Chapter 490

Shortest Route

Introduction

Given a directed network defined by nodes and arcs, this procedure finds the shortest route between two specified nodes. One example of the need for such an algorithm is to be used in a GPS device to find the shortest route between two locations.

There are specialized algorithms that can be used to solve for the shortest path among several possible paths. Taha (2011) shows that the problem can be solved using a linear programming algorithm. This is the approach we have taken in **NCSS**.

Data Structure

This procedure requires a special data format in which the rows represent the arcs (branches) between two locations (nodes). Each branch is defined by two nodes and the positive distance between them. The flow is always from the first node to the second node.

Consider the following transportation example. Each row represents a road (arc) between two cities (nodes). The cities are identified by letters and the distance in miles between the cities is the given. For this example, we have to assume that all roads are one-way from the first listed city to the second. These data are stored in the dataset *Short Path*.

Short Path dataset

City1	City2	Distance
A	B	6.8
A	D	6.2
B	C	9.1
B	D	8.5
B	E	7.2
C	E	10.3
D	E	8.8
D	F	5.6
E	F	2.5
E	G	9.4
F	G	12.3

Linear Programming Formulation of the Shortest Path Problem

As stated earlier, we use a linear programming algorithm to solve for the shortest path. Rather than present all the equations, we show how the above example is translated into a linear programming tableau. The LP tableau for the above data is

		X_{AB}	X_{AD}	X_{BC}	X_{BD}	X_{BE}	X_{CE}	X_{DE}	X_{DF}	X_{EF}	X_{EG}	X_{FG}	RHS
Minimize	z =	6.8	6.2	9.1	8.5	7.2	10.3	8.8	5.6	2.5	9.4	12.3	
City													
A		1	1										= 1
B		-1		1	1	1							= 0
C				-1			1						= 0
D			-1		-1			1	1				= 0
E						-1	-1	-1		1	1		= 0
F									-1	-1		1	= 0
G											-1	-1	= -1

Procedure Options

This section describes the options available in this procedure.

Specifications Tab

Set the specifications for the analysis.

Starting and Ending Nodes of Each Arc

Node 1 (Starting) Column

Specify the column containing the starting node of an arc. Each row gives the specification of a single, directed arc by giving its beginning node and ending node as well as the length from the start to the end. Thus the order of the arcs is critical. The starting node must be entered first.

A node may be a number or a text value.

Other names for nodes are vertices and points.

Here, we define a *directed arc* as a beginning node and an ending node along with the distance between them. Other names for arcs are edges, roads, and arcs.

Node 2 (Ending) Column

Specify the column containing the ending node of an arc. Each row gives the specification of a single, directed arc by giving its beginning node and ending node as well as the length from the start to the end. Thus the order of the arcs is critical. The starting node must be entered first.

A node may be a number or a text value.

Other names for nodes are vertices and points.

Here, we define a *directed arc* as a beginning node followed by an ending node along with the distance between them. Other names for arcs are edges, lines, and links.

Arc Lengths

Length Column

Specify the column containing the length (distance or cost) between the two nodes of an arc. The values should be positive numbers.

Starting and Ending Nodes of the Path

Node Values Input Type

Specify how you will enter the node names of the starting and ending nodes between which the shortest path is to be determined. The possible selections are

Use the first and last node names after sorting

The names of the nodes will be sorted and the first node name will be designated as the starting node value and the last node name will be designated as the ending node value.

Enter custom starting and stopping values

Two text boxes will appear that will allow you to enter the names of the starting and ending nodes.

Path Starting Node Value

Enter the value (number or text) of the starting node of the path. The procedure will find the shortest path from this node to the ending node. Note that this node can be in the middle of the nodes.

Path Ending Node Value

Enter the value (number or text) of the ending node of the path. The procedure will find the shortest path from the starting node to this node. Note that this node can be in the middle of the nodes.

Reports Tab

Select Reports

Shortest Path Solution – Possible Network Arcs

Indicate which reports you want to view.

Report Options

Column Names

This option lets you select whether to display only variable names, variable labels, or both.

Example 1 – Shortest Route through Several Cities

This section presents an example of how to find the shortest route between cities A and G in the Short Path dataset. As you use this example, you need to keep in mind that all roads are one-way roads.

Short Path dataset

City1	City2	Distance
A	B	6.8
A	D	6.2
B	C	9.1
B	D	8.5
B	E	7.2
C	E	10.3
D	E	8.8
D	F	5.6
E	F	2.5
E	G	9.4
F	G	12.3

You may follow along here by making the appropriate entries or load the completed template **Example 1** by clicking on Open Example Template from the File menu of the Shortest Path window.

1 Open the Short Path dataset.

- From the **File** menu of the NCSS Data window, select **Open Example Data**.
- Click on the file **Short Path.NCSS**.
- Click **Open**.

2 Open the Shortest Path window.

- Using the Analysis menu or the Procedure Navigator, find and select the **Shortest Route** procedure.
- On the menus, select **File**, then **New Template**. This will fill the procedure with the default template.

3 Specify the problem.

- On the **Shortest Route** window, select the **Specifications tab**.
- Double-click in the **Node 1 (Starting) Column** text box. This will bring up the column selection window.
- Select **City1** from the list of columns and then click **Ok**. "City1" will appear in this box.
- Double-click in the **Node 2 (Ending) Column** text box. This will bring up the column selection window.
- Select **City2** from the list of columns and then click **Ok**. "City2" will appear in this box.
- Double-click in the **Length Column** text box. This will bring up the column selection window.
- Select **Distance** from the list of columns and then click **Ok**. "Distance" will appear in this box.
- Set the **Node Value Input Type** to **Enter custom starting and stopping values**.
- Set **Path Starting Node Value** to **A**.
- Set **Path Ending Node Value** to **G.**

4 Run the procedure.

- From the **Run** menu, select **Run Procedure**. Alternatively, just click the green Run button.

Shortest Path

Shortest Path

Arc (Row)	Distance (Length)	City1 (Start)	City2 (End)
1	6.8	A (Start)	B
5	7.2	B	E
10	9.4	E	G (End)
Total	23.4		

This report lists the solution by giving the arcs that form the shortest path. The starting node is identified with the word *Start*. The ending city is identified with the word *End*.

The shortest path is A, B, E, and G.

Possible Network Arcs

Arc (Row)	Distance (Length)	City1 (Start)	City2 (End)
1	6.8*	A (Start)	B
2	6.2	A (Start)	D
3	9.1	B	C
4	8.5	B	D
5	7.2*	B	E
6	10.3	C	E
7	8.8	D	E
8	5.6	D	F
9	2.5	E	F
10	9.4*	E	G (End)
11	12.3	F	G (End)

* The starred arcs are on the shortest path.

This report lists all the arcs from which the shortest path derived. The starting node is identified with the word *Start*. The ending node is identified with the word *End*.