with missing values in any of the selected variables are ignored.

- **Average**

The average-value imputation method is used. Each missing value is estimated by the average value of that variable. The process is iterated as many times as is indicated in the second box.

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- **Multivariate Normal**

The multivariate-normal method. Each missing value is estimated using a multiple regression of the missing variable(s) on the variables that contain data in that row. This process is iterated as many times as indicated. See the discussion of missing value imputation methods elsewhere in this chapter.

Maximum Iterations

This option specifies the number of iterations used by either Missing Value Imputation or Robust Covariance Estimation. Robust estimation usually requires only four or five iterations to converge. Missing value imputation may require as many as twenty iterations if there are a lot of missing values.

When using this option, it is better to specify too many iterations than too few. After considering the Percent Change values in the Iteration Report, you can decide upon an appropriate number of iterations and re-run the problem.

Factor Options

Factor Rotation

Specifies the type of rotation, if any, that should be used on the solution. If rotation is desired, either varimax or quartimax rotation is available.

Number of Factors

This option specifies the number of factors to be used. On the first run, you would set this rather large (say eight or so). After viewing the eigenvalues you would reset this appropriately and make a second run.

Communality Options

Communality Iterations

This option specifies how many iterations to use in estimating the communalities. Some authors suggest a value of one here. Others suggest as many as four or five.

Reports Tab

The following options control the format of the reports.

Select Reports

Descriptive Statistics - Scores Report

These options let you specify which reports are displayed.

Report Options

Minimum Loading

Specifies the minimum absolute value that a loading can have and still remain in the Variable List report.

Precision

Specify the precision of numbers in the report. A single-precision number will show seven-place accuracy, while a double-precision number will show thirteen-place accuracy. Note that the reports are formatted for single precision. If you select double precision, some numbers may run into others. Also note that all calculations are performed in double precision regardless of which option you select here. This is for reporting purposes only.

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Variable Names

This option lets you select whether to display only variable names, variable labels, or both.

Plots Tab

These sections specify the pair-wise plots of the scores and loadings.

Select Plots

Scores Plot - Loadings Plot

These options let you specify which reports and plots are displayed. Click the plot format button to change the plot settings.

Plot Options

Number Factors Plotted

You can limit the number of plots generated using this parameter. Usually, you will only have interest in the first three or four factors.

Storage Tab

The factor scores and/or the correlation matrix may be stored on the current dataset for further analysis. This group of options let you designate which statistics (if any) should be stored and which columns should receive these statistics. The selected statistics are automatically stored to the current dataset. Note that existing data are replaced.

Data Storage Columns

Factor Scores

You can automatically store the factor scores for each row into the columns specified here. These scores are generated for each row of data in which all independent variable values are nonmissing.

Correlation Matrix

You can automatically store the correlation matrix to the columns specified here.

Example 1 – Factor Analysis

This section presents an example of how to run a factor analysis. The data used are shown in the table above and found in the PCA2 dataset.

You may follow along here by making the appropriate entries or load the completed template **Example 1** by clicking on Open Example Template from the File menu of the Factor Analysis window.

1 Open the PCA2 dataset.

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Click on the file **PCA2.NCSS**.
- Click **Open**.

2 Open the Factor Analysis window.

- Using the Analysis menu or the Procedure Navigator, find and select the **Factor Analysis** procedure.
- On the menus, select **File**, then **New Template**. This will fill the procedure with the default template.

3 Specify the variables.

- On the Factor Analysis window, select the **Variables tab**.
- Double-click in the **Variables** box. This will bring up the variable selection window.
- Select **X1** through **X6** from the list of variables and then click **Ok**. “X1-X6” will appear in the Variables box.
- Check the **Robust Covariance Matrix Estimation** box.
- Enter **6** in the **Maximum Iterations** box.
- Select **Varimax** in the **Factor Rotation** list box.
- Enter **2** in the **Number of Factors** box.
- Enter **6** in the **Communality Iterations** box.

4 Specify which reports.

- Select the **Reports tab**.
- Check all reports and plots. Normally you would only view a few of these reports, but we are selecting them all so that we can document them.

5 Run the procedure.

- From the Run menu, select **Run Procedure**. Alternatively, just click the green Run button.

Robust and Missing-Value Iteration Section

This report is only produced when robust or missing value estimation is used.

Robust and Missing-Value Estimation Iteration Section

No.	Count	Trace of Covar Matrix	Percent Change
0	30	4907.795	0.00
1	30	4907.795	0.00
2	30	4423.718	-9.86
3	30	4423.718	0.00
4	30	4353.748	-1.58
5	30	4353.748	0.00
6	30	4335.77	-0.41

This report presents the progress of the robust iterations. The trace of the covariance matrix gives a measure of what is happening at each iteration. When this value stabilizes, the program has converged. The percent change is reported to let you determine how much the trace has changed. In this particular example, we see very little

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change between iterations five and six. We would feel comfortable stopping at this point. A look at the Descriptive Statistics section will let you see how much the means and standard deviations have changed.

A look at the Residual Section will let you see the robust weights that are assigned to each row. Those weights that are near zero indicate observations whose influence have been removed by the robust procedure.

Descriptive Statistics Section

Descriptive Statistics Section				
Variables	Count	Mean	Standard Deviation	Communality
X1	30	42.83667	23.18579	0.997983
X2	30	53.25062	27.93123	0.999791
X3	30	57.13034	26.3737	0.999585
X4	30	43.5617	24.56474	0.992023
X5	30	43.20835	25.75021	1.00007
X6	30	48.61827	32.49559	0.999944

Count, Mean, and Standard Deviation

These are the familiar summary statistics of each variable. They are displayed to allow you to make sure that you have specified the correct variables. Note that using missing value imputation or robust estimation will change these values.

Communality

The communality shows how well this variable is predicted by the retained factors. It is similar to the R-Squared that would be obtained if this variable were regressed on the factors that were kept. However, remember that this is not based directly on the correlation matrix. Instead, calculations are based on an adjusted correlation matrix.

Correlation Section

Correlation Section

Variables	X1	X2	X3	X4	X5
X1	1.000000	0.271780	0.127016	0.881604	0.814686
X2	0.271780	1.000000	0.988909	0.683206	0.778649
X3	0.127016	0.988909	1.000000	0.568933	0.677480
X4	0.881604	0.683206	0.568933	1.000000	0.986945
X5	0.814686	0.778649	0.677480	0.986945	1.000000
X6	0.484907	0.973093	0.928454	0.831949	0.901975

Phi=0.769781 Log(Det|R)=-29.547320 Bartlett Test=773.15 DF=15 Prob=0.000000

Variables	X6
X1	0.484907
X2	0.973093
X3	0.928454
X4	0.831949
X5	0.901975
X6	1.000000

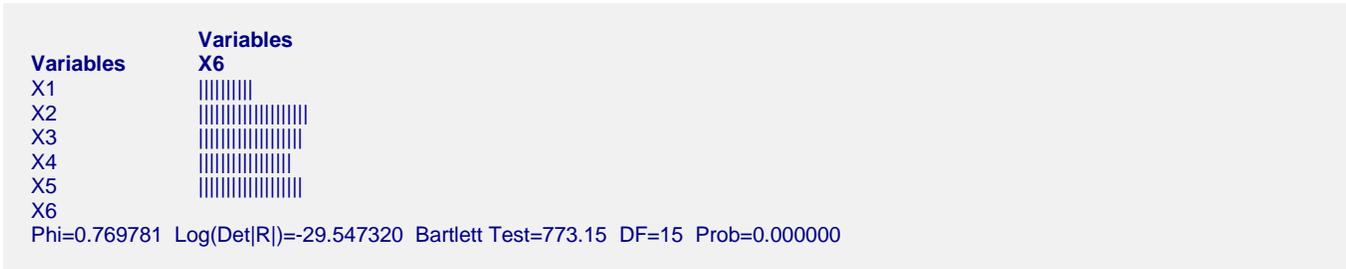
Phi=0.769781 Log(Det|R)=-29.547320 Bartlett Test=773.15 DF=15 Prob=0.000000

Bar Chart of Absolute Correlation Section

Variables	X1	X2	X3	X4	X5
X1					
X2					
X3					
X4					
X5					
X6					

Phi=0.769781 Log(Det|R)=-29.547320 Bartlett Test=773.15 DF=15 Prob=0.000000

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This report gives the correlations alone for a test of the overall correlation structure in the data. In this example, we notice several high correlation values. The Gleason-Staelin redundancy measure, phi, is 0.736, which is quite large. There is apparently some correlation structure in this data set that can be modeled. If all the correlations are small (say less than .3), there would be no need for a factor analysis.

Correlations

The simple correlations between each pair of variables. Note that using the missing value imputation or robust estimation options will affect the correlations in this report. When the above options are not used, the correlations are constructed from those observations having no missing values in any of the specified variables.

Phi

This is the Gleason-Staelin redundancy measure of how interrelated the variables are. A zero value of ϕ^2 means that there is no correlation among the variables, while a value of one indicates perfect correlation among the variables. This coefficient may have a value less than 0.5 even when there is obvious structure in the data, so care should be taken when using it. This statistic is especially useful for comparing two or more sets of data. The formula for computing ϕ^3 is:

$$\phi = \sqrt{\frac{\sum_{i=1}^p \sum_{j=1}^p r_{ij}^2 - p}{p(p-1)}}$$

Log(Det|R)

This is the log (base e) of the determinant of the correlation matrix. If you used the covariance matrix, this is the log (base e) of the determinant of the covariance matrix.

Bartlett Test, df, Prob

This is Bartlett's sphericity test (Bartlett, 1950) for testing the null hypothesis that the correlation matrix is an identity matrix (all correlations are zero). If you get a probability (Prob) value greater than 0.05, you should not perform a factor analysis on the data. The test is valid for large samples ($N > 150$). It uses a Chi-square distribution with $p(p-1)/2$ degrees of freedom. Note that this test is only available when you analyze a correlation matrix. The formula for computing this test is:

$$\chi^2 = \frac{(11 + 2p - 6N)}{6} \text{Log}_e |R|$$

Bar Chart of Absolute Correlation Section

This chart graphically displays the absolute values of the correlations. It lets you quickly find high and low correlations.

Eigenvalues Section

Eigenvalues after Varimax Rotation				
No.	Eigenvalue	Individual Percent	Cumulative Percent	Scree Plot
1	3.288191	54.89	54.89	
2	2.701207	45.09	99.99	
3	0.001207	0.02	100.01	
4	-0.000099	0.00	100.01	
5	-0.000121	0.00	100.00	
6	-0.000295	0.00	100.00	

Eigenvalues

The eigenvalues of the $R-U$ matrix. Often, these are used to determine how many factors to retain. (In this example, we would retain the first two eigenvalues.)

One rule-of-thumb is to retain those factors whose eigenvalues are greater than one. The sum of the eigenvalues is equal to the number of variables. Hence, in this example, the first factor retains the information contained in 3.3 of the original variables.

Note that, unlike in PCA where all eigenvalues are positive, the eigenvalues may be negative in factor analysis. Usually, these factors would be discarded and the analysis would be re-run.

Individual and Cumulative Percents

The first column gives the percentage of the total variation in the variables accounted for by this factor. The second column is the cumulative total of the percentage. Some authors suggest that the user pick a cumulative percentage, such as 80% or 90%, and keep enough factors to attain this percentage.

Scree Plot

This is a rough bar plot of the eigenvalues. It enables you to quickly note the relative size of each eigenvalue. Many authors recommend it as a method of determining how many factors to retain.

The word *scree*, first used by Cattell (1966), is usually defined as the rubble at the bottom of a cliff. When using the scree plot, you must determine which eigenvalues form the “cliff” and which form the “rubble.” You keep the factors that make up the cliff. Cattell and Jaspers (1967) suggest keeping those that make up the cliff plus the first factor of the rubble.

Interpretation of the Example

The first question that we would ask is how many factors should be kept. The scree plot shows that the first two factors are indeed the largest. The cumulative percentages show that the first two factors account for over 99.99% of the variation.

Eigenvectors Section

Eigenvectors after Varimax Rotation		
Variables	Factors	
	Factor1	Factor2
X1	-0.303444	-0.662220
X2	-0.416551	0.378018
X3	-0.382768	0.491154
X4	-0.428167	-0.317929
X5	-0.448606	-0.204840
X6	-0.450912	0.185189

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Bar Chart of Absolute Eigenvectors after Varimax Rotation



Eigenvector

The eigenvectors of the R-U matrix.

Bar Chart of Absolute Eigenvectors

This chart graphically displays the absolute values of the eigenvectors. It lets you quickly interpret the eigenvector structure. By looking at which variables correlate highly with a factor, you can determine what underlying structure it might represent.

Factor Loadings Section

Factor Loadings after Varimax Rotation

Variables	Factors	
	Factor1	Factor2
X1	-0.019936	-0.998792
X2	-0.967470	-0.252572
X3	-0.994037	-0.107126
X4	-0.478418	-0.873578
X5	-0.594943	-0.803812
X6	-0.883654	-0.468080

Bar Chart of Absolute Factor Loadings after Varimax Rotation



Factor Loadings

These are the correlations between the variables and factors.

Bar Chart of Absolute Factor Loadings

This chart graphically displays the absolute values of the factor loadings. It lets you quickly interpret the correlation structure. By looking at which variables correlate highly with a factor, you can determine what underlying structure it might represent.

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Communality Section

Communality after Varimax Rotation

Variables	Factors		Communality
	Factor1	Factor2	
X1	0.000397	0.997586	0.997983
X2	0.935999	0.063793	0.999791
X3	0.988109	0.011476	0.999585
X4	0.228883	0.763139	0.992023
X5	0.353958	0.646114	1.000072
X6	0.780845	0.219099	0.999944

Bar Chart of Communalities after Varimax Rotation

Variables	Factors		Communality
	Factor1	Factor2	
X1			
X2			
X3			
X4			
X5			
X6			

Communality

The communality is the proportion of the variation of a variable that is accounted for by the factors that are retained. It is similar to the R-Squared value that would be achieved if this variable were regressed on the retained factors. This table value gives the amount added to the communality by each factor.

Bar Chart of Communalities

This chart graphically displays the values of the communalities.

Factor Structure Summary Section

Factor Structure Summary after Varimax Rotation

Factor1	Factors	
	Factor2	
X3	X1	
X2	X4	
X6	X5	
X5	X6	
X4		

Interpretation

This report is provided to summarize the factor structure. Variables with an absolute loading greater than the amount set in the Minimum Loading option are listed under each factor. Using this report, you can quickly see which variables are related to each factor. Note that it is possible for a variable to have high loadings on several factors, although varimax rotation makes this very unlikely.

Score Coefficients Section

Score Coefficients after Varimax Rotation

Variables	Factors	
	Factor1	Factor2
X1	0.268188	-0.5366089
X2	-0.3613275	0.1312937
X3	-0.4135219	0.2176106
X4	2.901571E-02	-0.3414549
X5	-0.0422971	-0.2712604
X6	-0.2641693	-8.934696E-03

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Score Coefficients

These are the coefficients that are used to form the factor scores. The factor scores are the values of the factors for a particular row of data. These score coefficients are similar to the eigenvectors. They have been scaled so that the scores produced have a variance of one rather than a variance equal to the eigenvalue. This causes each of the factors to have the same variance.

You would use these scores if you wanted to calculate the factor scores for new rows not included in your original analysis.

Factor Scores Section

Factor Scores after Varimax Rotation

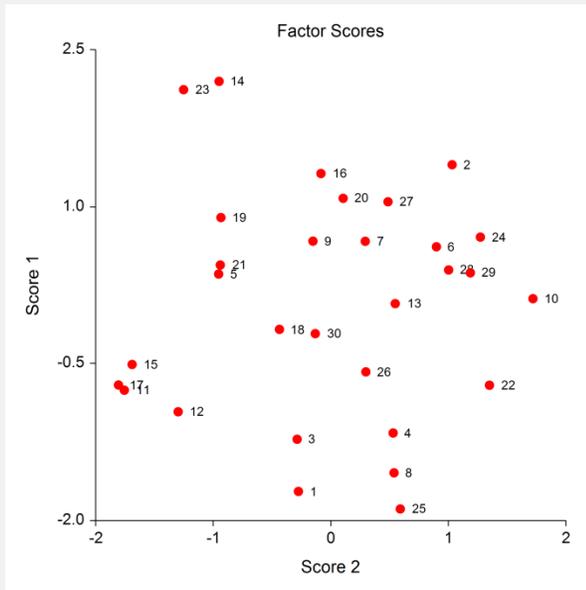
Row	Factor1	Factor2
1	-1.7219	-0.2752
2	1.4002	1.0317
3	-1.2231	-0.2862
4	-1.1632	0.5302

(report continues through all thirty rows)

Factor1 - Factor2

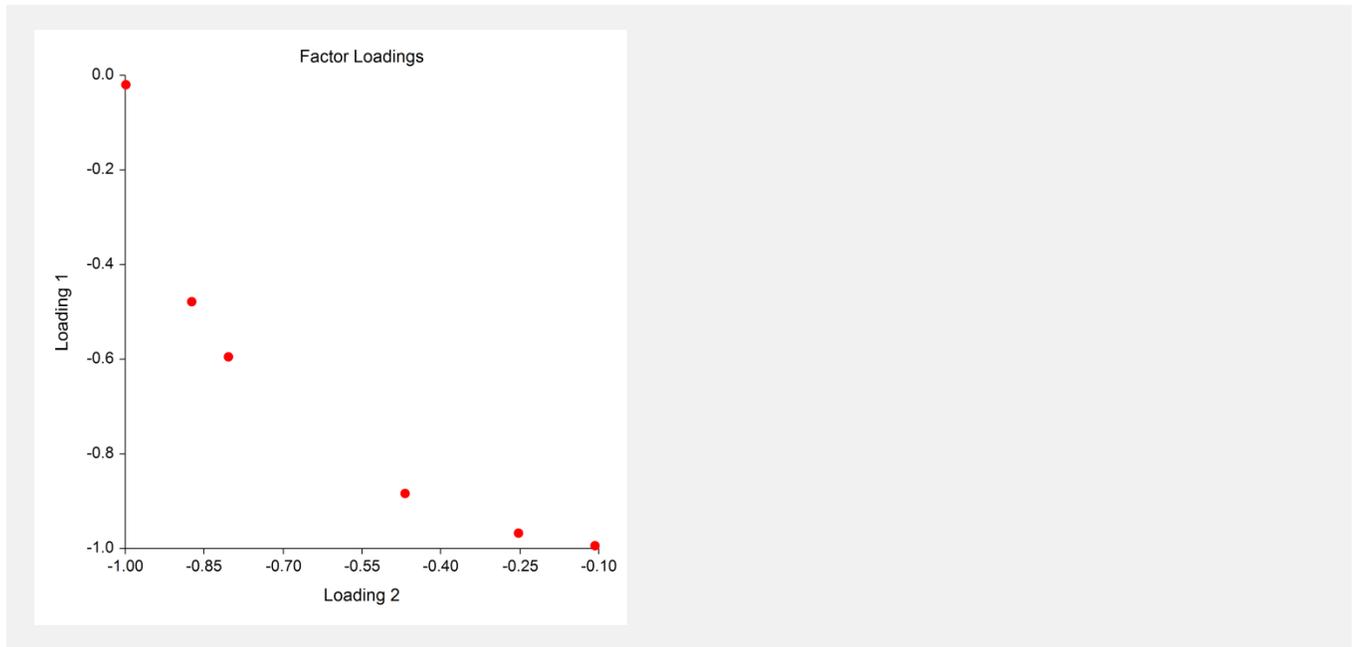
The factor scores are the values of the factors for a particular row of data. They have been scaled so they have a variance of one.

Factor Score Plots



This set of plots shows each factor plotted against every other factor.

Factor Loading Plots



This set of plots shows each of the factor loading columns plotted against each other.