

Chapter 585

Tolerance Intervals

Introduction

This procedure calculates one-, and two-, sided tolerance intervals based on either a distribution-free (nonparametric) method or a method based on a normality assumption (parametric). A two-sided *tolerance interval* consists of two limits between which a given proportion β of the population falls with a given confidence level $1 - \alpha$. A one-sided tolerance interval is similar, but consists of a single upper or lower limit.

Technical Details

Let X_1, X_2, \dots, X_n be a random sample for a population with distribution function $F(X)$. A $(\beta, 1 - \alpha)$ two-sided β -content tolerance interval (T_L, T_U) is defined by

$$\Pr[F(T_U) - F(T_L) \geq \beta] \geq 1 - \alpha$$

A $(\beta, 1 - \alpha)$ lower, one-sided β -content tolerance bound T_L is defined by

$$\Pr[1 - F(T_L) \geq \beta] \geq 1 - \alpha$$

A $(\beta, 1 - \alpha)$ upper, one-sided β -content tolerance bound T_U is defined by

$$\Pr[F(T_U) \geq \beta] \geq 1 - \alpha$$

Note that a one-sided tolerance limit is the same as the one-sided confidence limit of the quantile of F .

Distribution-Free Tolerance Intervals

The definition of two-sided distribution-free tolerance intervals is found in many places. We use the formulation given by Bury (1999). The only distributional assumption made about F is that it is a continuous, non-decreasing, probability distribution. That is, these intervals should not be used with discrete data. Given this, the tolerance limits are

$$T_L = X_{(r)}, \quad T_U = X_{(s)}$$

where r and s are two order indices. The values of r and s are determined using the formula

$$\sum_{i=0}^{n-2c} \binom{n}{i} \beta^i (1 - \beta)^{n-i} \geq 1 - \alpha$$

where

$$r = c$$

$$s = n - c + 1$$

The value of c is found as the largest value for which the above inequality is true.

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A lower, one-sided tolerance bound is $X_{(r)}$ where r is the largest value for with the following inequality is true.

$$\sum_{i=0}^{n-r} \binom{n}{i} \beta^i (1-\beta)^{n-i} \geq 1-\alpha$$

An upper, one-sided tolerance bound is $X_{(s)}$ where s is the largest value for with the following inequality is true.

$$\sum_{i=0}^{s-1} \binom{n}{i} \beta^i (1-\beta)^{n-i} \geq 1-\alpha$$

Normal-Distribution Tolerance Interval

The limits discussed in this section are based on the assumption that F is the normal distribution.

Two-Sided Limits

In this case, the two-sided tolerance interval is defined by the interval

$$T_L = \bar{x} - ks, \quad T_U = \bar{x} + ks$$

The construction reduces to the determination of the constant k . Howe (1969) provides the following approximation which is ‘nearly’ exact for all values of n greater than one

$$k = uvw$$

where

$$u = z_{\frac{1+\beta}{2}} \sqrt{1 + \frac{1}{n}}$$

$$v = \sqrt{\frac{n-1}{\chi_{n-1,\alpha}^2}}$$

$$w = \sqrt{1 + \frac{n-3 - \chi_{n-1,\alpha}^2}{2(n+1)^2}}$$

Note that originally, Howe (1969) used $n-2$ in the above definition of w . But Guenther (1977) gives the corrected version using $n-3$ shown above.

One-Sided Bound

A one-sided tolerance bound (‘bound’ is used instead of ‘limit’ in the one-sided case) is given by

$$T_U = \bar{x} + ks$$

Here k is selected so that

$$\Pr(t'_{n-1,\delta} = k\sqrt{n}) = 1-\alpha$$

where $t'_{f,\delta}$ represents a noncentral t distribution with f degrees of freedom and noncentrality

$$\delta = z_\beta \sqrt{n}.$$

Data Structure

The data are contained in a single column.

Procedure Options

This section describes the options available in this procedure. To find out more about using a procedure, turn to the Procedures chapter.

Following is a list of the procedure's options.

Variables Tab

The options on this panel specify which variables to use.

Data Variables

Variables

Specify a list of one or more variables for which tolerance intervals are to be generated. You can double-click the field or single click the button on the right of the field to bring up the Variable Selection window.

Group Variable

You can specify a grouping variable. When specified, a separate set of reports is generated for each unique value of this variable.

Frequency Variable

Frequency Variable

This optional variable specifies the number of observations (counts) that each row represents. When omitted, each row represents a single observation. If your data is the result of a previous summarization, you may want certain rows to represent several observations. Note that negative values are treated as a zero count and are omitted.

Population Percentages

Population Percentages for Tolerances

Specify a list of percentages for which tolerance intervals are to be calculated. Note that a tolerance interval is a pair of numbers between which a specified percentage of the population falls. This value is that specified percentage.

In the list, numbers are separated by blanks or commas. Specify sequences with a colon, putting the increment inside parentheses. For example: 5:25(5) means 5 10 15 20 25.

All values in the list must be between 1 and 99.

Data Transformation Options

Exponent

Occasionally, you might want to obtain a statistical report on the square root or log of your variable. This option lets you specify an on-the-fly transformation of the variable. The form of this transformation is $X = Y^A$, where Y is the original value, A is the selected exponent, and X is the resulting value.

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Additive Constant

Occasionally, you might want to obtain a statistical report on a transformed version of a variable. This option lets you specify an on-the-fly transformation of the variable. The form of this transformation is $X = Y+B$, where Y is the original value, B is the specified constant, and X is the value that results.

Note that if you apply both the *Exponent* and the *Additive Constant*, the form of the transformation is

$$X = (Y + B)^A .$$

Reports Tab

The options on this panel control the reports and plots displayed.

Select Reports

Descriptive Statistics ... Normality Tests

Indicate whether to display the indicated reports.

Report Options

Alpha Level

This is the value of alpha for the confidence limits and rejection decisions. Usually, this number will range from 0.1 to 0.001. The default value of 0.05 results in 95% tolerance limits.

Precision

Specify the precision of numbers in the report. A single-precision number will show seven-place accuracy, while a double-precision number will show thirteen-place accuracy. The double-precision option only works when the Decimals option is set to General.

Note that the reports were formatted for single precision. If you select double precision, some numbers may run into others. Also note that all calculations are performed in double precision regardless of which option you select here. This is for reporting purposes only.

Variable Names

This option lets you select whether to display only variable names, variable labels, or both.

Value Labels

This option applies to the *Group Variable*. It lets you select whether to display data values, value labels, or both. Use this option if you want the output to automatically attach labels to the values (like 1=Yes, 2=No, etc.). See the section on specifying *Value Labels* elsewhere in this manual.

Report Options – Decimal Places

Values ... Probabilities Decimals

Specify the number of digits after the decimal point to display on the output of values of this type. Note that this option in no way influences the accuracy with which the calculations are done.

Enter 'General' to display all digits available. The number of digits displayed by this option is controlled by whether the PRECISION option is SINGLE or DOUBLE.

Plots Tab

The options on this panel control the appearance of the histogram and probability plot.

Select Plots

Histogram and Probability Plot

Indicate whether to display these plots. Click the plot format button to change the plot settings.

Example 1 – Generating Tolerance Intervals

This section presents a detailed example of how to generate tolerance intervals for the *Height* variable in the Height dataset. To run this example, take the following steps (note that step 1 is not necessary if the Height dataset is open):

- 1 **Open the Height dataset.**
 - From the **File** menu of the NCSS Data window, select **Open Example Data**.
 - Click on the file **Height.NCSS**.
 - Click **Open**.
- 2 **Open the Tolerance Intervals window.**
 - Using the Analysis menu or the Procedure Navigator, find and select the **Tolerance Intervals** procedure.
 - On the menus, select **File**, then **New Template**. This will fill the procedure with the default template.
- 3 **Specify the options on the Variables tab.**
 - On the Tolerance Intervals window, select the **Variables tab**. (This is the default.)
 - Double-click in the **Variables** text box. This will bring up the variable selection window.
 - Select **Height** from the list of variables and then click **Ok**.
 - Set the **Population Percentages** to **50 75 80 90 95 99**.
- 4 **Specify the options on the Reports tab.**
 - On the Tolerance Intervals window, select the **Reports tab**.
 - Set the **Decimals-Values** to **3**.
 - Set the **Decimals-Means** to **3**.
 - Set the **Decimals-Probabilities** to **2**.
- 5 **Run the procedure.**
 - From the Run menu, select **Run Procedure**. Alternatively, just click the green Run button.

The following reports and charts will be displayed in the Output window.

Descriptive Statistics

Count	Mean	Standard Deviation	Standard Error	Minimum	Maximum	Range
20	62.100	8.441	1.887	51.000	79.000	28.000

This report was defined and discussed in the Descriptive Statistics procedure chapter. We refer you to the Summary Section of that chapter for details.

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Two-Sided Tolerance Intervals

Percent of Population Between Limits	Parametric Lower Tolerance Limit	Parametric Upper Tolerance Limit	Nonparametric Lower Tolerance Limit	Nonparametric Upper Tolerance Limit
50.00	54.074	70.126	52.000	73.000
75.00	48.411	75.789	51.000	79.000
80.00	46.850	77.350		
90.00	42.527	81.673		
95.00	38.777	85.423		
99.00	31.449	92.751		

This section gives the parametric and nonparametric two-sided tolerance intervals.

Percent of Population Between Limits

This is the percentage of population values that are contained in the tolerance interval.

Parametric Lower (Upper) Tolerance Limits

These are the values of the limits of a tolerance interval based on the assumption that the population is normally distributed.

Nonparametric Lower (Upper) Tolerance Limits

These are the values of the limits of a distribution-free tolerance interval. These intervals make no distributional assumption.

Lower One-Sided Tolerance Bounds

Percent of Population Greater Than Bound	Parametric Lower Tolerance Bound	Nonparametric Lower Tolerance Bound
50.00	60.264	56.000
75.00	52.254	52.000
80.00	50.524	51.000
90.00	45.842	
95.00	41.875	
99.00	34.285	

This section gives the parametric and nonparametric one-sided tolerance bounds.

Percent of Population Greater Than Bound

This is the percentage of population values that are above the tolerance bound.

Parametric Lower Tolerance Bound

This is the lower parametric (normal distribution) tolerance bound.

Nonparametric Lower (Upper) Tolerance Limits

This is the lower nonparametric (distribution-free) tolerance bound. Note that some values are missing because of the small sample size in this example.

Upper One-Sided Tolerance Bounds

Percent of Population Less Than Bound	Parametric Upper Tolerance Bound	Nonparametric Upper Tolerance Bound
50.00	63.936	65.000
75.00	71.946	73.000
80.00	73.676	76.000
90.00	78.358	79.000
95.00	82.325	79.000
99.00	89.915	79.000

This section gives the parametric and nonparametric one-sided tolerance bounds.

Percent of Population Less Than Bound

This is the percentage of population values that are below the tolerance bound.

Parametric Lower Tolerance Bound

This is the upper parametric (normal distribution) tolerance bound.

Nonparametric Lower (Upper) Tolerance Limits

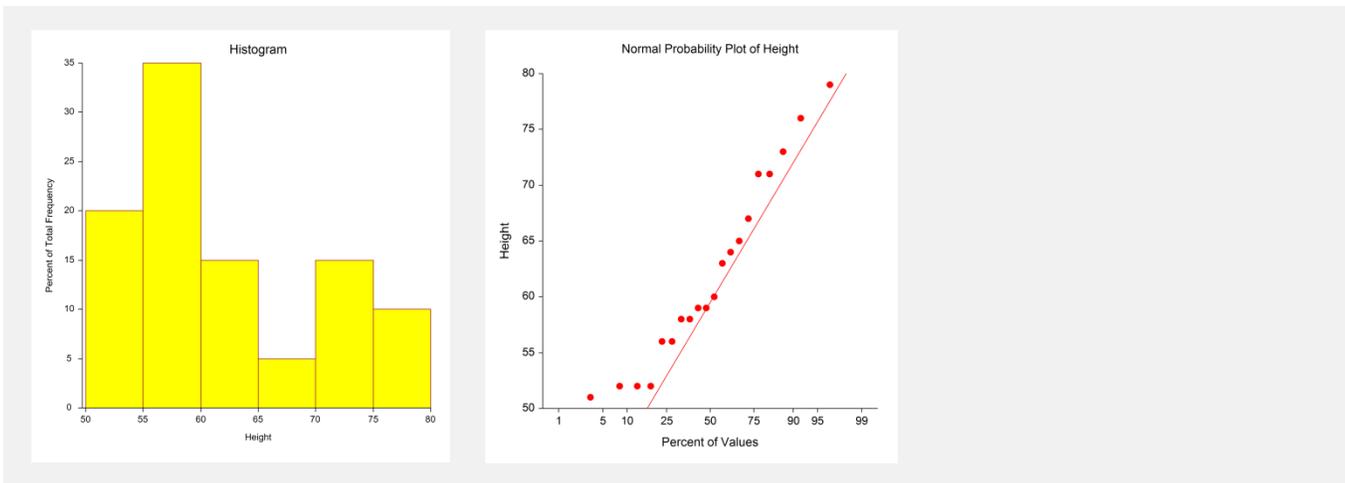
This is the upper nonparametric (distribution-free) tolerance bound.

Normality Test Section

Test Name	Test Value	Prob Level	10% Critical Value	5% Critical Value	Decision (5%)
Shapiro-Wilk W	0.937	0.21			Can't reject normality
Anderson-Darling	0.427	0.31			Can't reject normality
Kolmogorov-Smirnov	0.148		0.176	0.192	Can't reject normality
D'Agostino Skewness	1.037	0.30	1.645	1.960	Can't reject normality
D'Agostino Kurtosis	-0.786	0.43	1.645	1.960	Can't reject normality
D'Agostino Omnibus	1.6918	0.43	4.605	5.991	Can't reject normality

This report was defined and discussed in the Descriptive Statistics procedure chapter. We refer you to the Normality Test Section of that chapter for details.

Plots Section



The plots section displays a histogram and a probability plot to allow you to assess the accuracy of the normality assumption.