

Chapter 223

Mixed Models – Random Coefficients

Introduction

This specialized Mixed Models procedure analyzes random coefficient regression models. In this case, the regression coefficients (the intercepts and slopes) are unique to each subject. Since the subjects are a random sample from a population of subjects, this technique is called random coefficients. The technique is also known as multilevel modeling or hierarchical modeling.

This procedure uses the standard mixed model calculation engine to perform all calculations. However, the user-interface has been simplified to make specifying the random coefficients analysis much easier.

Random Coefficients Models

It is often important in a study to determine the relationship between the response and time. This is often done by including the measurement time as a covariate in the model, with a corresponding slope, say β_t . It is plausible and likely that the slope will vary with subject, so it might be useful to model a separate intercept and slope for each subject in the study. This is done by fitting the subject variable as the intercept and the subject*time interaction as the slope for each patient. These two terms could reasonably be assumed to arise at random from a distribution and, thus, would be specified as random effects. This gives rise to what is called a *random coefficients model*.

A random coefficients model is one in which the subject term and a subject*time interaction term are both included as random effects in the model. This type of model is different from an ordinary random effects model because when we fit a straight line, the estimates of the slope and intercept are not independent. Thus, the subject and subject*time effects in the model are correlated. The random effects model must be adapted to this situation to allow for correlation among these random effects. This is done using the bivariate normal distribution. The bivariate random effect becomes

$$\begin{pmatrix} \text{subject}_k \\ (\text{subject} * \text{time})_k \end{pmatrix} \sim N(0, \mathbf{G}),$$

where

$$\mathbf{G} = \begin{pmatrix} \sigma_{\text{subject}}^2 & \sigma_{\text{subject}, \text{subject} * \text{time}} \\ \sigma_{\text{subject}, \text{subject} * \text{time}} & \sigma_{\text{subject} * \text{time}}^2 \end{pmatrix}.$$

The random coefficients model is often used if the relationship with time is of interest or if the repeated measurements do not occur at fixed intervals.

Types of Factors

It is important to understand between-subject factors and within-subject factors.

Between-Subject Factors

Each subject is assigned to only one category of a each between-subject factor. For example, if 12 subjects are randomly assigned to three treatment groups (four subjects per group), treatment is a between-subject factor.

Within-Subject Factors

Within-subject factors are those in which the subject's response is measured at several time points.

Within-subject factors are those factors for which multiple levels of the factor are measured on the same subject. If each subject is measured at the low, medium, and high level of the treatment, treatment is a within-subject factor.

Determining the Correct Model of the Variance-Covariance of Y

Akaike Information Criterion (AIC) for Model Assessment

Akaike information criterion (AIC) is tool for assessing model fit (Akaike, 1973, 1974). The formula is

$$AIC = -2 \times L + 2p$$

where L is the (ML or REML) log-likelihood and p depends on the type of likelihood selected. If the ML method is used, p is the total number of parameters. If the REML method is used, p is the number of variance component parameters.

The formula is designed so that a smaller AIC value indicates a “better” model. AIC penalizes models with larger numbers of parameters. That is, if a model with a much larger number of parameters produces only a slight improvement in likelihood, the values of AIC for the two models will suggest that the more parsimonious (limited) model is still the “better” model.

As an example, suppose a researcher would like to determine the appropriate variance-covariance structure for a longitudinal model with four equal time points. The researcher uses REML as the likelihood type. The analysis is run five times, each with a different covariance pattern, and the AIC values are recorded as follows.

Pattern	Number of Parameters	-2 log-likelihood	AIC
Diagonal	1	214.43	216.43
Compound Symmetry	2	210.77	214.77
AR(1)	2	203.52	207.52
Toeplitz	4	198.03	206.03
Unstructured	7	197.94	211.94

The recommended variance-covariance structure among these five is the Toeplitz pattern, since it results in the smallest AIC value.

What to Do When You Encounter a Variance Estimate that is Equal to Zero

It is possible that a mixed models data analysis results in a variance component estimate that is negative or equal to zero. This is particularly true in the case of random coefficients models. When this happens, the component that has a variance estimate equal to zero should be removed from the random factors model statement (or, if possible, the repeated pattern should be simplified to ‘diagonal’), and the analysis should be rerun.

Fixed Effects

A fixed effect (or factor) is a variable for which levels in the study represent all levels of interest, or at least all levels that are important for inference (e.g., treatment, dose, etc.). The fixed effects in the model include those factor for which means, standard errors, and confidence intervals will be estimated and tests of hypotheses will be performed. Other variables for which the model is to be adjusted (that are not important for estimation or hypothesis testing) may also be included in the model as fixed factors. Fixed factors may be discrete variables or continuous covariates.

The correct model for fixed effects depends on the number of fixed factors, the questions to be answered by the analysis, and the amount of data available for the analysis. When more than one fixed factor may influence the response, it is common to include those factors in the model, along with their interactions (two-way, three-way, etc.). Difficulties arise when there are not sufficient data to model the higher-order interactions. In this case, some interactions must be omitted from the model. It is usually suggested that if you include an interaction in the model, you should also include the main effects (i.e. individual factors) involved in the interaction even if the hypothesis test for the main effects is not significant.

Covariates

Covariates are continuous measurements that may not be of primary interest in the study, but potentially have an influence on the response. Two types of covariates typically arise in mixed models designs: subject covariates and within-subject covariates.

Time as a Fixed Effects Factor vs. Time as a Covariate

Time is an essential measurement in many mixed model designs. In some analyses, time may be considered a fixed factor, while in others it is covariate. In random coefficient models, time is considered to be a covariate.

Multiple Comparisons of Fixed Effect Levels

If there is evidence that a fixed factor of a mixed model has difference responses among its levels, it is usually of interest to perform post-hoc pair-wise comparisons of the least-squares means to further clarify those differences. It is well-known that p-value adjustments need to be made when multiple tests are performed (see Hochberg and Tamhane, 1987, or Hsu, 1996, for general discussion and details of the need for multiplicity adjustment). Such adjustments are usually made to preserve the family-wise error rate (FWER), also called the experiment-wise error rate, of the group of tests. FWER is the probability of incorrectly rejecting at least one of the pair-wise tests.

We refer you to the Mixed Models chapter for more details on multiple comparisons.

Specifying the Within-Subjects Variance-Covariance Matrix

The \mathbf{R} Matrix

The \mathbf{R} matrix is the variance-covariance matrix for errors, $\boldsymbol{\varepsilon}$. When the \mathbf{R} matrix is used to specify the variance-covariance structure of \mathbf{y} , the \mathbf{G}_{sub} matrix is not used.

The full \mathbf{R} matrix is made up of N symmetric \mathbf{R} sub-matrices,

$$\mathbf{R} = \begin{pmatrix} \mathbf{R}_1 & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_2 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{R}_3 & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{R}_N \end{pmatrix}$$

where $\mathbf{R}_1, \mathbf{R}_2, \mathbf{R}_3, \dots, \mathbf{R}_N$ are all of the same structure, but, unlike the \mathbf{G}_{sub} matrices, differ according to the number of repeated measurements on each subject.

When the \mathbf{R} matrix is specified in *NCSS*, it is assumed that there is a fixed, known set of repeated measurement times. (If the repeated measurement times are random, specification of the \mathbf{G}_{sub} matrix with $\mathbf{R} = \sigma^2 \mathbf{I}$ should be used instead for specifying covariance structure.) Thus, the differences in the dimensions of the \mathbf{R} sub-matrices occur only when some measurements for a subject are missing.

As an example, suppose an \mathbf{R} sub-matrix is of the form

$$\mathbf{R}_{Sub} = \begin{pmatrix} \sigma_1^2 & & & & \\ & \sigma_2^2 & & & \\ & & \sigma_3^2 & & \\ & & & \sigma_4^2 & \\ & & & & \sigma_5^2 \end{pmatrix},$$

where there are five time points at which each subject is intended to be measured: 1 hour, 2 hours, 5 hours, 10 hours, and 24 hours. If the first subject has measurements at all five time points, then $n_1 = 5$, and the sub-matrix is identical to \mathbf{R}_{Sub} above, and $\mathbf{R}_1 = \mathbf{R}_{Sub}$.

Suppose the second subject is measured at 1 hour, 5 hours, and 24 hours, but misses the 2-hour and 10-hour measurements. The \mathbf{R}_2 matrix for this subject is

$$\mathbf{R}_2 = \begin{pmatrix} \sigma_1^2 & & \\ & \sigma_3^2 & \\ & & \sigma_5^2 \end{pmatrix}.$$

For this subject, $n_2 = 3$. That is, for the case when the time points are fixed, instead of having missing values in the \mathbf{R} sub-matrices, the matrix is collapsed to accommodate the number of realized measurements.

Structures of \mathbf{R}

There are many possible structures for the sub-matrices that make up the \mathbf{R} matrix. The \mathbf{R}_{Sub} structures that can be specified in *NCSS* are shown below.

Mixed Models – Random Coefficients

Diagonal

Homogeneous

$$\begin{pmatrix} \sigma^2 & & & \\ & \sigma^2 & & \\ & & \sigma^2 & \\ & & & \sigma^2 \end{pmatrix}$$

Heterogeneous

$$\begin{pmatrix} \sigma_1^2 & & & \\ & \sigma_2^2 & & \\ & & \sigma_3^2 & \\ & & & \sigma_4^2 \end{pmatrix}$$

Correlation

$$\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

Compound Symmetry

Homogeneous

$$\begin{pmatrix} \sigma^2 & \rho\sigma^2 & \rho\sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 & \rho\sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \rho\sigma^2 & \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \rho\sigma^2 & \rho\sigma^2 & \sigma^2 \end{pmatrix}$$

Heterogeneous

$$\begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 & \rho\sigma_1\sigma_3 & \rho\sigma_1\sigma_4 \\ \rho\sigma_2\sigma_1 & \sigma_2^2 & \rho\sigma_2\sigma_3 & \rho\sigma_2\sigma_4 \\ \rho\sigma_3\sigma_1 & \rho\sigma_3\sigma_2 & \sigma_3^2 & \rho\sigma_3\sigma_4 \\ \rho\sigma_4\sigma_1 & \rho\sigma_4\sigma_2 & \rho\sigma_4\sigma_3 & \sigma_4^2 \end{pmatrix}$$

Correlation

$$\begin{pmatrix} 1 & \rho & \rho & \rho \\ \rho & 1 & \rho & \rho \\ \rho & \rho & 1 & \rho \\ \rho & \rho & \rho & 1 \end{pmatrix}$$

AR(1)

Homogeneous

$$\begin{pmatrix} \sigma^2 & \rho\sigma^2 & \rho^2\sigma^2 & \rho^3\sigma^2 \\ \rho\sigma^2 & \sigma^2 & \rho\sigma^2 & \rho^2\sigma^2 \\ \rho^2\sigma^2 & \rho\sigma^2 & \sigma^2 & \rho\sigma^2 \\ \rho^3\sigma^2 & \rho^2\sigma^2 & \rho\sigma^2 & \sigma^2 \end{pmatrix}$$

Heterogeneous

$$\begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 & \rho^2\sigma_1\sigma_3 & \rho^3\sigma_1\sigma_4 \\ \rho\sigma_2\sigma_1 & \sigma_2^2 & \rho\sigma_2\sigma_3 & \rho^2\sigma_2\sigma_4 \\ \rho^2\sigma_3\sigma_1 & \rho\sigma_3\sigma_2 & \sigma_3^2 & \rho\sigma_3\sigma_4 \\ \rho^3\sigma_4\sigma_1 & \rho^2\sigma_4\sigma_2 & \rho\sigma_4\sigma_3 & \sigma_4^2 \end{pmatrix}$$

Correlation

$$\begin{pmatrix} 1 & \rho & \rho^2 & \rho^3 \\ \rho & 1 & \rho & \rho^2 \\ \rho^2 & \rho & 1 & \rho \\ \rho^3 & \rho^2 & \rho & 1 \end{pmatrix}$$

Toeplitz

Homogeneous

$$\begin{pmatrix} \sigma^2 & \rho_1\sigma^2 & \rho_2\sigma^2 & \rho_3\sigma^2 \\ \rho_1\sigma^2 & \sigma^2 & \rho_1\sigma^2 & \rho_2\sigma^2 \\ \rho_2\sigma^2 & \rho_1\sigma^2 & \sigma^2 & \rho_1\sigma^2 \\ \rho_3\sigma^2 & \rho_2\sigma^2 & \rho_1\sigma^2 & \sigma^2 \end{pmatrix}$$

Heterogeneous

$$\begin{pmatrix} \sigma_1^2 & \rho_1\sigma_1\sigma_2 & \rho_2\sigma_1\sigma_3 & \rho_3\sigma_1\sigma_4 \\ \rho_1\sigma_2\sigma_1 & \sigma_2^2 & \rho_1\sigma_2\sigma_3 & \rho_2\sigma_2\sigma_4 \\ \rho_2\sigma_3\sigma_1 & \rho_1\sigma_3\sigma_2 & \sigma_3^2 & \rho_1\sigma_3\sigma_4 \\ \rho_3\sigma_4\sigma_1 & \rho_2\sigma_4\sigma_2 & \rho_1\sigma_4\sigma_3 & \sigma_4^2 \end{pmatrix}$$

Correlation

$$\begin{pmatrix} 1 & \rho_1 & \rho_2 & \rho_3 \\ \rho_1 & 1 & \rho_1 & \rho_2 \\ \rho_2 & \rho_1 & 1 & \rho_1 \\ \rho_3 & \rho_2 & \rho_1 & 1 \end{pmatrix}$$

Mixed Models – Random Coefficients

Toeplitz(2)

Homogeneous

$$\begin{pmatrix} \sigma^2 & \rho_1\sigma^2 & & & \\ \rho_1\sigma^2 & \sigma^2 & \rho_1\sigma^2 & & \\ & \rho_1\sigma^2 & \sigma^2 & \rho_1\sigma^2 & \\ & & \rho_1\sigma^2 & \sigma^2 & \\ & & & \rho_1\sigma^2 & \sigma^2 \end{pmatrix}$$

Heterogeneous

$$\begin{pmatrix} \sigma_1^2 & \rho_1\sigma_1\sigma_2 & & & \\ \rho_1\sigma_2\sigma_1 & \sigma_2^2 & \rho_1\sigma_2\sigma_3 & & \\ & \rho_1\sigma_3\sigma_2 & \sigma_3^2 & \rho_1\sigma_3\sigma_4 & \\ & & \rho_1\sigma_4\sigma_3 & \sigma_4^2 & \\ & & & \rho_1\sigma_4\sigma_3 & \sigma_4^2 \end{pmatrix}$$

Correlation

$$\begin{pmatrix} 1 & \rho_1 & & & \\ \rho_1 & 1 & \rho_1 & & \\ & \rho_1 & 1 & \rho_1 & \\ & & \rho_1 & 1 & \\ & & & \rho_1 & 1 \end{pmatrix}$$

Note: This is the same as Banded(2).

Toeplitz(3)

Homogeneous

$$\begin{pmatrix} \sigma^2 & \rho_1\sigma^2 & \rho_2\sigma^2 & & & \\ \rho_1\sigma^2 & \sigma^2 & \rho_1\sigma^2 & \rho_2\sigma^2 & & \\ \rho_2\sigma^2 & \rho_1\sigma^2 & \sigma^2 & \rho_1\sigma^2 & & \\ & \rho_2\sigma^2 & \rho_1\sigma^2 & \sigma^2 & & \\ & & \rho_2\sigma^2 & \rho_1\sigma^2 & \sigma^2 & \\ & & & \rho_2\sigma^2 & \rho_1\sigma^2 & \sigma^2 \end{pmatrix}$$

Heterogeneous

$$\begin{pmatrix} \sigma_1^2 & \rho_1\sigma_1\sigma_2 & \rho_2\sigma_1\sigma_3 & & & \\ \rho_1\sigma_2\sigma_1 & \sigma_2^2 & \rho_1\sigma_2\sigma_3 & \rho_2\sigma_2\sigma_4 & & \\ \rho_2\sigma_3\sigma_1 & \rho_1\sigma_3\sigma_2 & \sigma_3^2 & \rho_1\sigma_3\sigma_4 & & \\ & \rho_2\sigma_4\sigma_2 & \rho_1\sigma_4\sigma_3 & \sigma_4^2 & & \\ & & \rho_2\sigma_4\sigma_2 & \rho_1\sigma_4\sigma_3 & \sigma_4^2 & \\ & & & \rho_2\sigma_4\sigma_2 & \rho_1\sigma_4\sigma_3 & \sigma_4^2 \end{pmatrix}$$

Correlation

$$\begin{pmatrix} 1 & \rho_1 & \rho_2 & & & \\ \rho_1 & 1 & \rho_1 & \rho_2 & & \\ \rho_2 & \rho_1 & 1 & \rho_1 & & \\ & \rho_2 & \rho_1 & 1 & & \\ & & \rho_2 & \rho_1 & 1 & \\ & & & \rho_2 & \rho_1 & 1 \end{pmatrix}$$

Toeplitz(4) and Toeplitz(5)

Toeplitz(4) and Toeplitz(5) follow the same pattern as Toeplitz(2) and Toeplitz(3), but with the corresponding numbers of bands.

Banded(2)

Homogeneous

$$\begin{pmatrix} \sigma^2 & \rho\sigma^2 & & & & \\ \rho\sigma^2 & \sigma^2 & \rho\sigma^2 & & & \\ & \rho\sigma^2 & \sigma^2 & \rho\sigma^2 & & \\ & & \rho\sigma^2 & \sigma^2 & \rho\sigma^2 & \\ & & & \rho\sigma^2 & \sigma^2 & \\ & & & & \rho\sigma^2 & \sigma^2 \end{pmatrix}$$

Heterogeneous

$$\begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 & & & & \\ \rho\sigma_2\sigma_1 & \sigma_2^2 & \rho\sigma_2\sigma_3 & & & \\ & \rho\sigma_3\sigma_2 & \sigma_3^2 & \rho\sigma_3\sigma_4 & & \\ & & \rho\sigma_4\sigma_3 & \sigma_4^2 & & \\ & & & \rho\sigma_4\sigma_3 & \sigma_4^2 & \\ & & & & \rho\sigma_4\sigma_3 & \sigma_4^2 \end{pmatrix}$$

Correlation

$$\begin{pmatrix} 1 & \rho & & & & \\ \rho & 1 & \rho & & & \\ & \rho & 1 & \rho & & \\ & & \rho & 1 & \rho & \\ & & & \rho & 1 & \\ & & & & \rho & 1 \end{pmatrix}$$

Note: This is the same as Toeplitz(1).

Mixed Models – Random Coefficients

Banded(3)

Homogeneous

$$\begin{pmatrix} \sigma^2 & \rho\sigma^2 & \rho\sigma^2 & \\ \rho\sigma^2 & \sigma^2 & \rho\sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \rho\sigma^2 & \sigma^2 & \rho\sigma^2 \\ & \rho\sigma^2 & \rho\sigma^2 & \sigma^2 \end{pmatrix}$$

Heterogeneous

$$\begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 & \rho\sigma_1\sigma_3 & \\ \rho\sigma_2\sigma_1 & \sigma_2^2 & \rho\sigma_2\sigma_3 & \rho\sigma_2\sigma_4 \\ \rho\sigma_3\sigma_1 & \rho\sigma_3\sigma_2 & \sigma_3^2 & \rho\sigma_3\sigma_4 \\ & \rho\sigma_4\sigma_2 & \rho\sigma_4\sigma_3 & \sigma_4^2 \end{pmatrix}$$

Correlation

$$\begin{pmatrix} 1 & \rho & \rho & \\ \rho & 1 & \rho & \rho \\ \rho & \rho & 1 & \rho \\ & \rho & \rho & 1 \end{pmatrix}$$

Banded(4) and Banded (5)

Banded(4) and Banded(5) follow the same pattern as Banded(2) and Banded(3), but with the corresponding numbers of bands.

Unstructured

Homogeneous

$$\begin{pmatrix} \sigma^2 & \rho_{12}\sigma^2 & \rho_{13}\sigma^2 & \rho_{14}\sigma^2 \\ \rho_{21}\sigma^2 & \sigma^2 & \rho_{23}\sigma^2 & \rho_{24}\sigma^2 \\ \rho_{31}\sigma^2 & \rho_{32}\sigma^2 & \sigma^2 & \rho_{34}\sigma^2 \\ \rho_{41}\sigma^2 & \rho_{42}\sigma^2 & \rho_{43}\sigma^2 & \sigma^2 \end{pmatrix}$$

Heterogeneous

$$\begin{pmatrix} \sigma_1^2 & \rho_{12}\sigma_1\sigma_2 & \rho_{13}\sigma_1\sigma_3 & \rho_{14}\sigma_1\sigma_4 \\ \rho_{21}\sigma_2\sigma_1 & \sigma_2^2 & \rho_{23}\sigma_2\sigma_3 & \rho_{24}\sigma_2\sigma_4 \\ \rho_{31}\sigma_3\sigma_1 & \rho_{32}\sigma_3\sigma_2 & \sigma_3^2 & \rho_{34}\sigma_3\sigma_4 \\ \rho_{41}\sigma_4\sigma_1 & \rho_{42}\sigma_4\sigma_2 & \rho_{43}\sigma_4\sigma_3 & \sigma_4^2 \end{pmatrix}$$

Correlation

$$\begin{pmatrix} 1 & \rho_{12} & \rho_{13} & \rho_{14} \\ \rho_{21} & 1 & \rho_{23} & \rho_{24} \\ \rho_{31} & \rho_{32} & 1 & \rho_{34} \\ \rho_{41} & \rho_{42} & \rho_{43} & 1 \end{pmatrix}$$

Partitioning the Variance-Covariance Structure with Groups

In the case where it is expected that the variance-covariance parameters are different across group levels of the data, it may be useful to specify a different set of **R** parameters for each level of a group variable. This produces a set of variance-covariance parameters that is different for each level of the chosen group variable, but each set has the same structure as the other groups.

Partitioning the R Matrix Parameters

Suppose the structure of **R** in a study with four time points is specified to be Toeplitz:

$$\mathbf{R} = \begin{pmatrix} \sigma^2 & \rho_1\sigma^2 & \rho_2\sigma^2 & \rho_3\sigma^2 \\ \rho_1\sigma^2 & \sigma^2 & \rho_1\sigma^2 & \rho_2\sigma^2 \\ \rho_2\sigma^2 & \rho_1\sigma^2 & \sigma^2 & \rho_1\sigma^2 \\ \rho_3\sigma^2 & \rho_2\sigma^2 & \rho_1\sigma^2 & \sigma^2 \end{pmatrix}.$$

Mixed Models – Random Coefficients

If there are sixteen subjects then

$$\mathbf{R} = \begin{pmatrix} \mathbf{R}_1 & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_2 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{R}_3 & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{R}_{16} \end{pmatrix}.$$

The total number of variance-covariance parameters is four: σ^2 , ρ_1 , ρ_2 , and ρ_3 .

Suppose now that there are two groups of eight subjects, and it is believed that the four variance parameters of the first group are different from the four variance parameters of the second group.

We now have

$$\mathbf{R}_1, \dots, \mathbf{R}_8 = \begin{pmatrix} \sigma_1^2 & \rho_{11}\sigma^2 & \rho_{12}\sigma^2 & \rho_{13}\sigma^2 \\ \rho_{11}\sigma^2 & \sigma_1^2 & \rho_{11}\sigma^2 & \rho_{12}\sigma^2 \\ \rho_{12}\sigma^2 & \rho_{11}\sigma^2 & \sigma_1^2 & \rho_{11}\sigma^2 \\ \rho_{13}\sigma^2 & \rho_{12}\sigma^2 & \rho_{11}\sigma^2 & \sigma_1^2 \end{pmatrix},$$

and

$$\mathbf{R}_9, \dots, \mathbf{R}_{16} = \begin{pmatrix} \sigma_2^2 & \rho_{21}\sigma^2 & \rho_{22}\sigma^2 & \rho_{23}\sigma^2 \\ \rho_{21}\sigma^2 & \sigma_2^2 & \rho_{21}\sigma^2 & \rho_{22}\sigma^2 \\ \rho_{22}\sigma^2 & \rho_{21}\sigma^2 & \sigma_2^2 & \rho_{21}\sigma^2 \\ \rho_{23}\sigma^2 & \rho_{22}\sigma^2 & \rho_{21}\sigma^2 & \sigma_2^2 \end{pmatrix}.$$

The total number of variance-covariance parameters is now eight.

It is easy to see how quickly the number of variance-covariance parameters increases when \mathbf{R} or \mathbf{G} is partitioned by groups.

Procedure Options

This section describes the options available in this procedure.

Variables

Response Variable

This variable contains the numeric responses (measurements) for each of the subjects. There is one measurement per subject per time point. Hence, all responses are in a single column (variable) of the spreadsheet.

Subject Variable

This variable contains an identification value for each subject. Each subject must have a unique identification number (or name). In a random coefficient design, several measurements are made on each subject.

Regressions

This option specifies the polynomial order of the regression fit for each subject. Usually, linear regression is used, but quadratic and cubic regressions are also possible.

Mixed Models – Random Coefficients

Covariances

If this box is checked, the G-matrix (covariance matrix) will include the covariance of each pair of variance components (diagonal element of the G-matrix). If the box is not checked, all off-diagonal elements will be set to zero (the G-matrix will be diagonal).

This option is commonly checked when fitting a random coefficient model.

Between and Within Fixed Factors

This section lets you specify all fixed factors whether they are between or within.

Number

Enter the number of factors (up to 6) that you want to use. This option controls how many factor variable entry boxes are displayed and used.

Note that if you select factor variables in the boxes below, and then reduce this number so those boxes are no longer visible, the hidden factors will not be used.

Fixed Factor Variables

Select a fixed factor (categorical or class) variable here. Capitalization is ignored when determining unique text values.

A categorical variable has only a few unique values (text or numeric) which are used to identify the categories (groups) into which the subject falls.

$\neq\sigma^2$ (Unequal Group Variance)

One factor variable can have a different variance in each group. Check this box to indicate that this factor should have unequal variances. Other factors will have equal variances.

This panel is used to specify multiple comparisons or custom contrasts for factor variables.

Comparison

This option specifies the set of multiple comparisons that will be computed for this factor. Several predefined sets are available or you can specify up to two of your own in the Custom (1-2) options.

For interactions, these comparisons are run for each category of the second factor.

Possible choices are:

- **First versus Each**

The multiple comparisons are each category tested against the first category. This option would be used when the first category is the control (standard) category. Note: the first is determined alphabetically.

- **2nd versus Each**

The multiple comparisons are each category tested against the second category. This option would be used when the second category is the control (standard) category.

- **3rd versus Each**

The multiple comparisons are each category tested against the third category. This option would be used when the third category is the control (standard) category.

- **Last versus Each**

The multiple comparisons are each category tested against the last category. This option would be used when the last category is the control (standard) category.

Mixed Models – Random Coefficients

- **Baseline versus Each**

The multiple comparisons are each category tested against the baseline category. This option would be used when the baseline category is the control (standard) category. The baseline category is entered to the right.

- **Ave versus Each**

The multiple comparisons are each category tested against the average of the other categories.

- **All Pairs**

The multiple comparisons are each category tested against every other category.

- **Custom**

The multiple comparisons are determined by the coefficients entered in the two Custom boxes to the right.

Baseline

Enter the level of the corresponding Factor Variable to which comparisons will be made. The Baseline is used only when Comparison is set to *Baseline vs Each*. The value entered here must be one of the levels of the Factor Variable. The entry is not case sensitive and values should not be entered with quotes.

Custom 1-2

This option specifies the weights of a comparison. It is used when the *Comparison* is set to *Custom*. There are no numerical restrictions on these coefficients. They do not even have to sum to zero. However, this is recommended. If the coefficients do sum to zero, the comparison is called a CONTRAST. The significance tests anticipate that only one or two of these comparisons are run. If you run several, you should make some type of Bonferroni adjustment to your alpha value.

Specifying the Coefficients

When you put in your own contrasts, you must be careful that you specify the appropriate number of coefficients. For example, if the factor has four levels, four coefficients must be specified, separated by blanks or commas. Extra coefficients are ignored. If too few coefficients are specified, the missing coefficients are assumed to be zero.

These comparison coefficients designate weighted averages of the level-means that are to be statistically tested. The null hypothesis is that the weighted average is zero. The alternative hypothesis is that the weighted average is nonzero. The coefficients are specified here in this box.

As an example, suppose you want to compare the average of the first two levels with the average of the last two levels in a six-level factor. You would enter -1 -1 0 0 1 1.

As a second example, suppose you want to compare the average of the first two levels with the average of the last three levels in a six-level factor. The custom contrast would be -3 -3 0 2 2 2.

Note that in each example, coefficients were used that sum to zero. Ones were not used in the second example because the result would not sum to zero.

Covariates

Number

Enter the number of covariates (up to six) that you want to use. This option controls how many covariate variable entry boxes are displayed and used. Since you have to specify a variable containing a time covariate, the minimum number of covariates is one.

Note that if you select covariate variables in the boxes below, and then reduce this number so those boxes are no longer visible, the hidden covariates will not be used.

Mixed Models – Random Coefficients

Covariate Variables

Specify each covariate variable here. The values in this variable must be numeric and should be at least ordinal. The first covariate becomes the time variable. It indicates the time values.

We have found it useful to scale the time values between -10 and 10. Doing this will avoid serious rounding errors that may occur otherwise.

Compute Means at these Values

Specify one or more values at which means and comparisons are to be calculated. A separate report is calculated for each unique set of covariate values.

Mean

Enter **Mean** to indicate that the covariate mean should be used as the value of the covariate in the various reports.

Within-Subject Variance-Covariance Matrix

The repeated component is used to specify the **R** matrix in the mixed model. Usually, the diagonal pattern is used.

Pattern

Specify the type of **R** (error covariance) matrix to be generated. This represents the relationship between observations from the same subject. The **R** structures that can be specified in *NCSS* are shown below. The usual type is the 'Diagonal' matrix.

The options are:

- Diagonal**

Homogeneous

$$\begin{pmatrix} \sigma^2 & & & \\ & \sigma^2 & & \\ & & \sigma^2 & \\ & & & \sigma^2 \end{pmatrix}$$

Heterogeneous

$$\begin{pmatrix} \sigma_1^2 & & & \\ & \sigma_2^2 & & \\ & & \sigma_3^2 & \\ & & & \sigma_4^2 \end{pmatrix}$$

- Compound Symmetry**

Homogeneous

$$\begin{pmatrix} \sigma^2 & \rho\sigma^2 & \rho\sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 & \rho\sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \rho\sigma^2 & \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \rho\sigma^2 & \rho\sigma^2 & \sigma^2 \end{pmatrix}$$

Heterogeneous

$$\begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 & \rho\sigma_1\sigma_3 & \rho\sigma_1\sigma_4 \\ \rho\sigma_2\sigma_1 & \sigma_2^2 & \rho\sigma_2\sigma_3 & \rho\sigma_2\sigma_4 \\ \rho\sigma_3\sigma_1 & \rho\sigma_3\sigma_2 & \sigma_3^2 & \rho\sigma_3\sigma_4 \\ \rho\sigma_4\sigma_1 & \rho\sigma_4\sigma_2 & \rho\sigma_4\sigma_3 & \sigma_4^2 \end{pmatrix}$$

- AR(1)**

Homogeneous

$$\begin{pmatrix} \sigma^2 & \rho\sigma^2 & \rho^2\sigma^2 & \rho^3\sigma^2 \\ \rho\sigma^2 & \sigma^2 & \rho\sigma^2 & \rho^2\sigma^2 \\ \rho^2\sigma^2 & \rho\sigma^2 & \sigma^2 & \rho\sigma^2 \\ \rho^3\sigma^2 & \rho^2\sigma^2 & \rho\sigma^2 & \sigma^2 \end{pmatrix}$$

Heterogeneous

$$\begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 & \rho^2\sigma_1\sigma_3 & \rho^3\sigma_1\sigma_4 \\ \rho\sigma_2\sigma_1 & \sigma_2^2 & \rho\sigma_2\sigma_3 & \rho^2\sigma_2\sigma_4 \\ \rho^2\sigma_3\sigma_1 & \rho\sigma_3\sigma_2 & \sigma_3^2 & \rho\sigma_3\sigma_4 \\ \rho^3\sigma_4\sigma_1 & \rho^2\sigma_4\sigma_2 & \rho\sigma_4\sigma_3 & \sigma_4^2 \end{pmatrix}$$

Mixed Models – Random Coefficients

- **AR(Time Diff)**

Homogeneous

$$\begin{pmatrix} \sigma^2 & \rho^{t_2-t_1}\sigma^2 & \rho^{t_3-t_1}\sigma^2 & \rho^{t_4-t_1}\sigma^2 \\ \rho^{t_2-t_1}\sigma^2 & \sigma^2 & \rho^{t_3-t_2}\sigma^2 & \rho^{t_4-t_2}\sigma^2 \\ \rho^{t_3-t_1}\sigma^2 & \rho^{t_3-t_2}\sigma^2 & \sigma^2 & \rho^{t_4-t_3}\sigma^2 \\ \rho^{t_4-t_1}\sigma^2 & \rho^{t_4-t_2}\sigma^2 & \rho^{t_4-t_3}\sigma^2 & \sigma^2 \end{pmatrix}$$

Heterogeneous

$$\begin{pmatrix} \sigma_1^2 & \rho^{t_2-t_1}\sigma_1\sigma_2 & \rho^{t_3-t_1}\sigma_1\sigma_3 & \rho^{t_4-t_1}\sigma_1\sigma_4 \\ \rho^{t_2-t_1}\sigma_2\sigma_1 & \sigma_2^2 & \rho^{t_3-t_2}\sigma_2\sigma_3 & \rho^{t_4-t_2}\sigma_2\sigma_4 \\ \rho^{t_3-t_1}\sigma_3\sigma_1 & \rho^{t_3-t_2}\sigma_3\sigma_2 & \sigma_3^2 & \rho^{t_4-t_3}\sigma_3\sigma_4 \\ \rho^{t_4-t_1}\sigma_4\sigma_1 & \rho^{t_4-t_2}\sigma_4\sigma_2 & \rho^{t_4-t_3}\sigma_4\sigma_3 & \sigma_4^2 \end{pmatrix}$$

- **Toeplitz (All)**

Homogeneous

$$\begin{pmatrix} \sigma^2 & \rho_1\sigma^2 & \rho_2\sigma^2 & \rho_3\sigma^2 \\ \rho_1\sigma^2 & \sigma^2 & \rho_1\sigma^2 & \rho_2\sigma^2 \\ \rho_2\sigma^2 & \rho_1\sigma^2 & \sigma^2 & \rho_1\sigma^2 \\ \rho_3\sigma^2 & \rho_2\sigma^2 & \rho_1\sigma^2 & \sigma^2 \end{pmatrix}$$

Heterogeneous

$$\begin{pmatrix} \sigma_1^2 & \rho_1\sigma_1\sigma_2 & \rho_2\sigma_1\sigma_3 & \rho_3\sigma_1\sigma_4 \\ \rho_1\sigma_2\sigma_1 & \sigma_2^2 & \rho_1\sigma_2\sigma_3 & \rho_2\sigma_2\sigma_4 \\ \rho_2\sigma_3\sigma_1 & \rho_1\sigma_3\sigma_2 & \sigma_3^2 & \rho_1\sigma_3\sigma_4 \\ \rho_3\sigma_4\sigma_1 & \rho_2\sigma_4\sigma_2 & \rho_1\sigma_4\sigma_3 & \sigma_4^2 \end{pmatrix}$$

- **Toeplitz(2)**

Homogeneous

$$\begin{pmatrix} \sigma^2 & \rho_1\sigma^2 & & \\ \rho_1\sigma^2 & \sigma^2 & \rho_1\sigma^2 & \\ & \rho_1\sigma^2 & \sigma^2 & \rho_1\sigma^2 \\ & & \rho_1\sigma^2 & \sigma^2 \end{pmatrix}$$

Heterogeneous

$$\begin{pmatrix} \sigma_1^2 & \rho_1\sigma_1\sigma_2 & & \\ \rho_1\sigma_2\sigma_1 & \sigma_2^2 & \rho_1\sigma_2\sigma_3 & \\ & \rho_1\sigma_3\sigma_2 & \sigma_3^2 & \rho_1\sigma_3\sigma_4 \\ & & \rho_1\sigma_4\sigma_3 & \sigma_4^2 \end{pmatrix}$$

Note: This is the same as Banded(2).

- **Toeplitz(3)**

Homogeneous

$$\begin{pmatrix} \sigma^2 & \rho_1\sigma^2 & \rho_2\sigma^2 & \\ \rho_1\sigma^2 & \sigma^2 & \rho_1\sigma^2 & \rho_2\sigma^2 \\ \rho_2\sigma^2 & \rho_1\sigma^2 & \sigma^2 & \rho_1\sigma^2 \\ & \rho_2\sigma^2 & \rho_1\sigma^2 & \sigma^2 \end{pmatrix}$$

Heterogeneous

$$\begin{pmatrix} \sigma_1^2 & \rho_1\sigma_1\sigma_2 & \rho_2\sigma_1\sigma_3 & \\ \rho_1\sigma_2\sigma_1 & \sigma_2^2 & \rho_1\sigma_2\sigma_3 & \rho_2\sigma_2\sigma_4 \\ \rho_2\sigma_3\sigma_1 & \rho_1\sigma_3\sigma_2 & \sigma_3^2 & \rho_1\sigma_3\sigma_4 \\ & \rho_2\sigma_4\sigma_2 & \rho_1\sigma_4\sigma_3 & \sigma_4^2 \end{pmatrix}$$

- **Toeplitz(4) and Toeplitz(5)**

Toeplitz(4) and Toeplitz(5) follow the same pattern as Toeplitz(2) and Toeplitz(3), but with the corresponding numbers of bands.

Mixed Models – Random Coefficients

- **Banded(2)**

Homogeneous

$$\begin{pmatrix} \sigma^2 & \rho\sigma^2 & & \\ \rho\sigma^2 & \sigma^2 & \rho\sigma^2 & \\ & \rho\sigma^2 & \sigma^2 & \rho\sigma^2 \\ & & \rho\sigma^2 & \sigma^2 \end{pmatrix}$$

Heterogeneous

$$\begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 & & \\ \rho\sigma_2\sigma_1 & \sigma_2^2 & \rho\sigma_2\sigma_3 & \\ & \rho\sigma_3\sigma_2 & \sigma_3^2 & \rho\sigma_3\sigma_4 \\ & & \rho\sigma_4\sigma_3 & \sigma_4^2 \end{pmatrix}$$

Note: This is the same as Toeplitz(1).

- **Banded(3)**

Homogeneous

$$\begin{pmatrix} \sigma^2 & \rho\sigma^2 & \rho\sigma^2 & \\ \rho\sigma^2 & \sigma^2 & \rho\sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \rho\sigma^2 & \sigma^2 & \rho\sigma^2 \\ & \rho\sigma^2 & \rho\sigma^2 & \sigma^2 \end{pmatrix}$$

Heterogeneous

$$\begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 & \rho\sigma_1\sigma_3 & \\ \rho\sigma_2\sigma_1 & \sigma_2^2 & \rho\sigma_2\sigma_3 & \rho\sigma_2\sigma_4 \\ \rho\sigma_3\sigma_1 & \rho\sigma_3\sigma_2 & \sigma_3^2 & \rho\sigma_3\sigma_4 \\ & \rho\sigma_4\sigma_2 & \rho\sigma_4\sigma_3 & \sigma_4^2 \end{pmatrix}$$

- **Banded(4) and Banded (5)**

Banded(4) and Banded(5) follow the same pattern as Banded(2) and Banded(3), but with the corresponding numbers of bands.

- **Unstructured**

Homogeneous

$$\begin{pmatrix} \sigma^2 & \rho_{12}\sigma^2 & \rho_{13}\sigma^2 & \rho_{14}\sigma^2 \\ \rho_{21}\sigma^2 & \sigma^2 & \rho_{23}\sigma^2 & \rho_{24}\sigma^2 \\ \rho_{31}\sigma^2 & \rho_{32}\sigma^2 & \sigma^2 & \rho_{34}\sigma^2 \\ \rho_{41}\sigma^2 & \rho_{42}\sigma^2 & \rho_{43}\sigma^2 & \sigma^2 \end{pmatrix}$$

Heterogeneous

$$\begin{pmatrix} \sigma_1^2 & \rho_{12}\sigma_1\sigma_2 & \rho_{13}\sigma_1\sigma_3 & \rho_{14}\sigma_1\sigma_4 \\ \rho_{21}\sigma_2\sigma_1 & \sigma_2^2 & \rho_{23}\sigma_2\sigma_3 & \rho_{24}\sigma_2\sigma_4 \\ \rho_{31}\sigma_3\sigma_1 & \rho_{32}\sigma_3\sigma_2 & \sigma_3^2 & \rho_{34}\sigma_3\sigma_4 \\ \rho_{41}\sigma_4\sigma_1 & \rho_{42}\sigma_4\sigma_2 & \rho_{43}\sigma_4\sigma_3 & \sigma_4^2 \end{pmatrix}$$

Force Positive Correlations

When checked, this option forces all covariances (off-diagonal elements of the R matrix) to be non-negative. When this option is not checked, covariances can be negative.

Usually, negative covariances are okay and should be allowed. However, some Repeated patterns such as *Compound Symmetry* assume that covariances (correlations) are positive. Checking this box will help those runs to converge.

Model (Fixed Terms)

Terms

This option specifies which terms (terms, powers, cross-products, and interactions) are included in the fixed portion of the mixed model.

The options are

- **1-Way**

All covariates and factors are included in the model. No interaction or power terms are included. Use this option when you just want to use the variables you have specified.

Mixed Models – Random Coefficients

- **Up to 2-Way**

All individual variables, two-way interactions, cross-products, and squared covariates are included.

For example, if you are analyzing four factors named A, B, C, and D, this option would generate the model as: $A + B + C + D + AB + AC + AD + BC + BD + CD$.

- **Up to 3-Way**

All individual variables, two-way interactions, three-way interactions, squared covariates, cross-products, and cubed covariates are included in the model.

For example, if you are analyzing two covariates called X1 and X2 and a factor named A, this option would generate the model as: $X1 + X2 + C + X1*X2 + X1*C + X2*C + X1*X1 + X2*X2 + X1*X1*C + X2*X2*C + X1*X2*C + X1*X1*X1 + X1*X1*X2 + X1*X2*X2 + X2*X2*X2$.

- **Up to 4-Way**

All individual variables, two-way interactions, three-way interactions, and four-way interactions, along with the squares, cubics, quartics, and cross-products of covariates and their interactions are included in the model.

- **Interaction Model**

All individual variables and their interactions are included. No powers of covariates are included. This requires a dataset in which all combinations of factor variables are present.

- **Custom**

The model specified in the **Custom** box is used.

Maximum Exponent of Covariates

This option specifies the maximum exponent of each covariate in the terms of the model. This maximum is further constrained by the setting of the **Terms** option.

Custom Model

Specify a custom statistical model for fixed effects here. Statistical hypothesis tests will be generated for each term in this model. Variables for which hypothesis tests are to be performed should be included in this model statement. You may also include variables in this model that are solely to be used for adjustment and not important for inference or hypothesis testing. For categorical factors, each term represents a set of indicator variables in the expanded design matrix.

The components of this model come from the variables listed in the factor and covariate variables. If you want to use them, they must be listed there.

Syntax

In the examples that follow each syntax description, A, B, C, and X represent variable names. Assume that A, B, and C are factors, and X is a covariate.

1. Specify main effects by specifying their variable names, separated by blanks or the '+' (plus) sign.

$A+B$ Main effects for A and B only

$A B C$ Main effects for A, B, and C only

$A B X$ Main effects for A and B, plus the covariate effect of X

2. Specify interactions and cross products using an asterisk (*) between variable names, such as Fruit*Nuts or A*B*C. When an interaction between a factor and a covariate is specified, a cross-product is generated for each value of the factor. For covariates, higher order (e.g. squared, cubic) terms may be added by repeating the covariate name. If X is a covariate, X*X represents the covariate squared, and X*X*X represents the covariate cubed, etc. Only covariates can be repeated. Factors cannot be squared or cubed. That is, if A is a factor, you would not include A*A nor A*A*A in your model.

Mixed Models – Random Coefficients

$A+B+A*B$	Main effects for A and B plus the AB interaction
$A+B+C+A*B+A*C+B*C+A*B*C$	Full model for factors A, B, and C
$A+B+C+A*X$	Main effects for A, B, and C plus the interaction of A with the covariate X
$A+X+X*X$	Main Effect for A plus X and the square of X
$A+B*B$	Not valid since B is categorical and cannot be squared

3. Use the '|' (bar) symbol as a shorthand technique for specifying large models quickly.

$$A|B = A+B+A*B$$

$$A|B|C = A+B+C+A*B+A*C+B*C+A*B*C$$

$$A|B C X*X = A+B+A*B+C+X*X$$

$$A|B C|X = A+B+A*B+C+X+C*X$$

4. You can use parentheses for multiplication.

$$(A+B)*(C+X) = A*C+A*X+B*C+B*X$$

$$(A+B)|C = A+B+C+(A+B)*C = A+B+C+A*C+B*C$$

5. Use the '@' (at) symbol to limit the order of interaction and cross-product terms in the model.

$$A|B|C @2 = A+B+C+A*B+A*C+B*C$$

$$A|B|X|X (@2) = A+B+X+A*B+A*X+B*X+X*X$$

Intercept

Check this box to include the intercept in the model. Under most circumstances, you should include the intercept term in your model.

Maximization Tab

This tab controls the Newton-Raphson, Fisher-Scoring, and Differential Evolution likelihood-maximization algorithms.

Options**Likelihood Type**

Specify the type of likelihood equation to be solved. The options are:

- **MLE**

The 'Maximum Likelihood' solution has become less popular.

- **REML (recommended)**

The 'Restricted Maximum Likelihood' solution is recommended. It is the default in other software programs (such as SAS).

Solution Method

Specify the method to be used to solve the likelihood equations. The options are:

- **Newton-Raphson**

This is an implementation of the popular 'gradient search' procedure for maximizing the likelihood equations. Whenever possible, we recommend that you use this method.

Mixed Models – Random Coefficients

- **Fisher-Scoring**

This is an intermediate step in the Newton-Raphson procedure. However, when the Newton-Raphson fails to converge, you may want to stop with this procedure.

- **MIVQUE**

This non-iterative method is used to provide starting values for the Newton-Raphson method. For large problems, you may want to investigate the model using this method since it is much faster.

- **Differential Evolution**

This grid search technique will often find a solution when the other methods fail to converge. However, it is painfully slow--often requiring hours to converge--and so should only be used as a last resort.

- **Read in from a Variable**

Use this option when you want to use a solution from a previous run or from another source. The solution is read in from the variable selected in the 'Read Solution From' variable.

Read Solution From (Variable)

This optional variable contains the variance-covariance parameter values of a solution that has been found previously. The order of the parameter values is the same as on the parameter reports.

This option is useful when problem requires a great deal of time to solve. Once you have achieved a solution, you can reuse it by entering this variable here and setting the 'Solution Method' option to 'Read in from a Variable'.

Write Solution To (Variable)

Select an empty variable into which the solution is automatically stored. Note that any previous information in this variable will be destroyed.

This option is useful when problem requires a great deal of time to solve. Once you have achieved a solution, you can then reuse it by entering this variable in the 'Read Solution From' variable box and setting the 'Solution Method' option to 'Read in from a Variable'.

Newton-Raphson / Fisher-Scoring Options

Max Fisher Scoring Iterations

This is the maximum number of Fisher Scoring iterations that occur in the maximum likelihood finding process. When Solution Method (Variables tab) is set to 'Newton-Raphson', up to this number of Fisher Scoring iterations occur before beginning Newton-Raphson iterations.

Max Newton-Raphson Iterations

This is the maximum number of Newton-Raphson iterations that occur in the maximum likelihood finding process. When Solution Method (Variables tab) is set to 'Newton-Raphson', Fisher-scoring iterations occur before beginning Newton-Raphson iterations.

Lambda

Each parameter's change is multiplied by this value at each iteration. Usually, this value can be set to one. However, it may be necessary to set this value to 0.5 to implement step-halving: a process that is necessary when the Newton-Raphson diverges.

Note: this parameter only used by the Fisher-Scoring and Newton-Raphson methods.

Mixed Models – Random Coefficients

Convergence Criterion

This procedure uses relative Hessian convergence (or the Relative Offset Orthogonality Convergence Criterion) as described by Bates and Watts (1981).

Recommended: The default value, 1E-8, will be adequate for many problems. When the routine fails to converge, try increasing the value to 1E-6.

Differential Evolution Options

Crossover Rate

This value controls the amount of movement of the differential evolution algorithm toward the current best. Larger values accelerate movement toward the current best, but reduce the chance of locating the global maximum. Smaller values improve the chances of finding the global, rather than a local, solution, but increase the number of iterations until convergence.

RANGE: Usually, a value between .5 and 1.0 is used.

RECOMMENDED: 0.9.

Mutation Rate

This value sets the mutation rate of the search algorithm. This is the probability that a parameter is set to a random value within the parameter space. It keeps the algorithm from stalling on a local maximum.

RANGE: Values between 0 and 1 are allowed.

RECOMMENDED: 0.9 for random coefficients (complex) models or 0.5 for random effects (simple) models.

Minimum Relative Change

This parameter controls the convergence of the likelihood maximizer. When the relative change in the likelihoods from one generation to the next is less than this amount, the algorithm concludes that it has converged. The relative change is $|L(g+1) - L(g)| / L(g)$ where $L(g)$ is absolute value of the likelihood at generation 'g'. Note that the algorithm also terminates if the Maximum Generations are reached or if the number of individuals that are replaced in a generation is zero. The value 0.0000000001 (ten zeros) seems to work well in practice. Set this value to zero to ignore this convergence criterion.

Solutions/Iteration

This is the number of trial points (solution sets) that are used by the differential evolution algorithm during each iteration. In the terminology of differential evolution, this is the population size.

RECOMMENDED: A value between 15 and 25 is recommended. More points may dramatically increase the running time. Fewer points may not allow the algorithm to converge.

Max Iterations

Specify the maximum number of differential evolution iterations used by the differential evolution algorithm. A value between 100 and 200 is usually adequate. For large datasets, i.e., number of rows greater than 1000, you may want to reduce this number.

Other Options

Zero (Algorithm Rounding)

This cutoff value is used by the least-squares algorithm to lessen the influence of rounding error. Values lower than this are reset to zero. If unexpected results are obtained, try using a smaller value, such as 1E-32. Note that 1E-5 is an abbreviation for the number 0.00001.

RECOMMENDED: 1E-10 or 1E-12.

RANGE: 1E-3 to 1E-40.

Mixed Models – Random Coefficients

Variance Zero

When an estimated variance component (diagonal element) is less than this value, the variance is assumed to be zero and all reporting is terminated since the algorithm has not converged properly.

To correct this problem, remove the corresponding term from the Random Factors Model or simplify the Repeated Variance Pattern. Since the parameter is zero, why would you want to keep it?

RECOMMENDED: 1E-6 or 1E-8.

RANGE: 1E-3 to 1E-40.

Correlation Zero

When an estimated correlation (off-diagonal element) is less than this value, the correlation is assumed to be zero and all reporting is terminated since the algorithm has not converged properly.

To correct this problem, remove the corresponding term from the Random Factors Model or simplify the Repeated Variance Pattern. Since the parameter is zero, why would you want to keep it?

RECOMMENDED: 1E-6 or 1E-8.

RANGE: 1E-3 to 1E-40.

Max Retries

Specify the maximum number of retries to occur. During the maximum likelihood search process, the search may lead to an impossible combination of variance-covariance parameters (as defined by a matrix of variance-covariance parameters that is not positive definite). When such a combination arises, the search algorithm will begin again. Max Retries is the maximum number of times the process will re-start to avoid such combinations.

Reports Tab

The following options control which reports are displayed.

Select Reports

Run Summary Report

Check this box to obtain a summary of the likelihood type, the model, the iterations, the resulting likelihood/AIC, and run time.

Variance Estimates Report

Check this box to obtain estimates of random and repeated components of the model.

Hypothesis Tests Report

Check this box to obtain F-Tests for all terms in the Fixed (Means) Specification (see Variables tab).

L-Matrices – Terms Report

Check this box to obtain L matrices for each term in the model. Each L matrix describes the linear combination of the betas that is used to test the corresponding term in the model.

Caution: Selecting this option can generate a very large amount of output, as the L matrices can be very numerous and lengthy.

Mixed Models – Random Coefficients

Comparisons by Fixed Effects Report

Check this box to obtain planned comparison tests, comparing levels of the fixed effects. Details of the comparisons to be made are specified under the Comparisons and Covariates tabs. When more than one covariate value is specified under the Covariates tab, the comparisons are grouped such that for each fixed effect, comparisons for all covariate(s) values are displayed.

Compare to Comparisons by Covariate Values.

Comparisons by Covariate Values Report

Check this box to obtain planned comparison tests, comparing levels of the fixed effects. Details of the comparisons to be made are specified under the Comparisons and Covariates tabs. When more than one covariate value is specified under the Covariates tab, the comparisons are grouped such that for each value of the covariate(s), a new set of comparisons is displayed.

Compare to Comparisons by Fixed Effects.

L-Matrices – Comparisons Report

Check this box to obtain L matrices for each planned comparison. Each L matrix describes the linear combination of the betas that is used to test the corresponding comparison.

Caution: Selecting this option can generate a very large amount of output, as the L matrices can be very numerous and lengthy.

Means by Fixed Effects Report

Check this box to obtain means and confidence limits for each fixed effect level. When more than one covariate value is specified under the Covariates tab, the means are grouped such that for each fixed effect, means for all covariate(s) values are displayed.

Compare to Means by Covariate Values.

Means by Covariate Values Report

Check this box to obtain means and confidence limits for each fixed effect level. When more than one covariate value is specified under the Covariates tab, the means are grouped such that for each value of the covariate(s), a new set of means is displayed.

Compare to Means by Fixed Effects.

L-Matrices – LS Means Report

Check this box to obtain L matrices for each least squares mean (of the fixed effects). Each L matrix describes the linear combination of the betas that is used to generate the least squares mean.

Caution: Selecting this option can generate a very large amount of output, as the L matrices can be very numerous and lengthy.

Fixed Effects Solution Report

Check this box to obtain estimates, P-values and confidence limits of the fixed effects and covariates (betas).

Asymptotic VC Matrix Report

Check this box to obtain the asymptotic variance-covariance matrix of the random (and repeated) components of the model.

Vi Matrices (1st 3 Subjects) Report

Check this box to display the Vi matrices of the first three subjects.

Hessian Matrix Report

Check this box to obtain the Hessian matrix. The Hessian matrix is directly associated with the variance-covariance matrix of the random (and repeated) components of the model.

Show Report Definitions

Indicate whether to show the definitions at the end of reports. Although these definitions are helpful at first, they may tend to clutter the output. This option lets you omit them.

Select Reports – Alpha

Alpha

Specify the alpha value (significance level) used for F-tests, T-tests, and confidence intervals. Alpha is the probability of rejecting the null hypothesis of equal means when it is actually true. Usually, an alpha of 0.05 is used. Typical choices for alpha range from 0.001 to 0.200.

Report Options

Stagger label and output of label length is \geq

When writing a row of information to a report, some variable names/labels may be too long to fit in the space allocated. If the name (or label) contains more characters than entered here, the rest of the output for that line is moved down to the next line. Most reports are designed to hold a label of up to 15 characters. Enter “1” when you always want each row's output to be printed on two lines. Enter “100” when you want each row printed on only one line. Note that this may cause some columns to be mis-aligned.

Report Options – Decimal Places

Precision

Specifies whether unformatted numbers are displayed as single (7-digit) or double (13-digit) precision numbers in the output. All calculations are performed in double precision regardless of the Precision selected here.

Single

Unformatted numbers are displayed with 7-digits. This is the default setting. All reports have been formatted for single precision.

Double

Unformatted numbers are displayed with 13-digits. This option is most often used when the extremely accurate results are needed for further calculation. For example, double precision might be used when you are going to use the Multiple Regression model in a transformation.

Double Precision Format Misalignment

Double precision numbers require more space than is available in the output columns, causing column alignment problems. The double precision option is for those instances when accuracy is more important than format alignment.

Effects/Betas ... DF Denominator

Specify the number of digits after the decimal point to display on the output of values of this type. Note that this option in no way influences the accuracy with which the calculations are done.

Enter 'General' to display all digits available. The number of digits displayed by this option is controlled by whether the PRECISION option is SINGLE or DOUBLE.

Plots Tab

These options specify the plots of group means and subjects. Click the plot format button to change the plot settings.

Select Plots

Means Plots

Check this box to obtain plots of means for each fixed effects term of the model. Details of the appearance of the plots are specified under the Means Plots and Symbols tabs.

Subject Plots

Check this box to obtain plots of the repeated values for each subject. Plots comparing main effects for each subject are also given. The repeated values for each subject are ordered according to the order the values appear in the data set. Details of the appearance of the plots are specified under the Subject Plots and Symbols tabs.

Y-Axis Scaling

Specify the method for calculating the minimum and maximum along the vertical axis. *Separate* means that each plot is scaled independently. *Uniform* means that all plots use the overall minimum and maximum of the data. This option is ignored if a minimum or maximum is specified.

These options specify the subject plots.

Example 1 – Random Coefficients Model with a Between Subjects Factor and Two, Within-Subjects Covariates

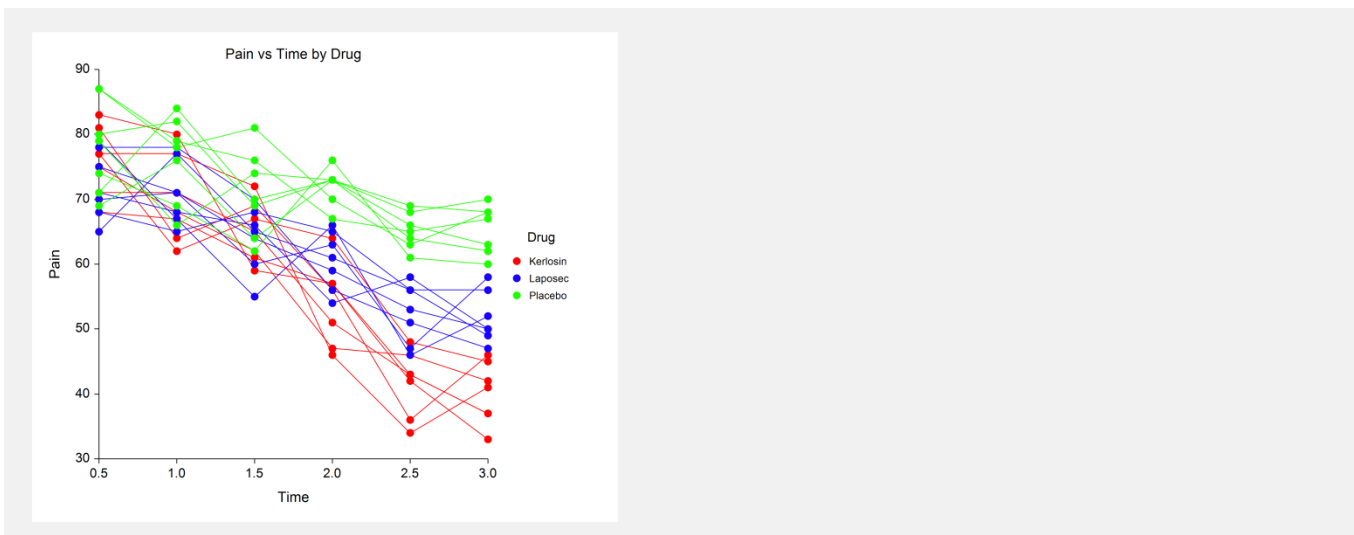
This example should acquaint the reader with the output for all output options. It presents an analysis of a longitudinal design in which there is one between-subjects factor (Drug) a time variable (Time), and a covariate (Cov). The response is a measure of pain (Pain).

Two drugs (Kerlosin and Laposec) are compared to a placebo for their effectiveness in reducing pain following a surgical eye procedure. A standard pain measurement for each patient is measured at 30 minute intervals following surgery and administration of the drug (or placebo). Six measurements, with the last at Time = 3 hours, are made for each of the 21 patients (7 per group). A blood pressure measurement of each individual at the time of pain measurement is measured as a covariate. The researchers wish to compare the drugs at the time value of 2.0 and the Cov value of 140.

Pain Dataset

Drug	Patient	Time	Cov	Pain
Kerlosin	1	0.5	125	68
Kerlosin	1	1	196	67
Kerlosin	1	1.5	189	61
Kerlosin	1	2	135	57
Kerlosin	1	2.5	128	43
Kerlosin	1	3	151	37
Kerlosin	2	0.5	215	75
Kerlosin	2	1	151	68
Kerlosin	2	1.5	191	62
Kerlosin	2	2	212	47
Kerlosin	2	2.5	127	46
Kerlosin	2	3	133	42
.
.
.
Placebo	21	2	129	73
Placebo	21	2.5	216	68
Placebo	21	3	158	70

The following plot shows the relationship among all variables except the covariate.



Mixed Models – Random Coefficients

To run the analysis using the Mixed Models – Random Coefficients procedure, you can enter the values according to the instructions below (beginning with Step 3) or load the completed template **Example 1** by clicking on Open Example Template from the File menu of the Mixed Models – Random Coefficients window.

1 Open the Pain dataset.

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Click on the file **Pain.NCSS**.
- Click **Open**.

2 Open the Mixed Models – Random Coefficients window.

- Using the Analysis menu or the Procedure Navigator, find and select the **Mixed Models – Random Coefficients** procedure.
- On the menus, select **File**, then **New Template**. This will fill the procedure with the default template.

3 Specify the variables.

- On the Mixed Models – Random Coefficients window, select the **Variables tab**.
- Double-click in the **Response Variable** text box. This will bring up the variable selection window.
- Select **Pain** from the list of variables and then click **Ok**. ‘Pain’ will appear in the Response Variable box.
- Double-click in the **Subject Variable** text box. This will bring up the variable selection window.
- Select **Patient** from the list of variables and then click **Ok**. ‘Patient’ will appear in the Subject Variable box.
- Set the **Regressions** option to **Linear**.
- Check the **Covariances** box.
- Set the number of Fixed Factors to **1**.
- Set the **Variable** to **Drug**.
- Set the **Comparison** to **All Pairs**.
- Set the number of Covariates to **2**.
- Set the first covariate to **Time**.
- Set the **Compute Means at these Values** to **2**.
- Set the second covariate to **Cov**.
- Set the **Compute Means at these Values** to **140**.
- Set the **Terms** to **Up to 2-Way**.
- Set the **Maximum Exponent of Covariates** to **1**.

4 Specify the reports.

- On the Mixed Models – Random Coefficients window, select the **Reports tab**.
- Check all report and plot checkboxes except **L Matrices – Comparisons** and **L Matrices – LS Means**.

5 Run the procedure.

- From the Run menu, select **Run Procedure**. Alternatively, just click the green Run button.

Run Summary Section

Parameter	Value
Likelihood Type	Restricted Maximum Likelihood
Fixed Model	COV+DRUG+TIME+COV*DRUG+COV*TIME+DRUG*TIME
Random Model	PATIENT+PATIENT*TIME
Repeated Pattern	Diagonal
Number of Rows	126
Number of Subjects	21
Solution Type	Newton-Raphson
Fisher Iterations	5 of a possible 5
Newton Iterations	40 of a possible 40
Max Retries	10
Lambda	1
Log Likelihood	-388.9034
-2 Log Likelihood	777.8067
AIC (Smaller Better)	785.8067
Convergence	Normal
Run Time (Seconds)	2.412

This section provides a summary of the model and the iterations toward the maximum log likelihood.

Likelihood Type

This value indicates that restricted maximum likelihood was used rather than maximum likelihood.

Fixed Model

The fixed model specified for this run. It includes fixed terms and covariates.

Random Model

The random model as specified by the Regressions setting.

Repeated Model

The pattern selected for the within-subjects variance-covariance matrix.

Number of Rows

The number of rows processed from the database.

Number of Subjects

The number of unique subjects from the database.

Solution Type

The solution type is method used for finding the maximum (restricted) maximum likelihood solution. Newton-Raphson is the recommended method.

Fisher Iterations

Some Fisher-Scoring iterations are used as part of the Newton-Raphson algorithm. The '5 of a possible 5' means five Fisher-Scoring iterations were used, while five was the maximum that were allowed (as specified on the Maximization tab).

Newton Iterations

The '40 of a possible 40' means that all forty Newton-Raphson iterations were used. You may want to increase this value and rerun so the algorithm has a chance to converge.

Mixed Models – Random Coefficients

Max Retries

The maximum number of times that lambda was changed and new variance-covariance parameters found during an iteration was ten. If the values of the parameters result in a negative variance, lambda is divided by two and new parameters are generated. This process continues until a positive variance occurs or until Max Retries is reached.

Lambda

Lambda is a parameter used in the Newton-Raphson process to specify the amount of change in parameter estimates between iterations. One is generally an appropriate selection. When convergence problems occur, reset this to 0.5.

If the values of the parameters result in a negative variance, lambda is divided by two and new parameters are generated. This process continues until a positive variance occurs or until Max Retries is reached.

Log Likelihood

This is the log of the likelihood of the data given the variance-covariance parameter estimates. When a maximum is reached, the algorithm converges.

-2 Log Likelihood

This is minus 2 times the log of the likelihood. When a minimum is reached, the algorithm converges.

AIC

The Akaike Information Criterion is used for comparing covariance structures in models. It gives a penalty for increasing the number of covariance parameters in the model.

Convergence

'Normal' convergence indicates that convergence was reached before the limit.

Run Time (Seconds)

The run time is the amount of time used to solve the problem and generate the output.

Random Component Parameter Estimates (G Matrix)

Random Component Parameter Estimates (G Matrix)

Parameter Number	Estimated Value	Model Term
1	9.4357	Patient
2	1.3770	Patient*Time
3	-3.6028	Patient, Patient*Time

This section gives the random component estimates.

Parameter Number

When the random component model results in more than one parameter for the component, the parameter number identifies parameters within the component.

Estimated Value

The estimated values of the three variance components.

Model Term

The name of the random term being presented on this line.

Repeated Component Parameter Estimates (R Matrix)

Repeated Component Parameter Estimates (R Matrix)

Parameter Number	Estimated Value	Parameter Type
1	27.6148	Diagonal (Variance)

This section gives the repeated component estimates according to the Repeated Variance Pattern specifications of the Variables tab.

Parameter Number

When the repeated pattern results in more than one parameter for the component, the parameter number identifies parameters within the component.

Estimated Value

The estimated value 27.6148 is the estimated residual (error) variance.

Parameter Type

The parameter type describes the structure of the R matrix.

Term-by-Term Hypothesis Test Results

Term-by-Term Hypothesis Test Results

Model Term	F-Value	Num DF	Denom DF	Prob Level
Cov	1.3193	1	86.7	0.253883
Drug	2.0662	2	55.4	0.136315
Time	3.0716	1	30.4	0.089767
Cov*Drug	0.7537	2	48.0	0.476088
Cov*Time	2.5462	1	29.8	0.121126
Drug*Time	25.3246	2	9.5	0.000155

This section contains an F-test for each fixed term in the model according to the methods described by Kenward and Roger (1997).

Model Term

This is the name of the term in the model.

F-Value

The F-Value corresponds to the L matrix used for testing this term in the model. The F-Value is based on the F approximation described in Kenward and Roger (1997).

Num DF

This is the numerator degrees of freedom for the corresponding term.

Denom DF

This is the approximate denominator degrees of freedom for this comparison as described in Kenward and Roger (1997).

Prob Level

The Probability Level (or P-value) gives the strength of evidence (smaller Prob Level implies more evidence) that a term in the model has differences among its levels, or a slope different from zero in the case of covariate. It is the probability of obtaining the corresponding F-Value (or greater) if the null hypothesis of equal means (or no slope) is true.

Individual Comparison Hypothesis Test Results

Individual Comparison Hypothesis Test Results						
Covariates: Cov=140.0000, Time=2.0000						
Comparison/ Covariate(s)	Comparison Mean Difference	F-Value	Num DF	Denom DF	Raw Prob Level	Bonferroni Prob Level
Drug		43.18	2	32.8	0.0000	
Drug: Kerlosin - Laposec	-4.9155	11.0898	1	4.5	0.024422	0.073266 [3]
Drug: Kerlosin - Placebo	-16.0780	117.3785	1	4.2	0.000318	0.000954 [3]
Drug: Laposec - Placebo	-11.1625	56.8153	1	4.3	0.001242	0.003727 [3]

This section shows the F-tests for comparisons of the levels of the fixed terms of the model according to the methods described by Kenward and Roger (1997). The individual comparisons are grouped into subsets of the fixed model terms.

Comparison/Covariate(s)

This is the comparison being made. The first line is 'Drug'. On this line, the levels of drug are compared when the covariate is equal to 140. The second line is 'Drug: Placebo – Kerlosin'. On this line, Kerlosin is compared to Placebo when the covariate is equal to 140.

Comparison Mean Difference

This is the difference in the least squares means for each comparison.

F-Value

The F-Value corresponds to the L matrix used for testing this comparison. The F-Value is based on the F approximation described in Kenward and Roger (1997).

Num DF

This is the numerator degrees of freedom for this comparison.

Denom DF

This is the approximate denominator degrees of freedom for this comparison as described in Kenward and Roger (1997).

Raw Prob Level

The Raw Probability Level (or Raw P-value) gives the strength of evidence for a single comparison, unadjusted for multiple testing. It is the single test probability of obtaining the corresponding difference if the null hypothesis of equal means is true.

Bonferroni Prob Level

The Bonferroni Prob Level is adjusted for multiple tests. The number of tests adjusted for is enclosed in brackets following each Bonferroni Prob Level. For example, 0.073266 [3] signifies that the probability the means are equal, given the data, is 0.073266, after adjusting for 3 tests.

Least Squares (Adjusted) Means

Least Squares (Adjusted) Means					
Covariates: Cov=140.0000, Time=2.0000					
Name	Mean	Standard Error of Mean	95.0% Lower Conf. Limit for Mean	95.0% Upper Conf. Limit for Mean	DF
Intercept	61.5145	0.6054	59.8674	63.1615	4.2
Drug					
Kerlosin	54.5166	1.0426	51.7155	57.3178	4.4
Laposec	59.4322	1.0436	56.6756	62.1887	4.6
Placebo	70.5946	1.0558	67.6454	73.5439	3.9

This section gives the adjusted means for the levels of each fixed factor when Cov = 140 and Time = 2.

Name

This is the level of the fixed term that is estimated on the line.

Mean

The mean is the estimated least squares (adjusted or marginal) mean at the specified value of the covariate.

Standard Error of Mean

This is the standard error of the mean.

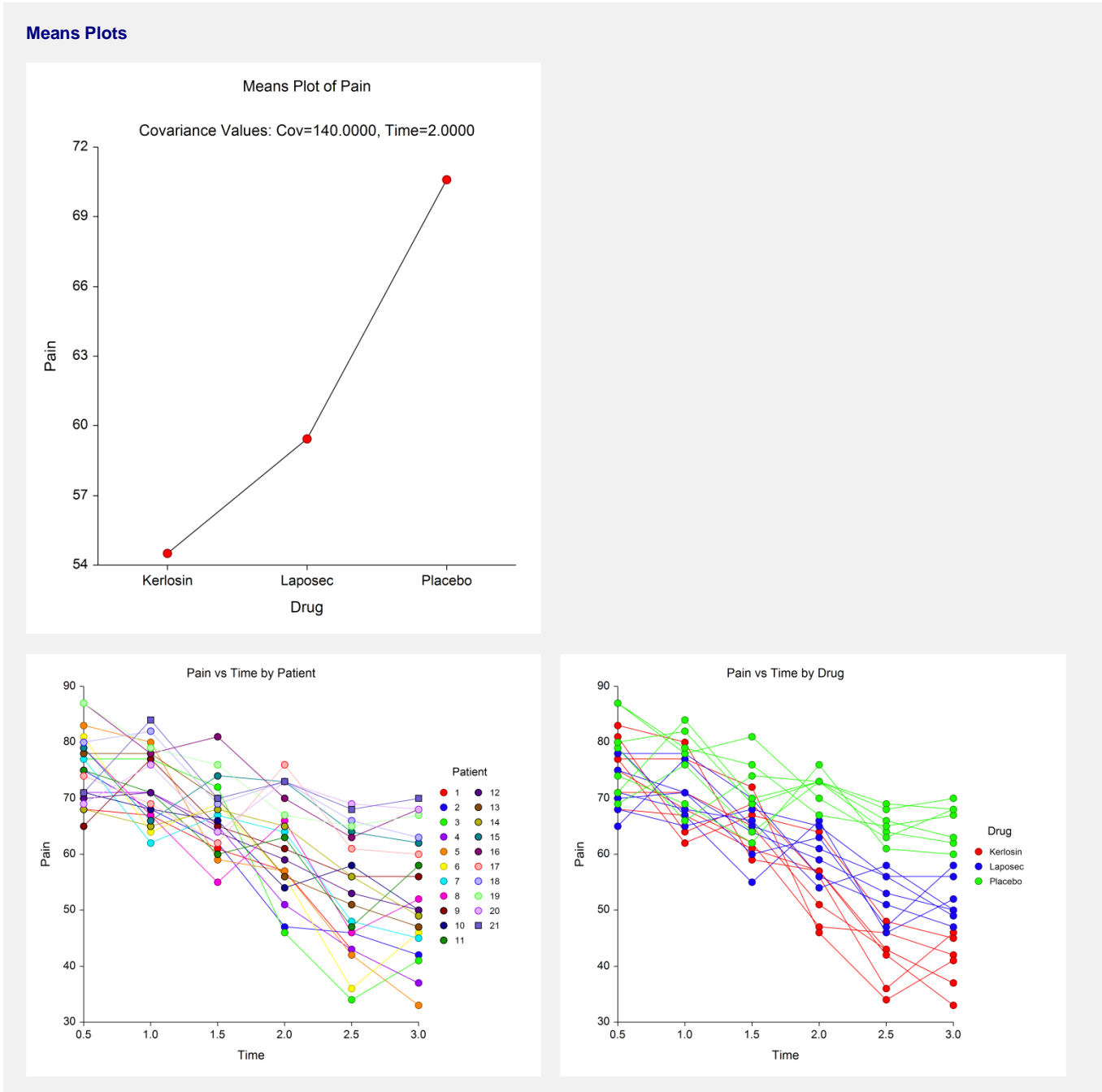
95.0% Lower (Upper) Conf. Limit for Mean

These limits give a 95% confidence interval for the mean.

DF

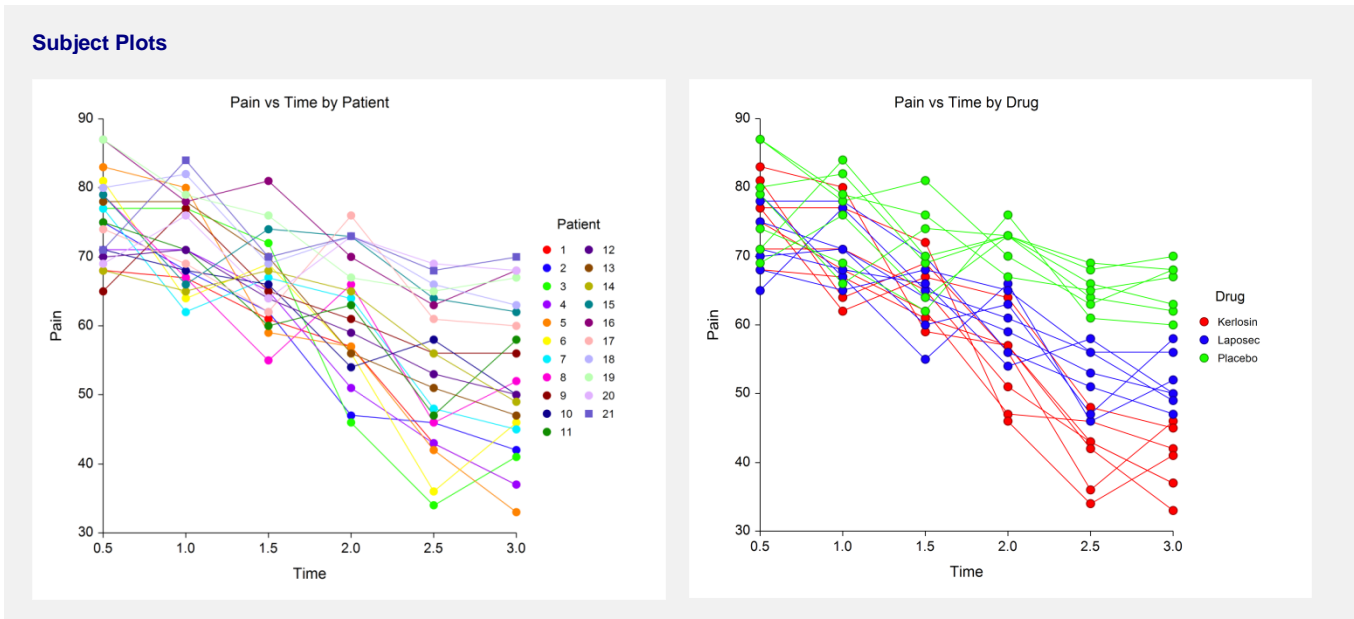
The degrees of freedom used for the confidence limits are calculated using the method of Kenward and Roger (1997).

Means Plots



These plots show the means broken up into the categories of the fixed effects of the model. Some general trends that can be seen are those of pain decreasing with time and lower pain for the two drugs after two hours.

Subject Plots



Each set of connected dots of the Subject plots show the response trajectory of particular subject.

Solution for Fixed Effects

Solution for Fixed Effects

Effect Name	Effect Estimate (Beta)	Effect Standard Error	Prob Level	95.0% Lower Conf. Limit of Beta	95.0% Upper Conf. Limit of Beta	DF	Effect No.
Intercept	75.3756	6.8605	0.000000	61.7020	89.0493	72.8	1
Cov (Drug="Kerlosin")	0.0340	0.0419	0.419993	-0.0495	0.1174	77.5	2
(Drug="Laposec")	7.7959	7.1744	0.282843	-6.6441	22.2359	46.1	3
(Drug="Placebo")	-7.9409	7.2768	0.279642	-22.5052	6.6233	58.3	4
Time	0.0000	0.0000					5
Cov*(Drug="Kerlosin")	-0.5691	3.1987	0.860185	-7.1478	6.0096	25.7	6
Cov*(Drug="Laposec")	-0.0179	0.0388	0.647188	-0.0963	0.0606	39.0	7
Cov*(Drug="Placebo")	0.0303	0.0401	0.453369	-0.0502	0.1108	49.7	8
Cov*Time	0.0000	0.0000					9
(Drug="Kerlosin")*Time	-0.0300	0.0188	0.121126	-0.0684	0.0084	29.8	10
(Drug="Laposec")*Time	-10.6849	1.5189	0.000048	-14.0966	-7.2731	9.4	11
(Drug="Placebo")*Time	-3.7308	1.5091	0.034477	-7.1245	-0.3371	9.4	12
	0.0000	0.0000					13

This section shows the model estimates for all the model terms (betas).

Effect Name

The Effect Name is the level of the fixed effect that is examine on the line.

Mixed Models – Random Coefficients

Effect Estimate (Beta)

The Effect Estimate is the beta-coefficient for this effect of the model. For main effects terms the number of effects per term is the number of levels minus one. An effect estimate of zero is given for the last effect(s) of each term. There may be several zero estimates for effects of interaction terms.

Effect Standard Error

This is the standard error for the corresponding effect.

Prob Level

The Prob Level tests whether the effect is zero.

95.0% Lower (Upper) Conf. Limit of Beta

These limits give a 95% confidence interval for the effect.

DF

The degrees of freedom used for the confidence limits and hypothesis tests are calculated using the method of Kenward and Roger (1997).

Effect No.

This number identifies the effect of the line.

Asymptotic Variance-Covariance Matrix of Variance Estimates

Asymptotic Variance-Covariance Matrix of Variance Estimates

Parm	G(1,1)	G(1,2)	G(1,3)	R(1,1)
G(1,1)	155.0994	35.0388	-72.6433	-16.9227
G(1,2)	35.0388	14.3116	-21.5071	-4.4822
G(1,3)	-72.6433	-21.5071	39.1741	7.9258
R(1,1)	-16.9227	-4.4822	7.9258	18.9471

This section gives the asymptotic variance-covariance matrix of the variance components of the model. Note that the component number is always '1' in this procedure.

Parm

Parm is the heading for both the row variance parameters and column variance parameters.

G(1,1)

The two elements of G(1,1) refer to the component number and parameter number of the covariance parameter in G.

R(1,1)

The two elements of R(1,1) refer to the component number and parameter number of the covariance parameter in R.

Estimated Vi Matrix of Subject = X

Estimated Vi Matrix of Subject = 1

Vi	1	2	3	4	5	6
1	33.7920	4.7200	3.2628	1.8057	0.3485	-1.1087
2	4.7200	31.2219	2.4942	1.3813	0.2684	-0.8445
3	3.2628	2.4942	29.3404	0.9569	0.1882	-0.5804
4	1.8057	1.3813	0.9569	28.1473	0.1081	-0.3163
5	0.3485	0.2684	0.1882	0.1081	27.6428	-0.0522
6	-1.1087	-0.8445	-0.5804	-0.3163	-0.0522	27.8268

Estimated Vi Matrix of Subject = 2

Vi	1	2	3	4	5	6
1	33.7920	4.7200	3.2628	1.8057	0.3485	-1.1087
2	4.7200	31.2219	2.4942	1.3813	0.2684	-0.8445
3	3.2628	2.4942	29.3404	0.9569	0.1882	-0.5804
4	1.8057	1.3813	0.9569	28.1473	0.1081	-0.3163
5	0.3485	0.2684	0.1882	0.1081	27.6428	-0.0522
6	-1.1087	-0.8445	-0.5804	-0.3163	-0.0522	27.8268

Estimated Vi Matrix of Subject = 3

Vi	1	2	3	4	5	6
1	33.7920	4.7200	3.2628	1.8057	0.3485	-1.1087
2	4.7200	31.2219	2.4942	1.3813	0.2684	-0.8445
3	3.2628	2.4942	29.3404	0.9569	0.1882	-0.5804
4	1.8057	1.3813	0.9569	28.1473	0.1081	-0.3163
5	0.3485	0.2684	0.1882	0.1081	27.6428	-0.0522
6	-1.1087	-0.8445	-0.5804	-0.3163	-0.0522	27.8268

This section gives the estimated variance-covariance matrix for each of the first three subjects.

1 – 6

Each of the 6 levels shown here represents one of the time values. That is 1 is for 0.5 hours, 2 is for 1 hour, 3 is for 1.5 hours, and so on.

Hessian Matrix of Variance Estimates

Hessian Matrix of Variance Estimates

Parm	G(1,1)	G(1,2)	G(1,3)	R(1,1)
G(1,1)	0.0937	0.1820	0.2710	0.0134
G(1,2)	0.1820	0.7529	0.7450	0.0290
G(1,3)	0.2710	0.7450	0.9313	0.0287
R(1,1)	0.0134	0.0290	0.0287	0.0596

The Hessian Matrix is directly related to the asymptotic variance-covariance matrix of the variance estimates. Note that the component number is always '1' in this procedure.

Parm

Parm is the heading for both the row variance parameters and column variance parameters.

G(1,1)

The two elements of G(1,1) refer to the component number and parameter number of the covariance parameter in G.

R(1,1)

The two elements of R(1,1) refer to the component number and parameter number of the covariance parameter in R.

L Matrices

L Matrix for Cov			
No.	Effect	Drug	L1
1	Intercept		
2	Cov		1.0000
3	Drug	Kerlosin	
4	Drug	Laposec	
5	Drug	Placebo	
6	Time		
7	Cov*Drug	Kerlosin	0.3333
8	Cov*Drug	Laposec	0.3333
9	Cov*Drug	Placebo	0.3333
10	Cov*Time		
11	Drug*Time	Kerlosin	
12	Drug*Time	Laposec	
13	Drug*Time	Placebo	

L Matrix for Drug				
No.	Effect	Drug	L1	L2
1	Intercept			
2	Cov			
3	Drug	Kerlosin	1.0000	1.0000
4	Drug	Laposec	-1.0000	
5	Drug	Placebo		-1.0000
6	Time			
7	Cov*Drug	Kerlosin		
8	Cov*Drug	Laposec		
9	Cov*Drug	Placebo		
10	Cov*Time			
11	Drug*Time	Kerlosin		
12	Drug*Time	Laposec		
13	Drug*Time	Placebo		

The L matrices are used to form a linear combination of the betas corresponding to a specific hypothesis test or mean estimate. The L matrix in the second report is used for testing whether there is a difference among the three levels of Drug.

No.

This number is used for identifying the corresponding beta term.

Effect

This column gives the model term.

Factor Variables (e.g. Drug, Time)

These columns identify the level of each fixed effect to which the coefficients of the L matrix of the same line correspond.

L1, L2, L3, ...

L1, L2, L3, ... are a group of column vectors that combine to form an L matrix. The L matrix in this example is used for testing whether there is a difference among the three levels of Drug.