

Chapter 546

One ROC Curve and Cutoff Analysis

Introduction

This procedure generates empirical (nonparametric) and Binormal ROC curves. It also gives the area under the ROC curve (AUC), the corresponding confidence interval of AUC, and a statistical test to determine if AUC is greater than a specified value. Summary measures for a desired (user-specified) list of cutoff values are also available. Some of these measures include sensitivity, specificity, proportion correctly specified, table counts, positive predictive value, cost analysis, likelihood ratios, and the Youden index. These measure are often used to determine the optimal cutoff value (optimal decision threshold).

Discussion and Technical Details

Although ROC curve analysis can be used for a variety of applications across a number of research fields, we will examine ROC curves through the lens of diagnostic testing. In a typical diagnostic test, each unit (e.g., individual or patient) is measured on some scale or given a score with the intent that the measurement or score will be useful in classifying the unit into one of two conditions (e.g., Positive / Negative, Yes / No, Diseased / Non-diseased). Based on a (hopefully large) number of individuals for which the score and condition is known, researchers may use ROC curve analysis to determine the ability of the score to classify or predict the condition. The analysis may also be used to determine the optimal cutoff value (optimal decision threshold).

For a given cutoff value, a positive or negative diagnosis is made for each unit by comparing the measurement to the cutoff value. If the measurement is less (or greater, as the case may be) than the cutoff, the predicted condition is negative. Otherwise, the predicted condition is positive. However, the predicted condition doesn't necessarily match the true condition of the experimental unit (patient). There are four possible outcomes: true positive, true negative, false positive, false negative.

Classification Table

		Predicted Condition	
		Positive	Negative
True Condition	Positive	True Positive	False Negative
	Negative	False Positive	True Negative

One ROC Curve and Cutoff Analysis

Each unit falls into only one of the four outcomes. When all of the units are assigned to the four outcomes for a given cutoff, a count for each outcome is produced. The four counts are labeled A, B, C, and D in the table below.

Classification Table (Counts)

		Predicted Condition		Total
		Positive	Negative	
True Condition	Positive	True Positive (A)	False Negative (C)	A + C
	Negative	False Positive (B)	True Negative (D)	B + D
Total		A + B	C + D	A + B + C + D

Various rates (proportions) can be used to describe a classification table. Some rates are based on the true condition, some rates are based on the predicted condition, and some rates are based on the whole table. These rates will be described in the following sections.

Rates Assuming a True Condition

The following rates assume one of the two true conditions.

True Positive Rate (TPR) or Sensitivity = $A / (A + C)$

The true positive rate is the proportion of the units with a known positive condition for which the predicted condition is positive. This rate is often called the sensitivity, and constitutes the Y axis on the ROC curve.

		Predicted Condition		
		Positive	Negative	
True Condition	Positive	True Positive (A)	False Negative (C)	$TPR = A / (A + C)$ (Y Axis on ROC Curve)
	Negative	False Positive (B)	True Negative (D)	

One ROC Curve and Cutoff Analysis

True Negative Rate (TNR) or Specificity = $D / (B + D)$

The true negative rate is the proportion of the units with a known negative condition for which the predicted condition is negative. This rate is often called the specificity. One minus this value constitutes the X axis on the ROC curve.

		Predicted Condition		
		Positive	Negative	
True Condition	Positive	True Positive (A)	False Negative (C)	$TNR = D / (B + D)$
	Negative	False Positive (B)	True Negative (D)	

False Negative Rate (FNR) or Miss Rate = $C / (A + C)$

The false negative rate is the proportion of the units with a known positive condition for which the predicted condition is negative. This rate is sometimes called the miss rate.

		Predicted Condition		
		Positive	Negative	
True Condition	Positive	True Positive (A)	False Negative (C)	$FNR = C / (A + C)$
	Negative	False Positive (B)	True Negative (D)	

False Positive Rate (FPR) or Fall-out = $B / (B + D)$

The false positive rate is the proportion of the units with a known negative condition for which the predicted condition is positive. This rate is sometimes called the fall-out, and constitutes the X axis on the ROC curve.

		Predicted Condition		
		Positive	Negative	
True Condition	Positive	True Positive (A)	False Negative (C)	$FPR = B / (B + D)$ (X Axis on ROC Curve)
	Negative	False Positive (B)	True Negative (D)	

One ROC Curve and Cutoff Analysis

Rates Assuming a Predicted Condition

The following rates assume one of the two predicted conditions.

Positive Predictive Value (PPV) or Precision = A / (A + B)

The positive predictive value is the proportion of the units with a predicted positive condition for which the true condition is positive. This rate is sometimes called the precision.

		Predicted Condition	
		Positive	Negative
True Condition	Positive	True Positive (A)	False Negative (C)
	Negative	False Positive (B)	True Negative (D)

$$PPV = A / (A + B)$$

Positive Predictive Value Adjusted for Known Prevalence

When the prevalence (or pre-test probability of a positive condition) is known for the experimental units, an adjusted formula for positive predictive value, based on the known prevalence value, can be used.

Using Bayes theorem, adjusted values of PPV are calculated based on known prevalence values as follows:

$$Adjusted\ PPV = \frac{sensitivity \times known\ prevalence}{sensitivity \times known\ prevalence + (1 - specificity) \times (1 - known\ prevalence)}$$

Negative Predictive Value (NPV) = D / (C + D)

The negative predictive value is the proportion of the units with a predicted negative condition for which the true condition is negative.

		Predicted Condition	
		Positive	Negative
True Condition	Positive	True Positive (A)	False Negative (C)
	Negative	False Positive (B)	True Negative (D)

$$NPV = D / (C + D)$$

Negative Predictive Value Adjusted for Known Prevalence

When the prevalence (or pre-test probability of a positive condition) is known for the experimental units, an adjusted formula for negative predictive value, based on the known prevalence value, can be used.

Using Bayes theorem, adjusted values of NPV are calculated based on known prevalence values as follows:

$$Adjusted\ NPV = \frac{specificity \times (1 - known\ prevalence)}{(1 - sensitivity) \times known\ prevalence + specificity \times (1 - known\ prevalence)}$$

One ROC Curve and Cutoff Analysis

False Omission Rate (FOR) = $C / (C + D)$

The false omission rate is the proportion of the units with a predicted negative condition for which the true condition is positive.

		Predicted Condition	
		Positive	Negative
True Condition	Positive	True Positive (A)	False Negative (C)
	Negative	False Positive (B)	True Negative (D)

$FOR = C / (C + D)$

False Discovery Rate (FDR) = $B / (A + B)$

The false discovery rate is the proportion of the units with a predicted positive condition for which the true condition is negative.

		Predicted Condition	
		Positive	Negative
True Condition	Positive	True Positive (A)	False Negative (C)
	Negative	False Positive (B)	True Negative (D)

$FDR = B / (A + B)$

Whole Table Rates

The following rates are proportions based on all the units.

Prevalence = $(A + C) / (A + B + C + D)$

The prevalence may be estimated from the table if all the units are randomly sampled from the population.

		Predicted Condition		
		Positive	Negative	
True Condition	Positive	True Positive (A)	False Negative (C)	Prevalence = $(A + C) / (A + B + C + D)$
	Negative	False Positive (B)	True Negative (D)	

One ROC Curve and Cutoff Analysis

Accuracy or Proportion Correctly Classified = $(A + D) / (A + B + C + D)$

The accuracy reflects the total proportion of units that are correctly predicted or classified.

		Predicted Condition		
		Positive	Negative	
True Condition	Positive	True Positive (A)	False Negative (C)	Accuracy = $(A + D) / (A + B + C + D)$
	Negative	False Positive (B)	True Negative (D)	

Proportion Incorrectly Classified = $(B + C) / (A + B + C + D)$

The proportion incorrectly classified reflects the total proportion of units that are incorrectly predicted or classified.

		Predicted Condition		
		Positive	Negative	
True Condition	Positive	True Positive (A)	False Negative (C)	PIC = $(B + C) / (A + B + C + D)$
	Negative	False Positive (B)	True Negative (D)	

Confidence Intervals for Rates (Proportions)

Confidence limits for the above rates are calculated using the exact (Binomial distribution) methods described in the One Proportion chapter of the documentation.

Other Diagnostic Accuracy Indices

Over the past several decades, a number of table summary indices have been considered, above those described above. Those available in NCSS are described below.

Youden Index

Conceptually, the Youden index is the vertical distance between the 45 degree line and the point on the ROC curve. The formula for the Youden index is

$$\text{Youden Index} = \text{sensitivity} + \text{specificity} - 1$$

Higher values of the Youden index are better than lower values.

Sensitivity + Specificity

The addition of the sensitivity and the specificity gives essentially the same information as the Youden index, but may be slightly more intuitive for interpretation.

Higher values of sensitivity plus specificity are better than lower values.

One ROC Curve and Cutoff Analysis

Distance to Corner

The distance to the top-left corner of the ROC curve for each cutoff value is given by

$$d = \sqrt{(1 - \text{sensitivity})^2 + (1 - \text{specificity})^2}$$

Lower distances to the corner are better than higher distances.

Positive Likelihood Ratio (LR+) = TPR / FPR

The positive likelihood ratio is the ratio of the true positive rate (sensitivity) to the false positive rate (1 – specificity). This likelihood ratio statistic measures the value of the test for increasing certainty about a positive diagnosis.

$$\text{LR+} = \text{TPR} / \text{FPR}$$

Negative Likelihood Ratio (LR-) = FNR / TNR

The negative likelihood ratio is the ratio of the false negative rate to the true negative rate (specificity).

$$\text{LR-} = \text{FNR} / \text{TNR}$$

Diagnostic Odds Ratio (DOR) = LR+ / LR-

The diagnostic odds ratio is the ratio of the positive likelihood ratio to the negative likelihood ratio.

$$\text{DOR} = \text{LR+} / \text{LR-}$$

Cost

A cost approach is sometimes used when seeking to determine the optimal cutoff value. This approach is based on an analysis of the costs of the four possible outcomes of a diagnostic test: true positive (TP), true negative (TN), false positive (FP), and false negative (FN). If the cost of each of these outcomes is known. The average overall cost C of performing a test at a given cutoff is given by

$$\text{Cost} = C_0 + C_{TP}P(TP) + C_{TN}P(TN) + C_{FP}P(FP) + C_{FN}P(FN)$$

Here, C_0 is the fixed cost of performing the test, C_{TP} is the cost associated with a true positive, $P(TP)$ is the proportion of TP's in the population, and so on.

Metz (1978) showed that the slope of the ROC curve at the optimal cutoff value is

$$m = \frac{1 - \text{Prevalence}}{\text{Prevalence}} \times \frac{C_{FP} - C_{TN}}{C_{FN} - C_{TP}}$$

Zweig and Campbell (1993) showed that the point along the ROC curve where the average cost is minimum corresponds to the cutoff value where

$$f_m = \text{sensitivity} - m(1 - \text{specificity})$$

is maximized. We refer to f_m as the Cost Index.

In order to make these cost calculations, known prevalence and cost values (or cost ratio) must be supplied.

One ROC Curve and Cutoff Analysis

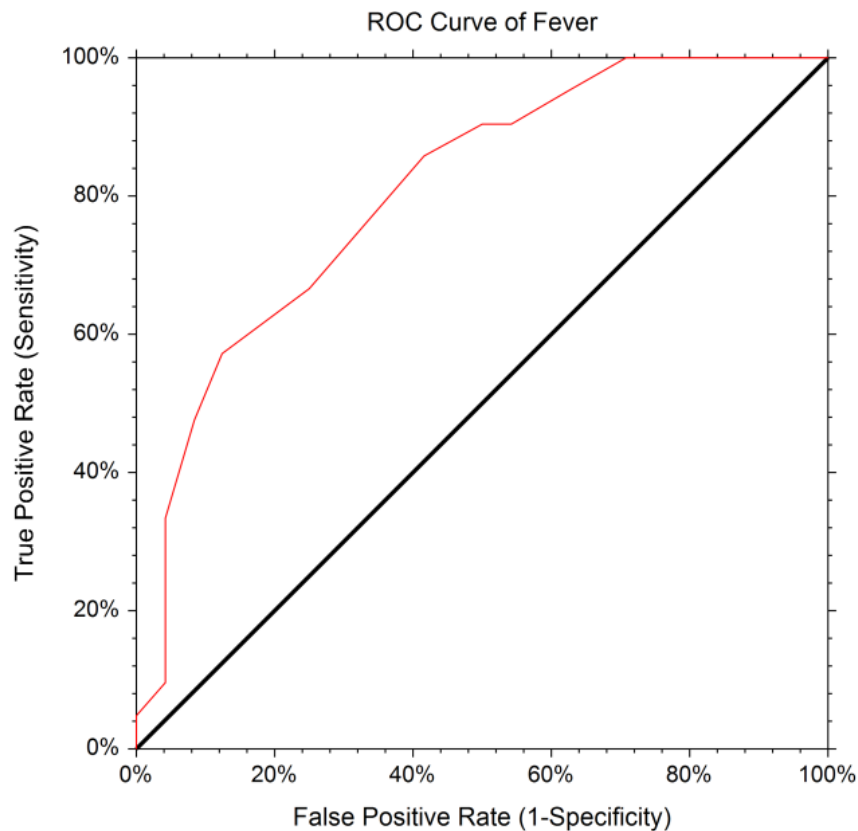
ROC Curves

Each of the rates above are calculated for a given table, based on a single cutoff value. A receiver operating characteristic (ROC) curve plots the true positive rate (sensitivity) against the false positive rate ($1 - \text{specificity}$) for *all possible* cutoff values. General discussions of ROC curves can be found in Altman (1991), Swets (1996), Zhou et al (2002), and Krzanowski and Hand (2009). Gehlbach (1988) provides an example of its use.

Two types of ROC curves can be generated in NCSS: the empirical ROC curve and the binormal ROC curve.

Empirical ROC Curve

The empirical ROC curve is the more common version of the ROC curve. The empirical ROC curve is a plot of the true positive rate versus the false positive rate for all possible cut-off values.



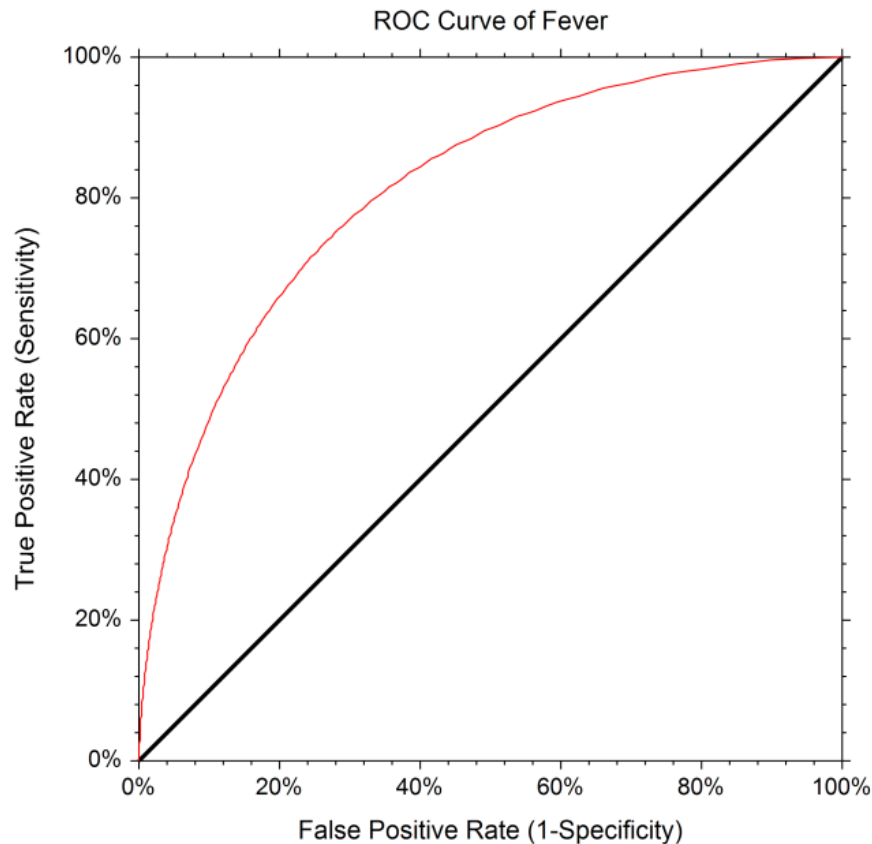
That is, each point on the ROC curve represents a different cutoff value. The points are connected to form the curve. Cutoff values that result in low false-positive rates tend to result low true-positive rates as well. As the true-positive rate increases, the false positive rate increases. The better the diagnostic test, the more quickly the true positive rate nears 1 (or 100%). A near-perfect diagnostic test would have an ROC curve that is almost vertical from (0,0) to (0,1) and then horizontal to (1,1). The diagonal line serves as a reference line since it is the ROC curve of a diagnostic test that randomly classifies the condition.

Binormal ROC Curve

The Binormal ROC curve is based on the assumption that the diagnostic test scores corresponding to the positive condition and the scores corresponding to the negative condition can each be represented by a Normal distribution. To estimate the Binormal ROC curve, the sample mean and sample standard deviation are estimated

One ROC Curve and Cutoff Analysis

from the known positive group, and again for the known negative group. These sample means and sample standard deviations are used to specify two Normal distributions. The Binormal ROC curve is then generated from the two Normal distributions. When the two Normal distributions closely overlap, the Binormal ROC curve is closer to the 45 degree diagonal line. When the two Normal distributions overlap only in the tails, the Binormal ROC curve has a much greater distance from the 45 degree diagonal line.



It is recommended that researchers identify whether the scores for the positive and negative groups need to be transformed to more closely follow the Normal distribution before using the Binormal ROC Curve methods.

Area under the ROC Curve (AUC)

The area under an ROC curve (AUC) is a popular measure of the accuracy of a diagnostic test. In general higher AUC values indicate better test performance. The possible values of AUC range from 0.5 (no diagnostic ability) to 1.0 (perfect diagnostic ability).

The AUC has a physical interpretation. The AUC is the probability that the criterion value of an individual drawn at random from the population of those with a positive condition is larger than the criterion value of another individual drawn at random from the population of those where the condition is negative.

Another interpretation of AUC is the average true positive rate (average sensitivity) across all possible false positive rates.

Two methods are commonly used to estimate the AUC. One method is the empirical (nonparametric) method by DeLong et al. (1988). This method has become popular because it does not make the strong normality assumptions that the Binormal method makes. The other method is the Binormal method presented by Metz (1978) and McClish (1989). This method results in a smooth ROC curve from which the complete (and partial) AUC may be calculated.

One ROC Curve and Cutoff Analysis

AUC of an Empirical ROC Curve

The empirical (nonparametric) method by DeLong et al. (1988) is a popular method for computing the AUC. This method has become popular because it does not make the strong Normality assumptions that the Binormal method makes.

The value of AUC using the empirical method is calculated by summing the area of the trapezoids that are formed below the connected points making up the ROC curve. From DeLong et al. (1988), define the T_1 component of the i^{th} subject, $V(T_{1i})$ as

$$V(T_{1i}) = \frac{1}{n_0} \sum_{j=1}^{n_0} \Psi(T_{1i}, T_{0j})$$

and define the T_0 component of the j^{th} subject, $V(T_{0j})$ as

$$V(T_{0j}) = \frac{1}{n_1} \sum_{i=1}^{n_1} \Psi(T_{1i}, T_{0j})$$

where

$$\begin{aligned} \Psi(X, Y) &= 0 \text{ if } Y > X, \\ \Psi(X, Y) &= 1/2 \text{ if } Y = X, \\ \Psi(X, Y) &= 1 \text{ if } Y < X \end{aligned}$$

The empirical AUC is estimated as

$$A_{Emp} = \sum_{i=1}^{n_1} V(T_{1i})/n_1 = \sum_{j=1}^{n_0} V(T_{0j})/n_0$$

The variance of the estimated AUC is estimated as

$$V(A_{Emp}) = \frac{1}{n_1} S_{T_1}^2 + \frac{1}{n_0} S_{T_0}^2$$

where $S_{T_1}^2$ and $S_{T_0}^2$ are the variances

$$S_{T_i}^2 = \frac{1}{n_i - 1} \sum_{i=1}^{n_i} [V(T_{1i}) - A_{Emp}]^2, \quad i = 0, 1$$

AUC of a Binormal ROC Curve

The formulas that we use here come from McClish (1989). Suppose there are two populations, one made up of individuals with the condition being positive and the other made up of individuals with the negative condition. Further, suppose that the value of a criterion variable is available for all individuals. Let X refer to the value of the criterion variable in the negative population and Y refer to the value of the criterion variable in the positive population. The binormal model assumes that both X and Y are normally distributed with different means and variances. That is,

$$X \sim N(\mu_x, \sigma_x^2), \quad Y \sim N(\mu_y, \sigma_y^2)$$

The ROC curve is traced out by the function

$$\{FP(c), TP(c)\} = \left\{ \Phi\left(\frac{\mu_x - c}{\sigma_x}\right), \Phi\left(\frac{\mu_y - c}{\sigma_y}\right) \right\}, \quad -\infty < c < \infty$$

where $\Phi(z)$ is the cumulative normal distribution function.

One ROC Curve and Cutoff Analysis

The area under the whole ROC curve is

$$\begin{aligned} A &= \int_{-\infty}^{\infty} TP(c)FP'(c) dc \\ &= \int_{-\infty}^{\infty} \left[\Phi\left(\frac{\mu_y - c}{\sigma_y}\right) \phi\left(\frac{\mu_x - c}{\sigma_x}\right) \right] dc \\ &= \Phi\left[\frac{a}{\sqrt{1+b^2}}\right] \end{aligned}$$

where

$$a = \frac{\mu_y - \mu_x}{\sigma_y} = \frac{\Delta}{\sigma_y}, \quad b = \frac{\sigma_x}{\sigma_y}, \quad \Delta = \mu_y - \mu_x$$

The area under a portion of the AUC curve is given by

$$\begin{aligned} A &= \int_{c_1}^{c_2} TP(c)FP'(c) dc \\ &= \frac{1}{\sigma_x} \int_{c_2}^{c_1} \left[\Phi\left(\frac{\mu_y - c}{\sigma_y}\right) \phi\left(\frac{\mu_x - c}{\sigma_x}\right) \right] dc \end{aligned}$$

The partial area under an ROC curve is usually defined in terms of a range of false-positive rates rather than the criterion limits c_1 and c_2 . However, the one-to-one relationship between these two quantities, given by

$$c_i = \mu_x + \sigma_x \Phi^{-1}(FP_i)$$

allows the criterion limits to be calculated from desired false-positive rates.

The MLE of A is found by substituting the MLE's of the means and variances into the above expression and using numerical integration. When the area under the whole curve is desired, these formulas reduce to

$$\hat{A} = \Phi\left[\frac{\hat{a}}{\sqrt{1+\hat{b}^2}}\right]$$

Note that for ease of reading we will often omit the use of the *hat* to indicate an MLE in the following.

The variance of \hat{A} is derived using the method of differentials as

$$V(\hat{A}) = \left(\frac{\partial A}{\partial \Delta}\right)^2 V(\hat{\Delta}) + \left(\frac{\partial A}{\partial \sigma_x^2}\right)^2 V(s_x^2) + \left(\frac{\partial A}{\partial \sigma_y^2}\right)^2 V(s_y^2)$$

where

$$\begin{aligned} \frac{\partial A}{\partial \Delta} &= \frac{E}{\sqrt{2\pi(1+b^2)}\sigma_y^2} [\Phi(\tilde{c}_1) - \Phi(\tilde{c}_0)] \\ \frac{\partial A}{\partial \sigma_x^2} &= \frac{E}{4\pi(1+b^2)\sigma_x\sigma_y} [e^{-k_0} - e^{-k_1}] - \frac{abE}{2\sigma_x\sigma_y\sqrt{2\pi}(1+b^2)^{3/2}} [\Phi(\tilde{c}_1) - \Phi(\tilde{c}_0)] \end{aligned}$$

One ROC Curve and Cutoff Analysis

$$E = \exp\left(-\frac{a^2}{2(1+b^2)}\right)$$

$$\frac{\partial A}{\partial \sigma_y^2} = -\frac{a}{2\sigma_y} \left(\frac{\partial A}{\partial \Delta}\right) - b^2 \left(\frac{\partial A}{\partial \sigma_x^2}\right)$$

$$\tilde{c}_i = \left[\Phi^{-1}(FP_i) + \frac{ab}{(1+b^2)} \right] \sqrt{(1+b^2)}$$

$$k_i = \frac{\tilde{c}_i^2}{2}$$

$$V(\hat{\Delta}) = \frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}$$

$$V(s_x^2) = \frac{2\sigma_x^4}{n_x - 1}$$

$$V(s_y^2) = \frac{2\sigma_y^4}{n_y - 1}$$

AUC Confidence Limits

Once estimates of AUC and the variance of AUC are calculated, hypothesis tests and confidence intervals can be calculated using standard methods. However, following the advice of Zhou et al. (2002) page 125, we use the following transformation which results in statistics that are closer to normality and ensures confidence limits that are inside the zero-one range. The transformation is

$$\hat{\Psi} = \ln\left(\frac{1 + \hat{A}}{1 - \hat{A}}\right)$$

The variance of $\hat{\Psi}$ is estimated using

$$V(\hat{\Psi}) = \frac{4}{(1 - \hat{A}^2)^2} V(\hat{A})$$

An $100(1 - \alpha)\%$ confidence interval for Ψ may then be constructed as

$$L, U = \hat{\Psi} \mp z_{1-\alpha/2} \sqrt{V(\hat{\Psi})}$$

Using the inverse transformation, the confidence interval for AUC is given by the two limits

$$\frac{1 - e^{-L}}{1 + e^{-L}} \quad \text{and} \quad \frac{1 - e^{-U}}{1 + e^{-U}}$$

One ROC Curve and Cutoff Analysis

AUC Hypothesis Tests

A statistical test to determine whether the test has any classification utility is to test the AUC against the value 0.5, which corresponds to the AUC of the 45 degree diagonal line, and represents random classification. For larger sample sizes, the estimated AUC can be assumed to follow the Normal distribution. The test statistic is

$$Z = \frac{A - 0.5}{\sqrt{V(A)}}$$

where A is the AUC estimate. More generally, the AUC estimate may be compared to any value using

$$Z = \frac{A - A_0}{\sqrt{V(A)}}$$

This general form may be used for both the empirical and Binormal methods for estimating AUC.

Finding the Optimal Cutoff Value (Optimal Decision Threshold)

While the ROC curve and corresponding AUC give an overall picture of the behavior of a diagnostic test across all cutoff values, there remains a practical need to determine the specific cutoff value that should be used for individuals requiring diagnosis. If the cost of each diagnostic decision is known, as well as the positive condition prevalence, the optimal cutoff value is the one that minimizes cost, as described under Cost in the Other Diagnostic Accuracy Indices section. However, cost and prevalence values are typically unknown and unattainable. In this case, a recommended approach is to find the cutoff with highest Youden Index, or equivalently, the highest Sensitivity + Specificity (see Krzanowski and Hand, 2009). These indices are also described in the Other Diagnostic Accuracy Indices section.

Data Structure

The data are entered in two or more columns. One column specifies the true condition of the individual. The other column(s) contain the criterion value(s) for the tests being examined.

Criterion dataset

Condition	Score
0	1
0	3
0	4
1	7
0	4
0	5
1	9
1	4
.	.
.	.
.	.

Procedure Options

This section describes the options available in this procedure.

Variables Tab

This panel specifies which variables are used in the analysis.

Variables – Condition Variable

Condition Variable

Specify a binary (two unique values) column which designates whether the individual has the condition of interest. The value representing a positive condition is specified in the Positive Condition Value box. Often a column containing the values 0 and 1 is used. You may type the column name or number directly, or you may use the column selection tool by clicking the column selection button to the right.

Positive Condition Value

Enter the value of the Condition Variable that indicates that the subject has a positive condition. All other values of the Condition Variable are considered to not have a positive condition. Often, the positive value is set to 1 (implying a negative value of 0), but any binary scheme may be used.

Criterion Variable(s)

Criterion Variable(s)

Specify one or more columns giving the scores for each subject. These scores are to be used as criteria for classification of positive (and negative) conditions. If more than one column is listed, a separate analysis is made for each column. You may type the column name(s) or number(s) directly, or you may use the column selection tool by clicking the column selection button to the right.

Criterion Direction

This option indicates whether low or high values of the criterion variable are associated with a positive condition. For example, low values of one criterion variable may indicate the presence of a disease, while high values of another criterion variable may be associated with the presence of the disease.

- **Lower values indicate a Positive Condition**

This selection indicates that a low value of the criterion variable should indicate a positive condition.

- **Higher values indicate a Positive Condition**

This selection indicates that a high value of the criterion variable should indicate a positive condition.

Frequency (Count) Variable

Frequency Variable

Specify an **optional** frequency (count) variable. This data column contains integers that represent the number of observations (frequency) associated with each row of the dataset. If this option is left blank, each dataset row has a frequency of one.

Cutoff Reports Tab

The following options control the cutoff value reports that are displayed.

Cutoff Values Reports

Cutoff Value List

Specify the criterion value cutoffs to be examined on the reports. If **Data** is entered, cutoff reports will be based on all unique criterion values. If you wish to designate a specific set of cutoff values, you can use any of the following methods of entry:

Enter a list using numbers separated by blanks: 1 2 3 4 5

Use the TO BY syntax: xx TO yy BY inc

Use the Colon(Increment) syntax: xx:yy(inc)

For example, entering 1 TO 10 BY 3 or 1:10(3) is the same as entering 1 4 7 10.

Cutoff Values Report

Check this box to obtain a report giving the listed statistics for each of the designated cutoff values.

Known Prevalence for Adjustment

Prevalence is defined as the proportion of individuals in the population that have the condition of interest. The calculations of positive predictive value and negative predictive value in this report require a user-supplied prevalence value. The estimated prevalence from this procedure should only be used here if the entire sample is a random sample of the population.

Known Prevalence for PPV and NPV

Prevalence is defined as the proportion of individuals in the population that have the condition of interest. The calculations of positive predictive value and negative predictive value in this report require a user-supplied prevalence value. The estimated prevalence from this procedure should only be used here if the entire sample is a random sample of the population.

Known Prevalence for Cost

Prevalence is defined as the proportion of individuals in the population that have the condition of interest. The cost index calculations in this report require a user-supplied prevalence value. The estimated prevalence from this procedure should only be used here if the entire sample is a random sample of the population.

Cost Specification

Check to display all reports about a single AUC.

Specify whether costs will be entered directly or as a cost ratio.

- **Enter Costs Directly**

Enter the following costs directly:

Cost of False Positive, C(FP)

Cost of True Negative, C(TN)

One ROC Curve and Cutoff Analysis

Cost of False Negative, C(FN)

Cost of True Positive, C(TP)

These four costs are used in a formula with C(FN) - C(TP) in the denominator, such that C(FN) cannot be equal to C(TP).

- **Enter Cost Ratio(s)**

Up to four cost ratios may be entered. The cost ratio entered here is

$$(C(FP) - C(TN))/(C(FN) - C(TP))$$

More than one cost ratio can be entered as a list: 0.5 0.8 0.9 1.0, or using the special list format: 0.5:0.8(0.1).

Cost of False Positive, C(FP)

Enter the (relative) cost of a false positive result. The four costs are used in the formula

$$(C(FP) - C(TN))/(C(FN) - C(TP))$$

as part of the calculation of the Cost Index. The costs should be chosen such that C(FN) - C(TP) is not 0.

Cost of True Negative, C(TN)

Enter the (relative) cost of a true negative result. The four costs are used in the formula

$$(C(FP) - C(TN))/(C(FN) - C(TP))$$

as part of the calculation of the Cost Index. The costs should be chosen such that C(FN) - C(TP) is not 0.

Cost of False Negative, C(FN)

Enter the (relative) cost of a false negative result. The four costs are used in the formula

$$(C(FP) - C(TN))/(C(FN) - C(TP))$$

as part of the calculation of the Cost Index. The costs should be chosen such that C(FN) - C(TP) is not 0.

Cost of True Positive, C(TP)

Enter the (relative) cost of a true positive result. The four costs are used in the formula

$$(C(FP) - C(TN))/(C(FN) - C(TP))$$

as part of the calculation of the Cost Index. The costs should be chosen such that C(FN) - C(TP) is not 0.

Cost Ratio(s)

Enter up to four values for the cost ratio:

$$(C(FP) - C(TN))/(C(FN) - C(TP))$$

More than one cost ratio can be entered as a list: 0.5 0.8 0.9 1.0, or using the special list format: 0.5:0.8(0.1)

AUC Reports Tab

The following options control the area under the ROC curve reports that are displayed.

AUC Reports

Area Under Curve (AUC) Analysis (Empirical Estimation)

Check this box to obtain a report with the area under the ROC curve (AUC) estimate, as well as the hypothesis test and confidence interval for the AUC. This report is based on the commonly-used empirical (nonparametric) estimation methods.

Area Under Curve (AUC) Analysis (Binormal Estimation)

Check this box to obtain a report with the area under the ROC curve (AUC) estimate, as well as the hypothesis test and confidence interval for the AUC. This report is based on the Binormal estimation methods. The Binormal report permits restricting the curve estimation to a partial area, if desired. The partial area is defined by the Lower and Upper FPR Boundaries.

AUC Reports – AUC Test Details

Test Direction

Specify the direction of the alternative hypothesis of the AUC test.

- **Upper One-Sided**
H0: $AUC = AUC_0$
H1: $AUC > AUC_0$
- **Lower One-Sided**
H0: $AUC = AUC_0$
H1: $AUC < AUC_0$
- **Two-Sided**
H0: $AUC = AUC_0$
H1: $AUC \neq AUC_0$

Null Hypothesis Value (AUC₀)

Enter the null hypothesis value of the AUC test. This is the value to which the AUC value is compared (or tested against).

Report Options Tab

The following options are used in several or all reports.

Confidence Level

Confidence Level

This confidence level is used for all reported confidence intervals in this procedure. Typical confidence levels are 90%, 95%, and 99%, with 95% being the most common.

One ROC Curve and Cutoff Analysis

Report Options

Label Length for New Line

When writing a row of a report, some variable names/labels may be too long to fit in the space allocated. If the name (or label) contains more characters than the value specified here, the remainder of the output for that line is moved to the next line. Most reports are designed to hold a label of up to about 12 characters.

Show Definitions and Notes

Check this box to display definitions and notes associated with each report.

Precision

Specify the precision of numbers in the report. Single precision will display seven-place accuracy, while the double precision will display thirteen-place accuracy. Note that all reports are formatted for single precision only.

Variable Names

Specify whether to use variable names, variable labels, or both to label output reports. In this discussion, the variables are the columns of the data table.

Report Options – Decimal Places

Cutoffs – Z-Values Decimal Places

Specify the number of decimal places used.

Report Options – Page Title

Report Page Title

Specify a page title to be displayed in report headings.

Plots Tab

The options on this panel control the inclusion and the appearance of the ROC Plot.

Select Plots

Empirical and Binormal ROC Plots

Check this box to obtain a ROC plot. Click the Plot Format button to edit the plot. Click the Plot Format button to include the Binormal estimation ROC curve on the plot. Check the box in the upper-right corner of the Plot Format button to edit the ROC plot with the actual ROC plot data, when the procedure is run.

Binormal Line Resolution

Enter the number of locations along the X-axis of the graph at which Binormal estimation should be made for the plot. A value of about 200 is generally acceptable.

One ROC Curve and Cutoff Analysis

Storage Tab

Various rates and statistics may be stored on the current dataset for further analysis. These options let you designate which statistics (if any) should be stored and which columns should receive these statistics. The selected statistics are automatically entered into the current dataset each time you run the procedure.

Note that the columns you specify must already have been named on the current dataset.

Note that existing data is replaced. Be careful that you do not specify columns that contain important data.

Criterion Values for Storage

Criterion Value (Cutoff) List for Storage

Specify the criterion value cutoffs for which values (as specified below) will be stored to the spreadsheet. If **Data** is entered, storage will be based on all unique criterion values. If you wish to designate a specific set of cutoff values, you can use any of the following methods of entry:

Enter a list using numbers separated by blanks: 1 2 3 4 5

Use the TO BY syntax: xx TO yy BY inc

Use the Colon(Increment) syntax: xx:yy(inc)

For example, entering 1 TO 10 BY 3 or 1:10(3) is the same as entering 1 4 7 10.

Storage Columns

Store Value in

Specify the column to which the corresponding values will be stored. Values will be stored for each criterion (cutoff) value specified for 'Criterion Value (Cutoff) List for Storage'. Existing data in the column will be replaced with the new values automatically when the analysis is run. You may type the column name or number directly, or you may use the column selection tool by clicking the column selection button to the right. Stored values are not saved with the spreadsheet until the spreadsheet is saved.

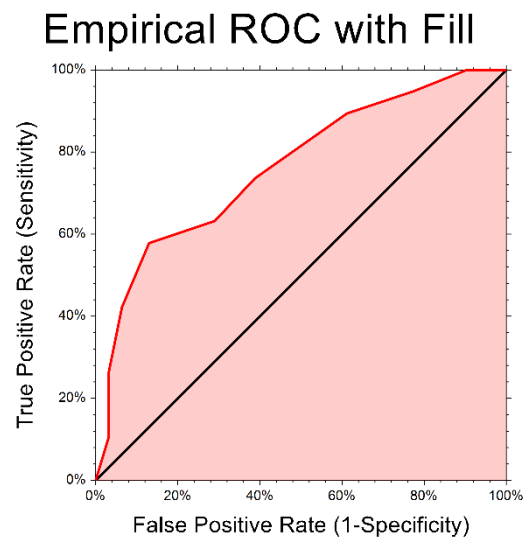
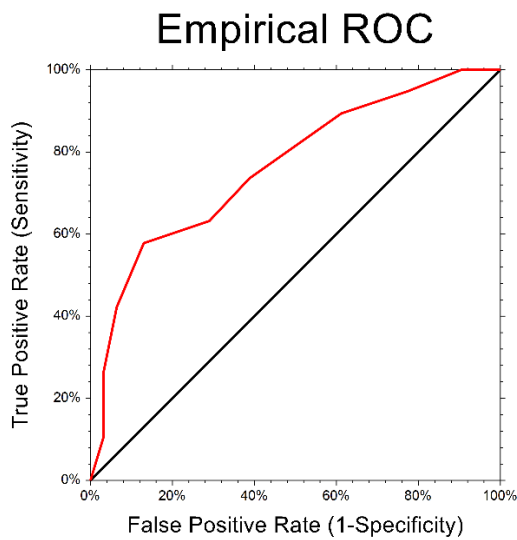
ROC Plot Format Window Options

This section describes some of the options available on the ROC Plot Format window, which is displayed when the ROC Plot Chart Format button is clicked. Common options, such as axes, labels, legends, and titles are documented in the Graphics Components chapter.

ROC Plot Tab

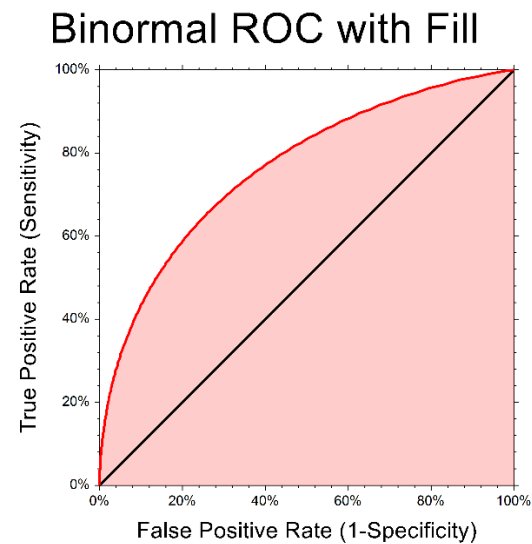
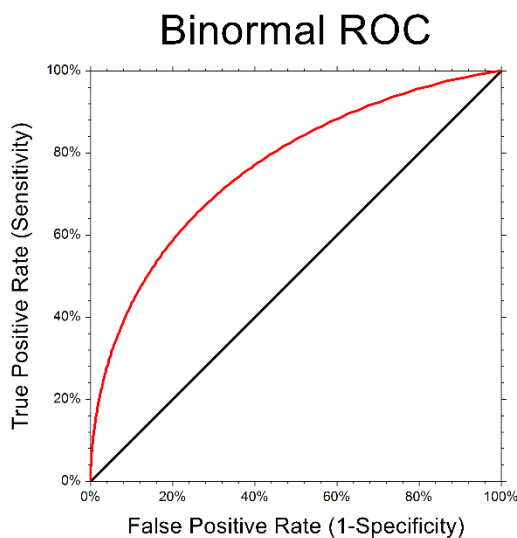
Empirical ROC Line Section

You can specify the format of the empirical ROC curve lines using the options in this section.



Binormal ROC Line Section

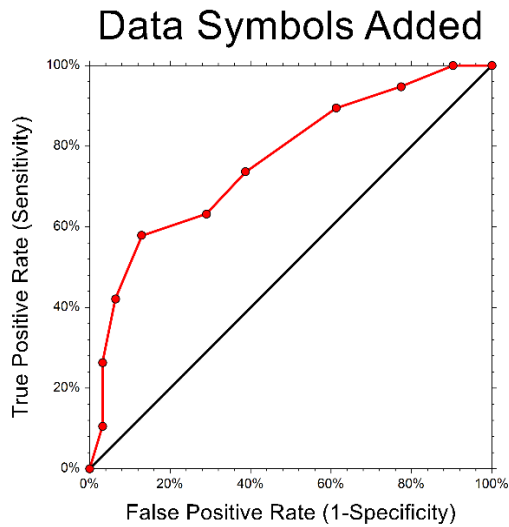
You can specify the format of the binormal ROC curves lines using the options in this section.



One ROC Curve and Cutoff Analysis

Symbols Section

You can modify the attributes of the symbols using the options in this section.



Reference Line Section

You can modify the attributes of the 45° reference line using the options in this section.

Titles, Legend, Numeric Axis, Group Axis, Grid Lines, and Background Tabs

Details on setting the options in these tabs are given in the Graphics Components chapter.

Example 1 – ROC Curve and General Cutoff Analysis

This section presents an example of producing a general cutoff analysis, AUC analysis, and ROC curve, based on the Criterion dataset. In this dataset, a 1 for Condition indicates the condition is present, while a 0 indicates the condition is absent. It is anticipated that higher Score values are associated with the condition being present.

You may follow along here by making the appropriate entries or load the completed template **Example 1** by clicking on Open Example Template from the File menu of the One ROC Curve and Cutoff Analysis window.

1 Open the Criterion dataset.

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Click on the file **Criterion.NCSS**.
- Click **Open**.

2 Open the One ROC Curve and Cutoff Analysis window.

- Using the Analysis menu or the Procedure Navigator, find and select the **One ROC Curve and Cutoff Analysis** procedure.
- On the menus, select **File**, then **New Template**. This will fill the procedure with the default template.

3 Specify the variables.

- On the One ROC Curve and Cutoff Analysis window, select the **Variables** tab.
- Set the **Condition Variable** to **Condition**.
- Set the **Positive Condition Value** to **1**.
- Set the **Criterion Variable** to **Score**.
- Set the **Criterion Direction** to **Higher values indicate a Positive Condition**.

4 Specify the cutoff reports.

- On the ROC Curves window, select the **Cutoff Reports** tab.
- For **Cutoff Value List**, enter **Data**.
- Leave all check boxes checked that are checked by default.

5 Run the procedure.

- From the Run menu, select **Run Procedure**. Alternatively, just click the green Run button.

One ROC Curve and Cutoff Analysis

Common Rates and Indices for each Cutoff Value

Criterion Variable: Score

Estimated Prevalence = $19 / 50 = 0.3800$

Estimated Prevalence is the proportion of the sample with a positive condition of 1, or $(A + C) / (A + B + C + D)$ for all cutoff values. The estimated prevalence should only be used as a valid estimate of the population prevalence when the entire sample is a random sample of the population.

Cutoff Value	Table Counts				TPR (Sens.)	TNR (Spec.)	PPV	Accuracy	TPR + TNR
	TPs A	FPs B	FNs C	TNs D					
≥ 1.00	19	31	0	0	1.0000	0.0000	0.3800	0.3800	1.0000
≥ 2.00	19	28	0	3	1.0000	0.0968	0.4043	0.4400	1.0968
≥ 3.00	18	24	1	7	0.9474	0.2258	0.4286	0.5000	1.1732
≥ 4.00	17	19	2	12	0.8947	0.3871	0.4722	0.5800	1.2818
≥ 5.00	14	12	5	19	0.7368	0.6129	0.5385	0.6600	1.3497
≥ 6.00	12	9	7	22	0.6316	0.7097	0.5714	0.6800	1.3413
≥ 7.00	11	4	8	27	0.5789	0.8710	0.7333	0.7600	1.4499
≥ 8.00	8	2	11	29	0.4211	0.9355	0.8000	0.7400	1.3565
≥ 9.00	5	1	14	30	0.2632	0.9677	0.8333	0.7000	1.2309
≥ 10.00	2	1	17	30	0.1053	0.9677	0.6667	0.6400	1.0730

Definitions:

Cutoff Value indicates the criterion value range that predicts a positive condition.

A is the number of True Positives.

B is the number of False Positives.

C is the number of False Negatives.

D is the number of True Negatives.

TPR is the True Positive Rate or Sensitivity = $A / (A + C)$.

TNR is the True Negative Rate or Specificity = $D / (B + D)$.

PPV is the Positive Predictive Value or Precision = $A / (A + B)$.

Accuracy is the Proportion Correctly Classified = $(A + D) / (A + B + C + D)$.

TPR + TNR is the Sensitivity + Specificity.

The report displays some of the more commonly used rates for each cutoff value.

Counts, TPR, TNR, FNR, FPR

Criterion Variable: Score

Cutoff Value	Table Counts				TPR (Sens.)	TNR (Spec.)	FNR (Miss)	FPR (Fall-out)
	TPs A	FPs B	FNs C	TNs D				
≥ 1.00	19	31	0	0	1.0000	0.0000	0.0000	1.0000
≥ 2.00	19	28	0	3	1.0000	0.0968	0.0000	0.9032
≥ 3.00	18	24	1	7	0.9474	0.2258	0.0526	0.7742
≥ 4.00	17	19	2	12	0.8947	0.3871	0.1053	0.6129
≥ 5.00	14	12	5	19	0.7368	0.6129	0.2632	0.3871
≥ 6.00	12	9	7	22	0.6316	0.7097	0.3684	0.2903
≥ 7.00	11	4	8	27	0.5789	0.8710	0.4211	0.1290
≥ 8.00	8	2	11	29	0.4211	0.9355	0.5789	0.0645
≥ 9.00	5	1	14	30	0.2632	0.9677	0.7368	0.0323
≥ 10.00	2	1	17	30	0.1053	0.9677	0.8947	0.0323

Definitions:

Cutoff Value indicates the criterion value range that predicts a positive condition.

A is the number of True Positives.

B is the number of False Positives.

C is the number of False Negatives.

D is the number of True Negatives.

TPR is the True Positive Rate or Sensitivity = $A / (A + C)$.

TNR is the True Negative Rate or Specificity = $D / (B + D)$.

FNR is the False Negative Rate or Miss Rate = $C / (A + C)$.

FPR is the False Positive Rate or Fall-out = $B / (B + D)$.

The report displays for each cutoff value the counts and rates assuming a true condition.

One ROC Curve and Cutoff Analysis

TPR (Sensitivity) and TNR (Specificity) Confidence Intervals

Criterion Variable: Score

Cutoff Value	TPR (Sens.)	95% Confidence Limits		TNR (Spec.)	95% Confidence Limits	
		Lower	Upper		Lower	Upper
≥ 1.00	1.0000	0.8235	1.0000	0.0000	0.0000	0.1122
≥ 2.00	1.0000	0.8235	1.0000	0.0968	0.0204	0.2575
≥ 3.00	0.9474	0.7397	0.9987	0.2258	0.0959	0.4110
≥ 4.00	0.8947	0.6686	0.9870	0.3871	0.2185	0.5781
≥ 5.00	0.7368	0.4880	0.9085	0.6129	0.4219	0.7815
≥ 6.00	0.6316	0.3836	0.8371	0.7097	0.5196	0.8578
≥ 7.00	0.5789	0.3350	0.7975	0.8710	0.7017	0.9637
≥ 8.00	0.4211	0.2025	0.6650	0.9355	0.7858	0.9921
≥ 9.00	0.2632	0.0915	0.5120	0.9677	0.8330	0.9992
≥ 10.00	0.1053	0.0130	0.3314	0.9677	0.8330	0.9992

Definitions:

Cutoff Value indicates the criterion value range that predicts a positive condition.

TPR is the True Positive Rate or Sensitivity = $A / (A + C)$.

Lower and Upper Confidence Limits form the confidence interval for TPR.

TNR is the True Negative Rate or Specificity = $D / (B + D)$.

Lower and Upper Confidence Limits form the confidence interval for TNR.

The report displays confidence intervals for the sensitivity (true positive rate) and specificity (true negative rate) for each cutoff value.

Counts, PPV, NPV, FOR, FDR

Criterion Variable: Score

Cutoff Value	----- Table Counts -----				PPV (Prec.)	NPV	FOR	FDR
	TPs A	FPs B	FNs C	TNs D				
≥ 1.00	19	31	0	0	0.3800			0.6200
≥ 2.00	19	28	0	3	0.4043	1.0000	0.0000	0.5957
≥ 3.00	18	24	1	7	0.4286	0.8750	0.1250	0.5714
≥ 4.00	17	19	2	12	0.4722	0.8571	0.1429	0.5278
≥ 5.00	14	12	5	19	0.5385	0.7917	0.2083	0.4615
≥ 6.00	12	9	7	22	0.5714	0.7586	0.2414	0.4286
≥ 7.00	11	4	8	27	0.7333	0.7714	0.2286	0.2667
≥ 8.00	8	2	11	29	0.8000	0.7250	0.2750	0.2000
≥ 9.00	5	1	14	30	0.8333	0.6818	0.3182	0.1667
≥ 10.00	2	1	17	30	0.6667	0.6383	0.3617	0.3333

Definitions:

Cutoff Value indicates the criterion value range that predicts a positive condition.

A is the number of True Positives.

B is the number of False Positives.

C is the number of False Negatives.

D is the number of True Negatives.

PPV is the Positive Predictive Value or Precision = $A / (A + B)$.

NPV is the Negative Predictive Value = $D / (C + D)$.

FOR is the False Omission Rate = $C / (C + D)$.

FDR is the False Discovery Rate = $B / (A + B)$.

The report displays for each cutoff value the counts and rates assuming a predicted condition.

One ROC Curve and Cutoff Analysis

Counts, Proportion Correctly Classified, Proportion Incorrectly Classified, Estimated Prevalence

Criterion Variable: Score

Estimated Prevalence = $19 / 50 = 0.3800$

Estimated Prevalence is the proportion of the sample with a positive condition of 1, or $(A + C) / (A + B + C + D)$ for all cutoff values. The estimated prevalence should only be used as a valid estimate of the population prevalence when the entire sample is a random sample of the population.

Cutoff Value	----- Table Counts -----				Proportion Correctly Classified	Proportion Incorrectly Classified
	TPs A	FPs B	FNs C	TNs D		
≥ 1.00	19	31	0	0	0.3800	0.6200
≥ 2.00	19	28	0	3	0.4400	0.5600
≥ 3.00	18	24	1	7	0.5000	0.5000
≥ 4.00	17	19	2	12	0.5800	0.4200
≥ 5.00	14	12	5	19	0.6600	0.3400
≥ 6.00	12	9	7	22	0.6800	0.3200
≥ 7.00	11	4	8	27	0.7600	0.2400
≥ 8.00	8	2	11	29	0.7400	0.2600
≥ 9.00	5	1	14	30	0.7000	0.3000
≥ 10.00	2	1	17	30	0.6400	0.3600

Definitions:

Cutoff Value indicates the criterion value range that predicts a positive condition.

A is the number of True Positives.

B is the number of False Positives.

C is the number of False Negatives.

D is the number of True Negatives.

Proportion Correctly Classified or Accuracy = $(A + D) / (A + B + C + D)$.

Proportion Incorrectly Classified = $(B + C) / (A + B + C + D)$.

The report displays the estimated prevalence, the counts, and the proportions correctly and incorrectly classified, for each cutoff value.

Counts, Youden Index, Sensitivity + Specificity, Distance to Corner, LR+, LR-, DOR

Criterion Variable: Score

Cutoff Value	----- Table Counts -----				Youden Index	Sens. + Spec.	Dist. to Corner	LR+	LR-	DOR (LR+/LR-)
	TPs A	FPs B	FNs C	TNs D						
≥ 1.00	19	31	0	0	0.0000	1.0000	1.0000	1.0000		
≥ 2.00	19	28	0	3	0.0968	1.0968	0.9032	1.1071	0.0000	
≥ 3.00	18	24	1	7	0.1732	1.1732	0.7760	1.2237	0.2331	5.2500
≥ 4.00	17	19	2	12	0.2818	1.2818	0.6219	1.4598	0.2719	5.3684
≥ 5.00	14	12	5	19	0.3497	1.3497	0.4681	1.9035	0.4294	4.4333
≥ 6.00	12	9	7	22	0.3413	1.3413	0.4691	2.1754	0.5191	4.1905
≥ 7.00	11	4	8	27	0.4499	1.4499	0.4404	4.4868	0.4834	9.2813
≥ 8.00	8	2	11	29	0.3565	1.3565	0.5825	6.5263	0.6189	10.5455
≥ 9.00	5	1	14	30	0.2309	1.2309	0.7375	8.1579	0.7614	10.7143
≥ 10.00	2	1	17	30	0.0730	1.0730	0.8953	3.2632	0.9246	3.5294

Definitions:

Cutoff Value indicates the criterion value range that predicts a positive condition.

A is the number of True Positives.

B is the number of False Positives.

C is the number of False Negatives.

D is the number of True Negatives.

Youden Index is the Sensitivity + Specificity - 1.

Sensitivity + Specificity is the True Positive Rate plus the True Negative Rate.

Distance to Corner is the distance from the top left corner of the ROC curve to the point on the ROC curve.

LR+ is the Positive Likelihood Ratio, or the ratio of TPR (Sensitivity) to FPR (1 - Specificity).

LR- is the Negative Likelihood Ratio, or the ratio of FNR to TNR.

DOR is the Diagnostic Odds Ratio, or the ratio of LR+ to LR-.

The report displays other useful diagnostic accuracy indices for each cutoff value.

One ROC Curve and Cutoff Analysis

Area Under Curve Analysis (Empirical Estimation)

Area Under Curve Analysis (Empirical Estimation)

Estimated Prevalence = 19 / 50 = 0.3800

Estimated Prevalence is the proportion of the sample with a positive condition of 1. The estimated prevalence should only be used as a valid estimate of the population prevalence when the entire sample is a random sample of the population.

Criterion	Count	AUC	Standard Error	Z-Value to Test AUC > 0.5	Upper 1-Sided P-Value	95% Confidence Limits	
Score	50	0.7640	0.0710	3.720	0.0001	Lower	Upper
						0.5860	0.8717

Definitions:

Criterion is the Criterion Variable containing the scores of the individuals.

Count is the number of the individuals used in the analysis.

AUC is the area under the ROC curve using the empirical (trapezoidal) approach.

Standard Error is the standard error of the AUC estimate.

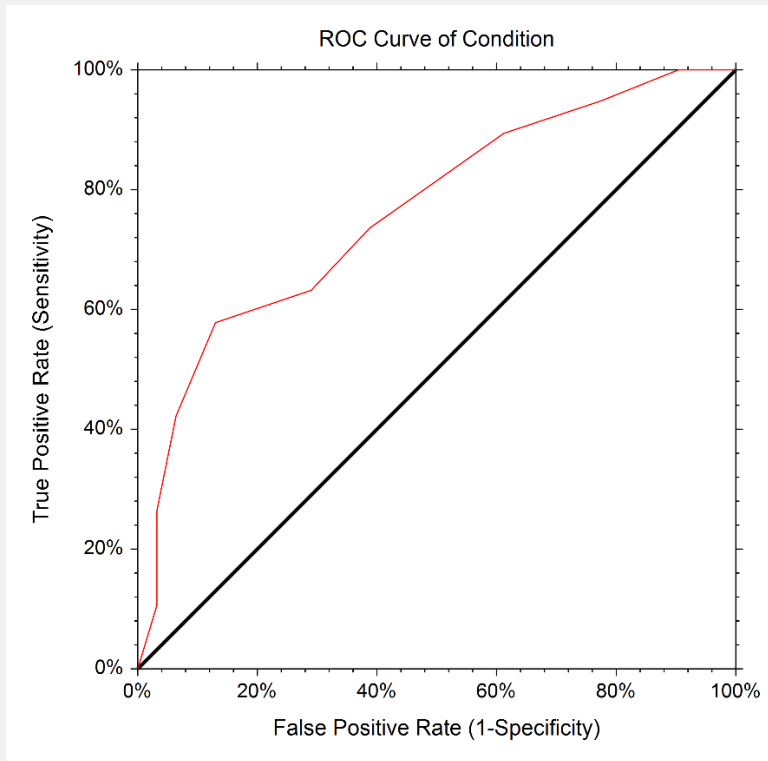
Z-Value is the Z-score for testing the designated hypothesis test.

P-Value is the probability level associated with the Z-Value.

The Lower and Upper Confidence Limits form the confidence interval for AUC.

This report gives a statistical test comparing the area under the curve to the value 0.5. The small P-value indicates a significant difference from 0.5. The report also gives the 95% confidence interval for the estimated AUC.

ROC Plot Section



The plot can be made to contain the empirical ROC curve, the Binormal ROC curve, or both, by making the proper selection after clicking the ROC Plot Format button.

The coordinates of the points of this ROC curve are the TPR and FPR for each of the unique Score values. The diagonal (45 degree) line is an ROC curve of random classification, and serves as a baseline. The ROC curve shows the overall ability of using the score to classify the condition. This curve shows a moderate classification ability of the score to classify the condition. The ROC curve for an ideal test would go up to 100% quickly on the left side and then stay level at 100% across the top.

Example 2 – Choosing the Optimal Cutoff Value

This section presents an example of choosing the optimal cutoff value, based on the Criterion dataset.

You may follow along here by making the appropriate entries or load the completed template **Example 2** by clicking on Open Example Template from the File menu of the One ROC Curve and Cutoff Analysis window.

1 Open the Criterion dataset.

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Click on the file **Criterion.NCSS**.
- Click **Open**.

2 Open the One ROC Curve and Cutoff Analysis window.

- Using the Analysis menu or the Procedure Navigator, find and select the **One ROC Curve and Cutoff Analysis** procedure.
- On the menus, select **File**, then **New Template**. This will fill the procedure with the default template.

3 Specify the variables.

- On the One ROC Curve and Cutoff Analysis window, select the **Variables** tab.
- Set the **Condition Variable** to **Condition**.
- Set the **Positive Condition Value** to **1**.
- Set the **Criterion Variable** to **Score**.
- Set the **Criterion Direction** to **Higher values indicate a Positive Condition**.

4 Specify the cutoff reports.

- On the ROC Curves window, select the **Cutoff Reports** tab.
- For **Cutoff Value List**, enter **Data**.
- Check the box next to **Counts, TPR (Sensitivity), TNR (Specificity), PPV, Accuracy, TPR + TNR, Prevalence**
- Check the box next to **Counts, Youden Index, Sensitivity + Specificity, Distance to Corner, LR+, LR-, DOR**

5 Run the procedure.

- From the Run menu, select **Run Procedure**. Alternatively, just click the green Run button.

One ROC Curve and Cutoff Analysis

Common Rates and Indices for each Cutoff Value

Criterion Variable: Score

Estimated Prevalence = $19 / 50 = 0.3800$

Estimated Prevalence is the proportion of the sample with a positive condition of 1, or $(A + C) / (A + B + C + D)$ for all cutoff values. The estimated prevalence should only be used as a valid estimate of the population prevalence when the entire sample is a random sample of the population.

Cutoff Value	----- Table Counts -----				TPR (Sens.)	TNR (Spec.)	PPV	Accuracy	TPR + TNR
	TPs A	FPs B	FNs C	TNs D					
≥ 1.00	19	31	0	0	1.0000	0.0000	0.3800	0.3800	1.0000
≥ 2.00	19	28	0	3	1.0000	0.0968	0.4043	0.4400	1.0968
≥ 3.00	18	24	1	7	0.9474	0.2258	0.4286	0.5000	1.1732
≥ 4.00	17	19	2	12	0.8947	0.3871	0.4722	0.5800	1.2818
≥ 5.00	14	12	5	19	0.7368	0.6129	0.5385	0.6600	1.3497
≥ 6.00	12	9	7	22	0.6316	0.7097	0.5714	0.6800	1.3413
≥ 7.00	11	4	8	27	0.5789	0.8710	0.7333	0.7600	1.4499
≥ 8.00	8	2	11	29	0.4211	0.9355	0.8000	0.7400	1.3565
≥ 9.00	5	1	14	30	0.2632	0.9677	0.8333	0.7000	1.2309
≥ 10.00	2	1	17	30	0.1053	0.9677	0.6667	0.6400	1.0730

Definitions:

Cutoff Value indicates the criterion value range that predicts a positive condition.

A is the number of True Positives.

B is the number of False Positives.

C is the number of False Negatives.

D is the number of True Negatives.

TPR is the True Positive Rate or Sensitivity = $A / (A + C)$.

TNR is the True Negative Rate or Specificity = $D / (B + D)$.

PPV is the Positive Predictive Value or Precision = $A / (A + B)$.

Accuracy is the Proportion Correctly Classified = $(A + D) / (A + B + C + D)$.

TPR + TNR is the Sensitivity + Specificity.

The cutoff value giving the highest accuracy (proportion correctly classified) and the highest TPR + TNR (Sensitivity + Specificity) is 7.

One ROC Curve and Cutoff Analysis

Counts, Youden Index, Sensitivity + Specificity, Distance to Corner, LR+, LR-, DOR

Criterion Variable: Score

Cutoff Value	----- Table Counts -----				Youden Index	Sens. + Spec.	Dist. to Corner	LR+	LR-	DOR (LR+/LR-)
	TPs A	FPs B	FNs C	TNs D						
≥ 1.00	19	31	0	0	0.0000	1.0000	1.0000	1.0000	0.0000	
≥ 2.00	19	28	0	3	0.0968	1.0968	0.9032	1.1071	0.0000	
≥ 3.00	18	24	1	7	0.1732	1.1732	0.7760	1.2237	0.2331	5.2500
≥ 4.00	17	19	2	12	0.2818	1.2818	0.6219	1.4598	0.2719	5.3684
≥ 5.00	14	12	5	19	0.3497	1.3497	0.4681	1.9035	0.4294	4.4333
≥ 6.00	12	9	7	22	0.3413	1.3413	0.4691	2.1754	0.5191	4.1905
≥ 7.00	11	4	8	27	0.4499	1.4499	0.4404	4.4868	0.4834	9.2813
≥ 8.00	8	2	11	29	0.3565	1.3565	0.5825	6.5263	0.6189	10.5455
≥ 9.00	5	1	14	30	0.2309	1.2309	0.7375	8.1579	0.7614	10.7143
≥ 10.00	2	1	17	30	0.0730	1.0730	0.8953	3.2632	0.9246	3.5294

Definitions:

Cutoff Value indicates the criterion value range that predicts a positive condition.

A is the number of True Positives.

B is the number of False Positives.

C is the number of False Negatives.

D is the number of True Negatives.

Youden Index is the Sensitivity + Specificity - 1.

Sensitivity + Specificity is the True Positive Rate plus the True Negative Rate.

Distance to Corner is the distance from the top left corner of the ROC curve to the point on the ROC curve.

LR+ is the Positive Likelihood Ratio, or the ratio of TPR (Sensitivity) to FPR (1 - Specificity).

LR- is the Negative Likelihood Ratio, or the ratio of FNR to TNR.

DOR is the Diagnostic Odds Ratio, or the ratio of LR+ to LR-.

The cutoff value corresponding to the point with the lowest distance to the upper-left corner of the ROC curve is also 7. Based on these criteria, the optimal cutoff value is 7. If the condition prevalence is known, as well as the relative cost of false positives, true negatives, false negatives, and true positives, a cost approach may then be used to determine the optimal cutoff value.

Example 3 – Choosing the Optimal Cutoff Value using the Cost Approach

When the condition prevalence is known, as well as the relative cost of false positives, true negatives, false negatives, and true positives, a cost approach may then be used to determine the optimal cutoff value. For details, see the Cost section under Other Diagnostic Accuracy Details of this chapter. In this example, suppose that condition prevalence is known to be 0.16. The relative cost of false positives, true negatives, false negatives, and true positives is estimated to be 4.2, 1, 7.7, and 1, respectively. The dataset used is the Criterion dataset.

You may follow along here by making the appropriate entries or load the completed template **Example 3** by clicking on Open Example Template from the File menu of the One ROC Curve and Cutoff Analysis window.

1 Open the Criterion dataset.

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Click on the file **Criterion.NCSS**.
- Click **Open**.

2 Open the One ROC Curve and Cutoff Analysis window.

- Using the Analysis menu or the Procedure Navigator, find and select the **One ROC Curve and Cutoff Analysis** procedure.
- On the menus, select **File**, then **New Template**. This will fill the procedure with the default template.

3 Specify the variables.

- On the One ROC Curve and Cutoff Analysis window, select the **Variables** tab.
- Set the **Condition Variable** to **Condition**.
- Set the **Positive Condition Value** to **1**.
- Set the **Criterion Variable** to **Score**.
- Set the **Criterion Direction** to **Higher values indicate a Positive Condition**.

4 Specify the cutoff reports.

- On the ROC Curves window, select the **Cutoff Reports** tab.
- For **Cutoff Value List**, enter **Data**.
- Check the box next to **Cost Analysis**.
- For **Known Prevalence for Cost**, enter **0.16**.
- For **C(FP)**, **C(TN)**, **C(FN)**, and **C(TP)**, enter **4.2**, **1**, **7.7**, and **1**, respectively.

5 Run the procedure.

- From the Run menu, select **Run Procedure**. Alternatively, just click the green Run button.

One ROC Curve and Cutoff Analysis

Cost Analysis

Cost Analysis

Criterion Variable: Score

Known Prevalence for Cost: 0.1600

Cost of False Positive, C(FP): 4.2

Cost of True Negative, C(TN): 1

Cost of False Negative, C(FN): 7.7

Cost of True Positive, C(TP): 1

Cost Ratio = $(C(\text{FP}) - C(\text{TN})) / (C(\text{FN}) - C(\text{TP})) = 0.4776$

Value	Sensitivity	Specificity	Cost Index
≥ 1.00	1.0000	0.0000	-1.5075
≥ 2.00	1.0000	0.0968	-1.2648
≥ 3.00	0.9474	0.2258	-0.9939
≥ 4.00	0.8947	0.3871	-0.6421
≥ 5.00	0.7368	0.6129	-0.2338
≥ 6.00	0.6316	0.7097	-0.0964
≥ 7.00	0.5789	0.8710	0.2554
≥ 8.00	0.4211	0.9355	0.2593
≥ 9.00	0.2632	0.9677	0.1823
≥ 10.00	0.1053	0.9677	0.0244

Definitions:

Cutoff Value indicates the criterion value range that predicts a positive condition.

Sensitivity is the True Positive Rate = $A / (A + C)$.

Specificity is the True Negative Rate = $D / (B + D)$.

Cost Index is derived from the Known Prevalence, Sensitivity, Specificity, and the user-input costs. The optimal cutoff value is the one with the highest Cost Index.

For this scenario, the Cost Index associated with the cutoff value of 8 is slightly higher than the Cost Index associated with the cutoff value of 7. Using the cost approach, given the prevalence value of 0.16 and the four specified relative costs, the optimal cutoff value is 8.

Example 4 – ROC Curve and Cutoff Analysis using Binormal Estimation

This section presents an example of producing a general cutoff analysis, AUC analysis, and ROC curve, using the Binormal estimation methods. The dataset used in this example is the Criterion dataset. In this dataset, a 1 for Condition indicates the condition is present, while a 0 indicates the condition is absent. It is anticipated that higher Score values are associated with the condition being present.

You may follow along here by making the appropriate entries or load the completed template **Example 4** by clicking on Open Example Template from the File menu of the One ROC Curve and Cutoff Analysis window.

1 Open the Criterion dataset.

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Click on the file **Criterion.NCSS**.
- Click **Open**.

2 Open the One ROC Curve and Cutoff Analysis window.

- Using the Analysis menu or the Procedure Navigator, find and select the **One ROC Curve and Cutoff Analysis** procedure.
- On the menus, select **File**, then **New Template**. This will fill the procedure with the default template.

3 Specify the variables.

- On the One ROC Curve and Cutoff Analysis window, select the **Variables** tab.
- Set the **Condition Variable** to **Condition**.
- Set the **Positive Condition Value** to **1**.
- Set the **Criterion Variable** to **Score**.
- Set the **Criterion Direction** to **Higher values indicate a Positive Condition**.

4 Specify the cutoff reports.

- On the ROC Curves window, select the **Cutoff Reports** tab.
- For **Cutoff Value List**, enter **Data**.
- Uncheck all check boxes that are checked by default.
- Check the box for **Rates from Binormal Estimation** under Other Diagnostic Accuracy Indices for each Cutoff Value.
- Enter **0.16** for **Known Prevalence for PPV and NPV**.

5 Specify the AUC reports.

- On the ROC Curves window, select the **AUC Reports** tab.
- Uncheck all check boxes that are checked by default.
- Check the box for **Area Under Curve (AUC) Analysis (Binormal Estimation)**.
- Leave the boundaries at 0 and 1.

6 Specify the ROC Plot.

- On the ROC Curves window, select the **Plots** tab.
- Click on the **Plot Format** button.
- Uncheck **Empirical ROC Line**.
- Check **Binormal ROC Line**.
- Click **OK**.

7 Run the procedure.

- From the Run menu, select **Run Procedure**. Alternatively, just click the green Run button.

One ROC Curve and Cutoff Analysis

Rates from Binormal Estimation

Criterion Variable: Score
 Known Prevalence for Adjustment: 0.1600

Cutoff Value	TPR (Sens.)	TNR (Spec.)	FNR (Miss)	FPR (Fall-out)	Known Prevalence Adjusted PPV	Known Prevalence Adjusted NPV	LR+	Sens. + Spec.
≥ 1.00	0.9891	0.0673	0.0000	1.0000	0.1680	0.9699	1.0604	1.0563
≥ 2.00	0.9700	0.1509	0.0000	0.9032	0.1787	0.9636	1.1425	1.1210
≥ 3.00	0.9293	0.2848	0.0526	0.7742	0.1984	0.9548	1.2993	1.2141
≥ 4.00	0.8553	0.4583	0.1053	0.6129	0.2312	0.9433	1.5789	1.3136
≥ 5.00	0.7417	0.6403	0.2632	0.3871	0.2820	0.9286	2.0618	1.3820
≥ 6.00	0.5940	0.7947	0.3684	0.2903	0.3554	0.9113	2.8940	1.3888
≥ 7.00	0.4313	0.9009	0.4211	0.1290	0.4533	0.8927	4.3538	1.3323
≥ 8.00	0.2797	0.9600	0.5789	0.0645	0.5712	0.8749	6.9924	1.2397
≥ 9.00	0.1599	0.9866	0.7368	0.0323	0.6946	0.8604	11.9418	1.1465
≥ 10.00	0.0799	0.9963	0.8947	0.0323	0.8046	0.8504	21.6148	1.0762

Definitions:

Cutoff Value indicates the criterion value range that predicts a positive condition.
 TPR is the Binormal-estimated True Positive Rate or Sensitivity.
 TNR is the Binormal-estimated True Negative Rate or Specificity.
 FNR is the Binormal-estimated False Negative Rate or Miss Rate.
 FPR is the Binormal-estimated False Positive Rate or Fall-out.
 Adjusted PPV is the Positive Predictive Value based on the Sensitivity, Specificity, and Known Prevalence (see documentation for formula).
 Adjusted NPV is the Negative Predictive Value based on the Sensitivity, Specificity, and Known Prevalence (see documentation for formula).
 LR+ is the Positive Likelihood Ratio, or the ratio of TPR (Sensitivity) to FPR (1 - Specificity).
 Sensitivity + Specificity is the True Positive Rate plus the True Negative Rate.

The report displays a variety of rates for each cutoff value. The highest Sensitivity + Specificity corresponds to a cutoff value of 6.

Area Under Curve Analysis (Binormal Estimation)

Estimated Prevalence = 19 / 50 = 0.3800
 Estimated Prevalence is the proportion of the sample with a positive condition of 1. The estimated prevalence should only be used as a valid estimate of the population prevalence when the entire sample is a random sample of the population.

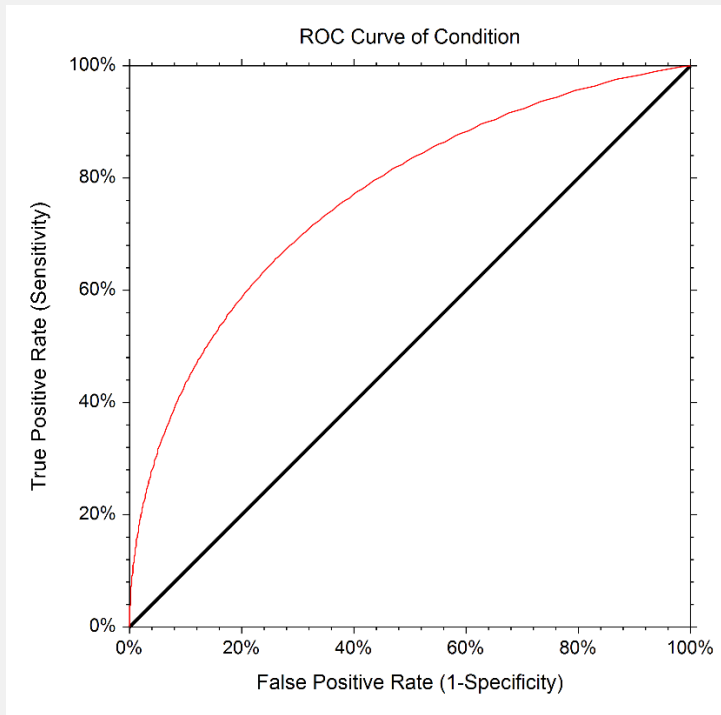
Criterion	Count	AUC	Standard Error	Z-Value to Test AUC > 0.5	Upper 1-Sided P-Value	95% Confidence Limits	
						Lower	Upper
Score	50	0.7654	0.0686	3.868	0.0001	0.5944	0.8702

Definitions:

Criterion is the Criterion Variable containing the scores of the individuals.
 Count is the number of the individuals used in the analysis.
 AUC is the area under the ROC curve using the Binormal estimation approach.
 Standard Error is the standard error of the AUC estimate.
 Z-Value is the Z-score for testing the designated hypothesis test.
 P-Value is the probability level associated with the Z-Value.
 The Lower and Upper Confidence Limits form the confidence interval for AUC.

This report gives a statistical test comparing the area under the curve to the value 0.5. The small P-value indicates a significant difference from 0.5. The report also gives the 95% confidence interval for the estimated AUC. These values are very close to those obtained when using empirical estimation.

ROC Plot Section



The plot can be made to contain the empirical ROC curve, the Binormal ROC curve, or both, by making the proper selection after clicking the ROC Plot Format button.

The Binormal estimation ROC plot is a smooth curve estimation of the true ROC curve. The diagonal (45 degree) line is an ROC curve of random classification, and serves as a baseline. The Binormal estimation ROC plot and the empirical estimation ROC plot can be superimposed in one plot using the plot format button (**Example 4b**):

