Chapter 901

Bayesian Adjustment using the Posterior Error Approach

Introduction

This procedure implements a general approach for determining sample size that combines both the frequentist and Bayesian framework. The approach was proposed by Lee and Zelen (2000) and summarized in Chow, Shoa, Wang, and Lokhnygina (2018).

The approach may be outlined as follows. Consider planning an experiment to evaluate a noncentrality parameter δ which is often a function of a difference and its associated standard deviation. Instead of basing the sample size on the usual frequentist error rates α and β, Lee and Zelen (2000) suggested that Bayesian posterior error rates could be used. These Bayesian error rates can then be transformed into frequentist error rates which can then be used in the frequentist sample size formulas. This approach assumes that the actual data analysis would follow the frequentist methods.

Error Rate Formulas

The frequentist approach searches for the minimum sample size so the following error rates are achieved

$$\alpha = P(C = +|T = -)$$

$$\beta = P(C = -|T = +)$$

where C is a binary variable (+ or -) that gives the result of the experiment. The positive result is to conclude that $\delta \neq 0$ and the negative result is to conclude that $\delta = 0$. Similarly, T is another binary variable (+ or -) that gives the true state of the hypotheses. That is, positive implies that actually $\delta \neq 0$ and the negative sign implies that $\delta = 0$.

The Bayesian approach defines two posterior error rates

$$\alpha^* = P(T = +|C = -)$$

$$\beta^* = P(T = -|C = +)$$

These are the posterior probabilities of the true state of nature being the opposite of the outcome of the trial.

The Bayesian approach defines another parameter θ which is the prior probability that T = +.

$$\theta = P(T = +).$$

With these definitions, Lee and Zelen (2000) formulate the relationship between these two sets of error rates as

$$1 - \alpha^* = \frac{(1-\alpha)(1-\theta)}{[(1-\alpha)(1-\theta) + \beta\theta]}$$

$$1 - \beta^* = \frac{(1-\beta)\theta}{[(1-\beta)\theta + \alpha(1-\theta)]}$$

These equations can be inverted to obtain conversion formulas from Bayesian to frequentist as follows

$$\alpha = \frac{(1-P_2)(\theta + P_1 - 1)}{(1-\theta)(P_1 + P_2 - 1)}$$

$$\beta = \frac{(1-P_1)(P_2 - \theta)}{\theta(P_1 + P_2 - 1)}$$

where $P_1 = 1 - \alpha^*$ and $P_2 = 1 - \beta^*$. To avoid negative probabilities, these formulas require $P_1 > 1 - \theta$ and $P_2 > \theta$ or $P_1 < 1 - \theta$ and $P_2 < \theta$.

Hence, one simply specifies $\alpha^*, \beta^*,$ and θ using the Bayesian approach and translates this to α and β in the frequentist approach. These frequentist error rates can used in any PASS procedure that calls for these error rates.

Procedure Options

This section describes the options that are specific to this procedure. These are located on the Design tab. For more information about the options of other tabs, go to the Procedure Window chapter.

Design Tab

The Design tab contains most of the parameters and options that you will be concerned with.

Solve For

Solve For

This option specifies the parameter to be calculated from the values of the other parameters. Under most conditions, you would either select *Frequentist* or *Bayesian* results.

Test Direction

Alternative Hypothesis

This option specifies the whether the null hypothesis is one-sided or two-sided.

Probabilities

θ [Prior Pr(T = +)]

Enter one or more values for the prior probability that T = + (or $\delta \neq 0$). If you don't have any idea, use $\theta = 0.5$.

Range

$0 < \theta < 1$.

α* [Posterior Error Pr(T = +|C = -)]

Enter one or more values for the posterior error probability that T = + (or $\delta \neq 0$) given that the conclusion of the experiment was the opposite. Lee and Zelen (2000) recommend values ≤ 0.15.

Range

$0 < \alpha^* < 1$. Note that $1 - \alpha^* > 1 - \theta$ and $1 - \beta^* > \theta$ or $1 - \alpha^* < 1 - \theta$ and $1 - \beta^* < \theta$.

β* [Posterior Error Pr(T = -|C = +)]

Enter one or more values for the posterior error probability that T = - (or $\delta = 0$) given that the conclusion of the experiment was the opposite. Lee and Zelen (2000) recommend values ≤ 0.05.

Range

$0 < \beta^* < 1$. Note that $1 - \alpha^* > 1 - \theta$ and $1 - \beta^* > \theta$ or $1 - \alpha^* < 1 - \theta$ and $1 - \beta^* < \theta$.

α [Type I Error Pr(C = +|T = -)]

Enter one or more values for the Type I error probability of reject a true null hypothesis.

Range

Typical value is 0.05. $0 < \alpha < 1$.

β [Type II Error Pr(C = -|T = +)]

Enter one or more values for the Type II error probability of failing to reject a false null hypothesis.

Note that Power = $1 - \beta$.

Range

Typical values are 0.1 or 0.2. $0 < \beta < 1$.

Example 1 – Validation: Finding α and β

Lee and Zelen (2000) on page 98 present a table of values of α and β given various values of α*, β*, and θ. A portion of that table is given next (in reverse order).

Θ	α*	β*	α	β
0.25	0.05	0.05	0.0148	0.1556
0.25	0.10	0.05	0.0118	0.3294
0.25	0.15	0.05	0.0083	0.5250

We will validate this procedure by duplicating these values.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Bayesian Adjustment using the Posterior Error Approach** procedure window. You may then make the appropriate entries as listed below, or open **Example 1** by going to the **File** menu and choosing **Open Example Template**.

Option   **Value**

Design Tab

Solve For .... **Frequentist Parameters (α,β)**
Alternative Hypothesis .... **Two-Sided**
θ [Prior Pr(T = +)] **0.25**
α* [Posterior Error Pr(T = +|C = -)] **0.05 0.1 0.15**
β* [Posterior Error Pr(T = -|C = +)] **0.05**

Annotated Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results ——
Two-sided alternative hypothesis

Prior P(T = +) θ	Posterior Error P(T=+|C=-) α*	Posterior Error P(T=-|C=+) β*	1-α* P1	1-β* P2	Type I Error P(C=+|T=-) α	Type II Error P(C=-|T=+) β	1-β Power
0.2500	0.050	0.050	0.950	0.950	0.0148	0.1556	0.8444
0.2500	0.100	0.050	0.900	0.950	0.0118	0.3294	0.6706
0.2500	0.150	0.050	0.850	0.950	0.0083	0.5250	0.4750

References

Lee, S.J. and Zelen, M. 2000. 'Clinical Trials and Sample Size Considerations: Another Perspective.'
    Statistical Science, Vol. 15, No. 2, Pages 95-110.
Chow, S.C., Shao, J., Wang, H., and Lokhnygina, Y. 2018. Sample Size Calculations in Clinical Research, Third
    Edition. Taylor & Francis/CRC. Boca Raton, Florida.

Bayesian Adjustment using the Posterior Error Approach

Report Definitions
C = Conclusion is δ≠0 (+ or -).
T = True State is δ≠0 (+ or -).
δ is the effect parameter of interest (not specified).
θ [Prior Prob(T = +)].
α* [Posterior Error Prob(T = +|C = -)].
β* [Posterior Error Prob(T = -|C = +)].
P1 [Posterior Correct Prob(T = -|C = -)].
P2 [Posterior Correct Prob(T = +|C = +)].
α [Type I Error Prob(C = +|T = -)].
β [Type II Error Prob(C = -|T = +)].
Power = 1 - β.

Summary Statements ───
This routine converts Bayesian design parameters to frequentist design parameters.
Specifically, the posterior error probabilities α* (0.050) and β* (0.050) along with the prior
probability θ (0.2500) that T = + (δ ≠ 0) are converted to the common frequentist error
probabilities α (0.0148) and β (0.1556).

This report duplicates the values from Lee and Zelen (2000) exactly, thus validating the procedure.