Chapter 480

Linear Programming with Tableau

Introduction

Linear programming maximizes a linear objective function subject to one or more constraints. The technique finds broad use in operations research and is included here because it is occasionally of use in statistical work.

The mathematical representation of the linear programming (LP) problem is

Maximize

$$Z = C_1 X_1 + C_2 X_2 + \cdots + C_n X_n$$

Subject to

$$X_1 \geq 0,\ X_1 \geq 0, \cdots,\ X_n \geq 0$$

$$a_{i1} X_1 + a_{i2} X_2 + \cdots + a_{in} X_n \ \{\leq, =, \geq\}\ b_i \geq 0 \quad i = 1, \cdots, m$$

The X's are called *decision variables* (the unknowns), the first equation is called the *objective function* and the *m* inequalities (and equalities) are called *constraints*. The b_i's are often called *right-hand sides* (RHS).

The *simplex* algorithm, which solves this problem, was discovered by George Dantzig in 1947. We use a modified version of the revised simplex algorithm given by Press, Teukolsky, Vetterling, and Flannery (1992).

Example

We will solve the following problem using **NCSS**:

Maximize Z=X1+X2+2X3-2X4

subject to

X1 + 2X3	≤	700
2X2 - 8X4	≤	0
X2 - 2X3 + X4	≥	1
X1 + X2 + X3 + X4	=	10

The solution is X1 = 9, X2 = 0.8, X3 = 0, and X4 = 0.2 which results in Z = 9.4.

Data Structure

This technique requires a special data format. The coefficients of the object function are stored in one (usually the first) row. The constraints are stored one to a row. The type of constraint (less than, greater than, or equal to) is stored in a column. Following is an example of how to store the above example in an ***NCSS*** dataset. This particular dataset is called LP.

LP dataset

X1	X2	X3	X4	Logic	RHS
1	1	2	-2	O	
1		2		LT	700
	2		-8	LT	0
	1	-2	1	GT	1
1	1	1	1	EQ	10

Procedure Options

This section describes the options available in this procedure.

Variables Tab

Specify the variables on which to run the analysis.

Variables

Constraint (A) Variables

Specify the variables containing the A matrix (the matrix of coefficients). Each variable will be solved for during the running of the program. The coefficients can be either positive or negative. If a particular coefficient is zero, you may enter a zero or leave the value blank. Usually the objective function coefficients are entered as the first row.

Logic Variable

Specify the variable containing the logic values for the constraints. Use 'LE' for less than or equal, 'EQ' for equal, 'GE' for greater than or equal, and 'O' to designate the row containing the coefficients of the objective function.

Bounds (R.H.S.) Variable

Specify the variable that contains the right-hand sides (the b's) for each constraint.

Zero

Zero

Because of the possibility of rounding error, a small value must be specified below which, all values will be treated as zero by the algorithm.

Reports Tab

Select Reports

Initial Tableau Report - Final Tableau Report

Indicate which reports you want to view.

Report Options

Variable Names

This option lets you select whether to display only variable names, variable labels, or both.

Precision

Specify the precision of numbers in the report. Single precision will display seven-place accuracy, while double precision will display thirteen-place accuracy.

Report Options – Decimal Places

Initial Tableau - Final Tableau

This option lets you designate the number of decimal places to be displayed on each report.

Example 1 – Linear Programming

This section presents an example of how to run the data presented in the example given above. The data are contained in the LP database.

You may follow along here by making the appropriate entries or load the completed template **Example 1** by clicking on Open Example Template from the File menu of the Linear Programming window.

1 Open the LP dataset.

- From the **File** menu of the NCSS Data window, select **Open Example Data**.
- Click on the file **LP.NCSS**.
- Click **Open**.

2 Open the Linear Programming window.

- Using the Analysis menu or the Procedure Navigator, find and select the **Linear Programming with Tableau** procedure.
- On the menus, select **File**, then **New Template**. This will fill the procedure with the default template.

3 Specify the variables.

- On the Linear Programming window, select the **Variables tab**.
- Double-click in the **Constraint (A) Variables** text box. This will bring up the variable selection window.
- Select variables **X1, X2, X3,** and **X4** from the list of variables and then click **Ok**. “X1-X4” will appear in this box.
- Double-click in the **Logic Variable** text box. This will bring up the variable selection window.
- Select **Logic** as the Logic Variable since this variable contains the logical sign of each constraint.
- Double-click in the **Bounds (R.H.S.) Variable** text box. This will bring up the variable selection window.
- Select **RHS** as the Bounds (R.H.S.) Variable since this variable contains the logical sign of each constraint.

4 Run the procedure.

- From the Run menu, select **Run Procedure**. Alternatively, just click the green Run button.

Initial Tableau Section

Initial Tableau Section

Row	X1	X2	X3	X4	RHS
1 Obj Fn	1.0000	1.0000	2.0000	-2.0000	0.0000
2 <=	1.0000	0.0000	2.0000	0.0000	700.0000
3 <=	0.0000	2.0000	0.0000	-8.0000	0.0000
4 >=	0.0000	1.0000	-2.0000	1.0000	1.0000
5 =	1.0000	1.0000	1.0000	1.0000	10.0000

This report lists the initial values so you can double check the input.

Optimal Solution Section

Optimal Solution Section

Variable	Optimal Value	Original Cost	Reduced Cost	Status
X1	9.0000	1.0000	0.0000	Basis
X2	0.8000	1.0000	0.0000	Basis
X3	0.0000	2.0000	-0.2000	Non Basis
X4	0.2000	-2.0000	0.0000	Basis
Obj. Fn.	9.4000			

This report presents the solution. It shows the optimal value of each variable.

Variable

The variables that are being solved for.

Optimal Value

The values of the independent variables that results in a maximum value of the objective function. The maximum value of the objective function is given as the last line of the report.

Original Cost

These are the values of the coefficients of the objective functions. These are the C's.

Reduced Cost

The reduced costs are an additional output of the simplex method.

Status

This column gives the status of each independent variable in final solution. The solution is found by ignoring some variables (setting their values to zeros). When a variable is ignored, it is said to be a "non basis" variable. When a variable is not ignored, it is said to be a "basis" variable.

Constraint Section

Constraint Section

Row No.	Type	RHS	Optimal RHS	Constraint
2	<=	700.0000	9.0000	X1+2X3
3	<=	0.0000	0.0000	2X2-8X4
4	>=	1.0000	1.0000	X2-2X3+X4
5	=	10.0000	10.0000	X1+X2+X3+X4

This report presents an analysis of each constraint when the variables are set to their optimal values.

Row No.

The row of the database from which this constraint comes.

Type

The type of constraint that this row represents.

RHS

The original value of the right-hand side of the constraint.

Optimal RHS

The value of this constraint at the optimal solution.

Constraint

The first forty characters of the constraint.

Final Tableau Section

Final Tableau Section

Variables	X3	Slack2	Art1	Slack3	RHS
Z	-0.2000	-0.3000	1.0000	-0.6000	9.4000
Slack1	-1.0000	0.0000	1.0000	-1.0000	691.0000
X2	-1.6000	0.1000	0.0000	-0.8000	0.8000
X4	-0.4000	-0.1000	0.0000	-0.2000	0.2000
X1	3.0000	0.0000	-1.0000	1.0000	9.0000

This report presents the final values of the simplex tableau. The variables listed down the left side are the basis variables. These are the variables that are active in the solution. The variables listed across the top are the non-basis variables. These variables were not in the solution.

A slack variable is generated for each inequality constraint. An artificial variable is generated for each equality constraint. The values in the RHS column are the solution values.