Chapter 482

Mixed Integer Programming

Introduction

Linear programming maximizes (or minimizes) a linear objective function subject to one or more constraints. Mixed integer programming adds one additional condition that at least one of the variables can only take on integer values. The technique finds broad use in operations research.

The mathematical representation of the mixed integer programming (MIP) problem is

Maximize (or minimize)

$$z = \boldsymbol{CX}$$

subject to

$$\mathbf{AX} \leq \mathbf{b},\ \mathbf{X} \geq \mathbf{0}, \text{ some } x_i \text{ are restricted to integer values.}$$

where

$$\boldsymbol{X} = (x_1, x_2, \dots, x_n)'$$

$$\boldsymbol{C} = (c_1, c_2, \dots, c_n)$$

$$\boldsymbol{b} = (b_1, b_2, \dots, b_m)'$$

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}$$

The x_i's are the *decision variables* (the unknowns), the first equation is called the *objective function* and the m inequalities (and equalities) are called *constraints*. The constraint bounds, the b_i's, are often called *right-hand sides* (RHS).

NCSS solves a particular MIP problem using the branch and bound algorithm available in the *Extreme Optimization* mathematical subroutine package.

Example

We will solve the following problem using **NCSS**:

Maximize

$$z = x_1 + x_2 + 2x_3 - 2x_4$$

subject to

$x_1 + 2x_3 \le 700$

$2x_2 - 8x_3 \le 0$

$x_2 - 2x_3 + x_4 \ge 1$

$x_1 + x_2 + x_3 + x_4 = 10$

$0 \le x_1 \le 10$

$0 \le x_2 \le 10$

$0 \le x_3 \le 10$

$0 \le x_4 \le 10$

All variables are integers.

The solution (see Example 1 below) is $x_1 = 3$, $x_2 = 4$, $x_3 = 2$, and x4 = 1 which results in z = 9.0. Note that this is quite different from the LP solution of $x_1 = 9$, $x_2 = 0.8$, $x_3 = 0$, and x4 = 0.2 which results in z = 9.4.

Data Structure

This technique requires a special data format which will be discussed under the *Specifications* tab. Here is the way the above example would be entered. It is stored in the dataset *LP 1*.

LP 1 dataset

Type	Logic	RHS	X1	X2	X3	X4
O			1	1	2	-2
C	<	700	1		2	
C	<	0		2		-8
C	>	1		1	-2	1
C	=	10	1	1	1	1
L			0	0	0	0
U			10	10	10	10

Example 1 – Mixed Integer Programming

This section presents an example of how to run the data presented in the example given above. The data are contained in the LP 1 database. Here is the specification of the problem.

Maximize

$$z = x_1 + x_2 + 2x_3 - 2x_4$$

subject to

$x_1 + 2x_3 \leq 700$

$2x_2 - 8x_3 \leq 0$

$x_2 - 2x_3 + x_4 \geq 1$

$x_1 + x_2 + x_3 + x_4 = 10$

$0 \leq x_1 \leq 10$

$0 \leq x_2 \leq 10$

$0 \leq x_3 \leq 10$

$0 \leq x_4 \leq 10$

All variables are integers.

Setup

To run this example, complete the following steps:

1 Open the LP 1 example dataset

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select **LP 1** and click **OK**.

2 Specify the Mixed Integer Programming procedure options

- Find and open the **Mixed Integer Programming** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 1** settings template. To load this template, click **Open Example Template** in the Help Center or File menu.

Option	Value
Specifications Tab	
Type of Optimum	**Maximum**
Row Type Column	**Type**
Integer Variables Columns	**X1-X4**
Labels of Constraints Column	**CLabel**
Logic Column	**Logic**
Constraint Bounds (RHS) Column	**RHS**

3 Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

Objective Function and Solution for Maximum

Objective Function and Solution for Maximum

Variable	Type	Objective Function Coefficient	Value at Maximum
X1	Integer	1.0	3.000
X2	Integer	1.0	4.000
X3	Integer	2.0	2.000
X4	Integer	-2.0	1.000

Maximum of Objective Function 9.000

Solution Status: The optimization model is optimal.

This report lists the linear portion of the objective function coefficients and the values of the variables at the maximum (that is, the solution). It also shows the value of the objective function at the solution as well as the status of the algorithm when it terminated.

Constraints

Constraints

Label, Logic	X1	X2	X3	X4	RHS
Con1, ≤	1.0	0.0	2.0	0.0	700.0
Con2, ≤	0.0	2.0	0.0	-8.0	0.0
Con3, ≥	0.0	1.0	-2.0	1.0	1.0
Con4, =	1.0	1.0	1.0	1.0	10.0

This report presents the coefficients of the constraints as they were input.

Values of Constraints at Solution for Maximum

Values of Constraints at Solution for Maximum

Label, Logic	RHS	RHS at Solution
Con1, ≤	700.0	7.000
Con2, ≤	0.0	0.000
Con3, ≥	1.0	1.000
Con4, =	10.0	10.000

This report presents the right-hand side of each constraint along with its value at the optimal values of the variables.