Chapter 255

Cochran-Armitage Test for Trend in Proportions

Introduction

This module computes power and sample size for the Cochran-Armitage test for a linear trend in proportions based on the results in Nam (1987). Asymptotic and exact power calculations for the uncorrected and continuity-corrected tests are available. The results assume that the proportions follow a linear trend on the logistic scale, with *X* being the covariate (or dose) variable, and that random samples are drawn from *k* separate populations.

Technical Details

Suppose we have *k* independent binomial variates y_i, with response probabilities p_i based on samples of size n_i at covariate (or dose) levels x_i, for *i* = 1, 2, …, *k*, where $x_1 < x_2 < ... < x_k$. Define the following:

$$N = \sum_{i=1}^{k} n_i$$

$$\bar{p} = \frac{1}{N}\sum_{i=1}^{k} y_i$$

$$\bar{q} = 1 - \bar{p}$$

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{k} n_i x_i$$

If we assume that the probability of response follows a linear trend on the logistic scale, then

$$p_i = \frac{\exp(\alpha + \beta x_i)}{1 + \exp(\alpha + \beta x_i)}.$$

Hypothesis Tests

The Cochran-Armitage test can be used to test the following hypotheses:

One-Sided (Increasing Trend) $H_0: p_1 = p_2 = ... = p_k$ vs. $H_1: p_1 < p_2 < ... < p_k$

One-Sided (Decreasing Trend) $H_0: p_1 = p_2 = ... = p_k$ vs. $H_1: p_1 > p_2 > ... > p_k$

Two-Sided $H_0: p_1 = p_2 = ... = p_k$ vs. $H_1: p_1 < p_2 < ... < p_k$ or $p_1 > p_2 > ... > p_k$

One-Sided Test of Increasing Linear Trend in Proportions

Continuity-Corrected Test

Nam (1987) presents the following continuity-corrected asymptotic test statistic for detecting an increasing linear trend in proportions

$$z_{c.c.} = \frac{\sum_{i=1}^{k} y_i (x_i - \bar{x}) - \frac{\Delta}{2}}{\sqrt{\bar{p}\bar{q}\left[\sum_{i=1}^{k} n_i (x_i - \bar{x})^2\right]}}.$$

The factor $\Delta/2$ is the continuity correction. If the covariates x_i are equally-spaced then

$$\Delta = x_{i+1} - x_i \text{ for all } i < k$$

or the interval between adjacent covariates. ***PASS*** computes Δ for unequally-spaced covariates as

$$\Delta = \frac{1}{k-1} \sum_{i=1}^{k-1} (x_{i+1} - x_i).$$

For the case of unequally-spaced covariates, Nam (1987) has this to say, "For unequally spaced doses, no constant correction is adequate for all outcomes." Therefore, we caution against the use of the continuity-corrected test statistic in the case of unequally-spaced covariates.

The test rejects H_0 if $z_{c.c.} \geq z_{1-\alpha}$, where $z_{1-\alpha}$ is the value that leaves $1 - \alpha$ in the upper tail of the standard normal distribution.

Uncorrected Test

The uncorrected test statistic is equivalent to the corrected test statistic except that $\Delta = 0$,

$$z = \frac{\sum_{i=1}^{k} y_i (x_i - \bar{x})}{\sqrt{\bar{p}\bar{q}\left[\sum_{i=1}^{k} n_i (x_i - \bar{x})^2\right]}}.$$

The test rejects H_0 if $z \geq z_{1-\alpha}$, where $z_{1-\alpha}$ is the value that leaves $1 - \alpha$ in the upper tail of the standard normal distribution.

One-Sided Test of Decreasing Linear Trend in Proportions

Continuity-Corrected Test

Nam (1987) presents a continuity-corrected asymptotic test statistic for detecting an increasing linear trend in proportions. The continuity-corrected test statistic for a decreasing trend is the same as that for an increasing trend, except that $\Delta/2$ is added in the numerator instead of subtracted

$$z_{c.c.} = \frac{\sum_{i=1}^{k} y_i (x_i - \bar{x}) + \frac{\Delta}{2}}{\sqrt{\overline{pq}\left[\sum_{i=1}^{k} n_i (x_i - \bar{x})^2\right]}}.$$

The factor Δ is defined the same as in the case of a test for increasing trend, and the caution about the use of the continuity-corrected test statistic in the case of unequally-spaced covariates also applies here.

The test rejects H_0 if $z_{c.c.} \le z_\alpha$, where z_α is the value that leaves α in the lower tail of the standard normal distribution.

Uncorrected Test

The uncorrected test statistic is equivalent to the corrected test statistic except that $\Delta = 0$,

$$z = \frac{\sum_{i=1}^{k} y_i (x_i - \bar{x})}{\sqrt{\overline{pq}\left[\sum_{i=1}^{k} n_i (x_i - \bar{x})^2\right]}}.$$

The test rejects H_0 if $z \le z_\alpha$, where z_α is the value that leaves α in the lower tail of the standard normal distribution.

Two-Sided Test for Linear Trend in Proportions

Continuity-Corrected Test

Nam (1987) presents a continuity-corrected asymptotic test statistic for detecting an increasing linear trend in proportions. A two-sided test statistic utilizes a combination of the upper- and lower-tailed test statistics.

$$z_{c.c.U} = \frac{\sum_{i=1}^{k} y_i (x_i - \bar{x}) - \frac{\Delta}{2}}{\sqrt{\overline{pq}\left[\sum_{i=1}^{k} n_i (x_i - \bar{x})^2\right]}} \quad \text{and} \quad z_{c.c.L} = \frac{\sum_{i=1}^{k} y_i (x_i - \bar{x}) + \frac{\Delta}{2}}{\sqrt{\overline{pq}\left[\sum_{i=1}^{k} n_i (x_i - \bar{x})^2\right]}}.$$

The factor Δ is defined the same as in the case of a test for increasing trend, and the caution about the use of the continuity-corrected test statistic in the case of unequally-spaced covariates also applies here.

The test rejects H_0 if $z_{c.c.U} \ge z_{1-\alpha/2}$ or if $z_{c.c.L} \le z_{\alpha/2}$.

Uncorrected Test

The uncorrected test statistic is the same as the corrected test statistic except that $\Delta = 0$, which reduces the upper- and lower-tailed statistics to a single test statistic

$$z = \frac{\sum_{i=1}^{k} y_i (x_i - \bar{x})}{\sqrt{\bar{p}\bar{q}\left[\sum_{i=1}^{k} n_i (x_i - \bar{x})^2\right]}}.$$

The test rejects H_0 if $|z| \geq z_{1-\alpha/2}$.

Exact Power Calculations

The power for the previous test statistics that are based on the normal approximation can be computed exactly using the binomial distribution. The following steps are taken to compute exact power.

One-Sided Test of Increasing Linear Trend in Proportions

1. Find the critical value using the standard normal distribution. The critical value, $z_{critical}$, is that value of z that leaves exactly the target value of alpha in the upper tail of the normal distribution. For example, for an upper-tailed test (increasing trend) with a target alpha of 0.05, the critical value is 1.645.
2. Compute the value of the test statistic, z_t, for every **y**, where $\mathbf{y} = (y_1, y_2, ..., y_k)$. Note that y_1 ranges from 0 to n_1, y_2 ranges from 0 to n_2, and so on. The test statistic z_t can be either the corrected or uncorrected test statistic.
3. If $z_t \geq z_{critical}$, the combination is in the rejection region. Call all **y** that lead to a rejection the set *A*.
4. Compute the power for given values of $\mathbf{p} = (p_1, p_2, ..., p_k)$ as

$$1 - \beta = \sum_{A} \left\{ \prod_{i=1}^{k} \left[\binom{n_i}{y_i} p_i^{y_i} (1 - p_i)^{n_i - y_i} \right] \right\}$$

When the values of n_i are large (say over 50) or *k* is large (say over 5), these formulas may take a little time to evaluate. In this case, a large sample approximation may be used.

One-Sided Test of Decreasing Linear Trend in Proportions

1. Find the critical value using the standard normal distribution. The critical value, $z_{critical}$, is that value of z that leaves exactly the target value of alpha in the upper tail of the normal distribution. For example, for a lower-tailed test (decreasing trend) with a target alpha of 0.05, the critical value is 1.645.
2. Compute the value of the test statistic, z_t, for every **y**, where $\mathbf{y} = (y_1, y_2, ..., y_k)$. Note that y_1 ranges from 0 to n_1, y_2 ranges from 0 to n_2, and so on. The test statistic z_t can be either the corrected or uncorrected test statistic.
3. If $z_t \leq -z_{critical}$, the combination is in the rejection region. Call all **y** that lead to a rejection the set *A*.
4. Compute the power for given values of $\mathbf{p} = (p_1, p_2, ..., p_k)$ as

$$1 - \beta = \sum_{A} \left\{ \prod_{i=1}^{k} \left[\binom{n_i}{y_i} p_i^{y_i} (1 - p_i)^{n_i - y_i} \right] \right\}$$

When the values of n_i are large (say over 50) or k is large (say over 5), these formulas may take a little time to evaluate. In this case, a large sample approximation may be used.

Two-Sided Test of Linear Trend in Proportions

1. Find the critical value using the standard normal distribution. The critical value, $z_{critical}$, is that value of z that leaves exactly alpha/2 in the upper tail of the normal distribution. For example, for a two-sided test with a target alpha of 0.05, the critical value is 1.96.
2. Compute the value of the test statistics, z_U and z_L, for every **y**, where $\mathbf{y} = (y_1, y_2, ..., y_k)$. Note that y_1 ranges from 0 to n_1, y_2 ranges from 0 to n_2, and so on. The test statistics z_U and z_L can be either the corrected or uncorrected test statistics. In the case of the uncorrected test, $z_U = z_L$.
3. If $z_U \geq z_{critical}$ or $z_L \leq -z_{critical}$, the combination is in the rejection region. Call all **y** that lead to a rejection the set *A*.
4. Compute the power for given values of $\mathbf{p} = (p_1, p_2, ..., p_k)$ as

$$1 - \beta = \sum_A \left\{ \prod_{i=1}^{k} \left[\binom{n_i}{y_i} p_i^{y_i} (1 - p_i)^{n_i - y_i} \right] \right\}$$

When the values of n_i are large (say over 50) or k is large (say over 5), these formulas may take a little time to evaluate. In this case, a large sample approximation may be used.

Approximate Power Calculation

The power for the Cochran-Armitage test can be computed quickly using the normal approximation to the binomial distribution. The following steps are taken to compute approximate power.

One-Sided Test of Increasing Linear Trend in Proportions

1. Find the critical value using the standard normal distribution. The critical value, $z_{critical}$, is that value of z that leaves exactly the target value of alpha in the upper tail of the normal distribution. For example, for an upper-tailed test with a target alpha of 0.05, the critical value is 1.645.
2. For a one-sided test of the alternative hypothesis that p_i is a monotone increasing function of x_i, compute the power for given values of $\mathbf{p} = (p_1, p_2, ..., p_k)$ as

$$\begin{aligned} 1 - \beta &= \Pr\left(z \geq z_{critical} \mid H_1\right) \\ &= 1 - \Phi\left(u_U\right) \end{aligned},$$

   where $\Phi()$ is the cumulative normal distribution and

$$u_U = \frac{-\left[\sum_{i=1}^{k} n_i p_i (x_i - \bar{x}) - \frac{\Delta}{2}\right] + z_{critical}\sqrt{p(1-p)\sum_{i=1}^{k} n_i (x_i - \bar{x})^2}}{\sqrt{\sum_{i=1}^{k} n_i p_i (1 - p_i)(x_i - \bar{x})^2}},$$

where

$$p = \frac{1}{N}\sum_{i=1}^{k} n_i p_i .$$

The power for the uncorrected test is computed with $\Delta = 0$.

One-Sided Test of Decreasing Linear Trend in Proportions

1. Find the critical value using the standard normal distribution. The critical value, $z_{critical}$, is that value of z that leaves exactly the target value of alpha in the upper tail of the normal distribution. For example, for a lower-tailed test with a target alpha of 0.05, the critical value is 1.645.

2. For a one-sided test of the alternative hypothesis that p_i is a monotone decreasing function of x_i, compute the power for given values of $\mathbf{p} = (p_1, p_2, ..., p_k)$ as

$$\begin{aligned} 1-\beta &= \Pr\left(z \le -z_{critical} \mid H_1\right) \\ &= \Phi\left(u_L\right) \end{aligned},$$

where $\Phi()$ is the cumulative normal distribution and

$$u_L = \frac{-\left[\sum_{i=1}^{k} n_i p_i \left(x_i - \bar{x}\right) + \frac{\Delta}{2}\right] - z_{critical}\sqrt{p(1-p)\sum_{i=1}^{k} n_i \left(x_i - \bar{x}\right)^2}}{\sqrt{\sum_{i=1}^{k} n_i p_i (1-p_i)(x_i - \bar{x})^2}},$$

where

$$p = \frac{1}{N}\sum_{i=1}^{k} n_i p_i .$$

The power for the uncorrected test is computed with $\Delta = 0$.

Two-Sided Test of Linear Trend in Proportions

1. Find the critical value using the standard normal distribution. The critical value, $z_{critical}$, is that value of z that leaves exactly alpha/2 in the upper tail of the normal distribution. For example, for a two-tailed test with a target alpha of 0.05, the critical value is 1.96.

2. For a two-sided test of the alternative hypothesis that p_i is a monotone decreasing or increasing function of x_i, compute the power for given values of $\mathbf{p} = (p_1, p_2, ..., p_k)$ as

$$\begin{aligned} 1-\beta &= \Pr\left(z_U \ge z_{critical} \mid H_1\right) + \Pr\left(z_L \le -z_{critical} \mid H_1\right) \\ &= 1 - \Phi\left(u_U\right) + \Phi\left(u_L\right) \end{aligned},$$

where $\Phi()$ is the cumulative normal distribution and u_U and u_L are as defined previously. The power for the uncorrected test is computed with $\Delta = 0$.

Procedure Options

This section describes the options that are specific to this procedure. These are located on the Design tab. For more information about the options of other tabs, go to the Procedure Window chapter.

Design Tab

The Design tab contains most of the parameters and options that you will be concerned with.

Solve For

Solve For

This option specifies the parameter to be solved for from the other parameters. The parameters that may be selected are *Power* or *Sample Size*.

Test

Test Type

Specify which type of test will be used in all searching and reporting.

The continuity correction refers to adding or subtracting $\Delta/2$ from the numerator of the *z*-value to bring the normal approximation closer to the binomial distribution. The factor Δ is calculated as the average difference between adjacent *x* values.

If the *x*'s are equally-spaced, Δ is equal to the difference between adjacent *x*'s

$$\Delta = x_{i+1} - x_i \text{ for all } i < k.$$

In the case of unequally-spaced *x*'s, Nam (1987) states, “For unequally spaced doses, no constant correction is adequate for all outcomes.” Therefore, we recommend using the continuity-corrected test in the case of equally-spaced *x*'s, but caution against its use in the case of unequally-spaced *x*'s.

Alternative Hypothesis (H1)

Specify the alternative hypothesis of the test. Since the null hypothesis is the opposite, specifying the alternative is all that is needed. The alternative hypothesis determines how the alternative proportions, P (Proportions), should be entered. Usually, the two-sided option is selected.

For a one-sided alternative hypothesis test of increasing trend, the proportions should be strictly increasing, e.g. *0.1 0.2 0.3*.

Power and Alpha

Power

This option specifies one or more values for power. Power is the probability of rejecting a false null hypothesis, and is equal to one minus Beta. Beta is the probability of a type-II error, which occurs when a false null hypothesis is not rejected. In this procedure, a type-II error occurs when you fail to reject the null hypothesis of equal means when in fact the means are different.

Values must be between zero and one. Historically, the value of 0.80 (Beta = 0.20) was used for power. Now, 0.90 (Beta = 0.10) is also commonly used.

A single value may be entered here or a range of values such as *0.8 to 0.95 by 0.05* may be entered.

Alpha

This option specifies one or more values for the probability of a type-I error. A type-I error occurs when a true null hypothesis is rejected. In this procedure, a type-I error occurs when you reject the null hypothesis of equal means when in fact the means are equal.

Values must be between zero and one. Historically, the value of 0.05 has been used for alpha. This means that about one test in twenty will falsely reject the null hypothesis. You should pick a value for alpha that represents the risk of a type-I error you are willing to take in your experimental situation.

You may enter a range of values such as *0.01 0.05 0.10* or *0.01 to 0.10 by 0.01*.

Sample Size / Groups – Sample Size Multiplier

n (Sample Size Multiplier)

This is the base, per group, sample size. One or more values, separated by blanks or commas, may be entered. A separate analysis is performed for each value listed here.

The group samples sizes are determined by multiplying this number by each of the Group Sample Size Pattern numbers. If the Group Sample Size Pattern numbers are represented by $m_1, m_2, \ldots, m_k$ and this value is represented by n, the group sample sizes $n_1, n_2, ..., n_k$ are calculated as follows:

$n_1=[n(m_1)]$

$n_2=[n(m_2)]$

$n_3=[n(m_3)]$

etc.

where the operator, $[X]$ means the next integer after X, e.g. [3.1]=4.

For example, suppose there are three groups and the Group Sample Size Pattern is set to *1,2,3*. If n is 5, the resulting sample sizes will be 5, 10, and 15. If n is 50, the resulting group sample sizes will be 50, 100, and 150. If n is set to *2,4,6,8,10*, five sets of group sample sizes will be generated and an analysis run for each. These sets are:

2	4	6
4	8	12
6	12	18
8	16	24
10	20	30

As a second example, suppose there are three groups and the Group Sample Size Pattern is *0.2,0.3,0.5*. When the fractional Pattern values sum to one, *N* can be interpreted as the total sample size of all groups and the Pattern values as the proportion of the total in each group.

If N is 10, the three group sample sizes would be 2, 3, and 5.

If N is 20, the three group sample sizes would be 4, 6, and 10.

If N is 12, the three group sample sizes would be

(0.2)12 = 2.4 which is rounded up to the next whole integer, 3.

(0.3)12 = 3.6 which is rounded up to the next whole integer, 4.

(0.5)12 = 6.

Note that in this case, 3+4+6 does not equal *N* (which is 12). This can happen because of rounding.

Sample Size / Groups – Groups

k (Number of Groups)

This is the number of groups being compared. Thus, it is the number of proportions and X values (or covariates). It must be greater than or equal to two.

The Cochran-Armitage method tests for a trend in proportions among these groups.

Note that the number of values used in the P (Proportions), X Values, and Group Sample Size Pattern boxes are all controlled by this number.

Group Sample Size Pattern

A set of positive, numeric values, one for each group, is entered here. The sample size of group *i* is found by multiplying the i^{th} number from this list times the value of *n* and rounding up to the next whole number. The number of values must match the number of groups, *k*. When too few numbers are entered, 1's are added. When too many numbers are entered, the extras are ignored.

- **Equal**

  If all sample sizes are to be equal, enter *Equal* here and the desired sample size in *n*. A set of *k* 1's will be used. This will result in $n_1 = n_2 = n_3 = n$. That is, all sample sizes are equal to *n*.

Effect Size – Proportions

P (Proportions)

Specify two or more proportions. These are the alternative proportions for the Cochran-Armitage test of trend. The proportions should be strictly increasing or decreasing (depending on the alternative hypothesis) and all values should be greater than zero and less than one. The number of proportions entered should equal the value of *k*, the number of groups. If the number of proportions entered is less than *k*, the last proportion is repeated. If the number is greater than *k*, the extra proportions are ignored.

Several sets of proportions can be entered by using the ***PASS*** spreadsheet. To launch the spreadsheet, click on the "Spreadsheet" button above the box. To select columns from the spreadsheet, click on the button with the arrow pointing down to the right. Specify the column (or columns) to be used by beginning your entry with an equals sign, e.g. enter *=C1-C3*.

- **List Input**

  Specify a single set of proportions as a list. For example, with three groups you might enter *0.1 0.2 0.3*.

- **Spreadsheet Column Input**

  Specify more than one set of proportions using the column input syntax

  =[column 1] [column 2] etc.

  For example, if you have three proportion sets stored in the spreadsheet in columns C1, C2, and C3, you would enter *=C1 C2 C3* in the P (Proportions) box.

  Each column in the spreadsheet corresponds to a single set of proportions. Missing cells are not allowed. The number of proportions entered in each column should equal the value of *k*. If the number of proportions entered is less than *k*, the last proportion is repeated. If the number is greater than *k*, the extra proportions are ignored.

Effect Size – X's (Covariate or Dose Values)

Equally-Spaced X Values

Check this box if the *x*'s (covariates or doses) are equally spaced. It is not necessary to specify the individual *x*'s if they are equally spaced, e.g. *x* value sets of 0, 1, 2 and 10, 20, 30 yield the same results.

If the covariates or doses are not equally spaced (e.g. *x*'s = 1, 3, 7), you should enter the individual values in the box below after unchecking this option. The continuity correction factor $\Delta/2$ and the power calculation then depend on the actual values entered.

X Values

Enter a list of *x* values if the covariates are unequally spaced (e.g. *x*'s = 1, 3, 7). The values should be strictly increasing. The continuity correction factor $\Delta/2$ and the power calculation then depend on the actual values entered here. The factor Δ is calculated as the average difference between adjacent *x* values.

If the "Equally-Spaced X Values" option is checked, these values are ignored.

For a one-sided alternative hypothesis test of decreasing trend, the proportions should be strictly decreasing, e.g. *0.5 0.4 0.3*.

For a two-sided test, the proportions can be either increasing or decreasing.

Exact Power Calculation

Maximum Group Sample Size for Exact Power Calculations

When all group sample sizes are less than or equal to this amount and the product of all group sample sizes is less than the Max Group Size Product for Exact Power Calculations, exact power calculations using the binomial distribution are made. Otherwise, the normal approximation to the binomial is used for calculating power.

Large values for this option can greatly increase the time required to calculate the power, especially when searching for sample size or when *k* is large. For larger sample sizes, the power based on the normal approximation is very close to the exact power. We recommend keeping this value less than 50.

Maximum Group Sample Size Product for Exact Power Calculations

When the product of all group sample sizes is less than this amount and all individual group sample sizes are less than the Maximum Group Sample Size for Exact Power Calculations, exact power calculations using the binomial distribution are made. Otherwise, the normal approximation to the binomial is used for calculating power.

This option is used to reduce computing time and to avoid running out of memory in the case of large sample sizes and/or large *k*. Raising this value will increase computing time.

Large values for this option can greatly increase the time required to calculate the power, especially when searching for sample size or when *k* is large. For larger sample sizes, the power based normal approximation is very close to the exact power. We recommend keeping this value less than 10 million when solving for power and beta and less than 1 million when solving for sample size.

Example 1 – Finding the Power

An experiment is being designed to determine if there exists a dose-response relationship for a particular drug. Researchers will administer the drug at three dose levels: control (no drug), low, and high. The low dose is exactly half of the high dose, so the dosage structure is equally spaced. They expect to find response proportions of 0.05, 0.15, and 0.25 corresponding to the three doses, control, low, and high, respectively. A two-sided test with an alpha level of 0.05 will be used along with the continuity-corrected Cochran-Armitage test. They wish to compute the power for conducting the study with equal-sized groups ranging from 30 to 70 subjects in size.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Cochran-Armitage Test for Trend in Proportions** procedure window by expanding **Proportions**, then clicking on **Trend**, and then clicking on **Cochran-Armitage Test for Trend in Proportions**. You may then make the appropriate entries as listed below, or open **Example 1** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Power**
Test Type	**Z test with continuity correction**
Alternative Hypothesis (H1)	**Two-Sided**
Alpha	**0.05**
n (Sample Size Multiplier)	**30 to 70 by 5**
k (Number of Groups)	**3**
Group Sample Size Pattern	**Equal**
P (Proportions)	**0.05 0.15 0.25**
Equally-Spaced X Values	**Checked**
Max Grp Sample Size for Exact Power	**20**
Max Grp Size Product for Exact Power	**1000000**

Annotated Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results
Test Type = Two-sided Z test with continuity correction. Correction Factor = 0.5.
H0: P1 = P2 = ... = Pk. H1: Increasing or Decreasing Trend.
Equally-Spaced X Values.

Power	Average n	k	Total N	Alpha	Beta	Proportions
0.51187	30.00	3	90	0.05000	0.48813	0.05, 0.15, 0.25
0.58893	35.00	3	105	0.05000	0.41107	0.05, 0.15, 0.25
0.65710	40.00	3	120	0.05000	0.34290	0.05, 0.15, 0.25
0.71640	45.00	3	135	0.05000	0.28360	0.05, 0.15, 0.25
0.76724	50.00	3	150	0.05000	0.23276	0.05, 0.15, 0.25
0.81029	55.00	3	165	0.05000	0.18971	0.05, 0.15, 0.25
0.84635	60.00	3	180	0.05000	0.15365	0.05, 0.15, 0.25
0.87629	65.00	3	195	0.05000	0.12371	0.05, 0.15, 0.25
0.90093	70.00	3	210	0.05000	0.09907	0.05, 0.15, 0.25

References

Nam, J. 1987. 'A Simple Approximation for Calculating Sample Sizes for Detecting Linear Trend in Proportions'. Biometrics 43, 701-705.

Report Definitions

Power is the probability of rejecting a false null hypothesis. It should be close to one.
Average n is the average group sample size.
k is the number of groups.
Total N is the total sample size of all groups combined.
Alpha is the probability of rejecting a true null hypothesis. It should be small.
Beta is the probability of accepting a false null hypothesis. It should be small.
Proportions lists the set of proportions used. The number of proportions is equal to k.

Summary Statements

In a Cochran-Armitage test for trend in proportions, sample sizes of 30, 30, and 30 are obtained from 3 groups with equally-spaced X values and proportions equal to 0.05, 0.15, and 0.25, respectively. The total sample of 90 subjects achieves 51% power to detect a linear trend using a two-sided Z test with continuity correction and a significance level of 0.05000.

This report shows the numeric results of this power study. Following are the definitions of the columns of the report.

Power

The probability of rejecting a false null hypothesis.

Average n

The average of the group sample sizes.

k

The number of groups.

Total N

The total sample size of the study.

Alpha

The probability of rejecting a true null hypothesis. This is often called the significance level.

Beta

The probability of accepting a false null hypothesis.

Proportions

The alternative proportions used to calculate the power.

Detailed Results Report

Details when Power = 0.51187 and Alpha = 0.05000

Group	Sample Size (Ni)	Percent Ni of Total N	Proportion (Pi)
1	30	33.33	0.05
2	30	33.33	0.15
3	30	33.33	0.25
ALL	90	100.00	

Details when Power = 0.58893 and Alpha = 0.05000

Group	Sample Size (Ni)	Percent Ni of Total N	Proportion (Pi)
1	35	33.33	0.05
2	35	33.33	0.15
3	35	33.33	0.25
ALL	105	100.00	

(More Reports Follow)

This report shows the details of each row of the previous report.

Group

The number of the group shown on this line. The last line, labeled *ALL*, gives the total sample size for the scenario.

Ni

This is the sample size of each group. This column is especially useful when the sample sizes are unequal.

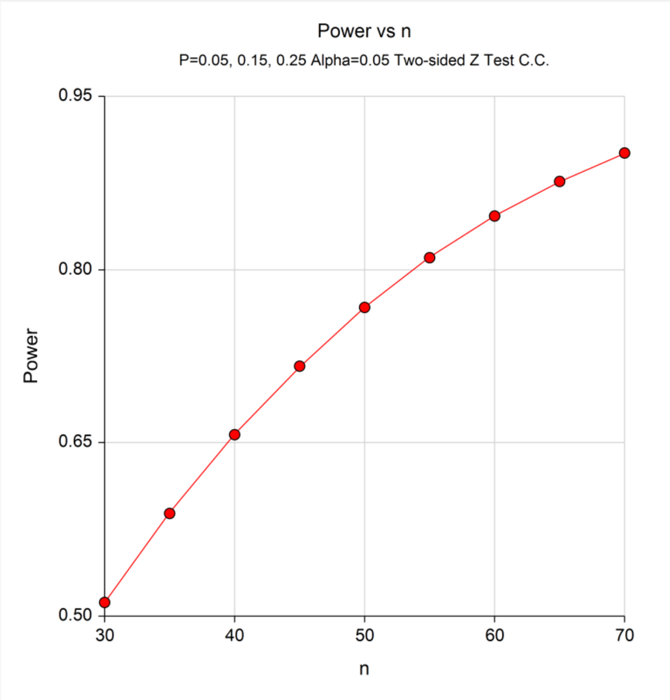
Percent Ni of Total Ni

This is the percentage of the total sample that is allocated to each group.

Pi

This is the value of the hypothesized proportion.

Plots Section



This plot gives a visual presentation to the results in the Numeric Report. We can quickly see the impact on the power of increasing the sample size.

When you create one of these plots, it is important to use trial and error to find an appropriate range for the horizontal variable so that you have results with both low and high power.

Exact Power Calculation

You can calculate the exact power for this scenario by setting the maximum group sample size for exact power calculations to *70*. You can do this yourself or load the completed template **Example1b** from the Template tab on the procedure window.

Numeric Results
Test Type = Two-sided Z test with continuity correction. Correction Factor = 0.5.
H0: P1 = P2 = ... = Pk. H1: Increasing or Decreasing Trend.
Equally-Spaced X Values.

Power	Average n	k	Total N	Alpha	Beta	Proportions
0.51173*	30.00	3	90	0.05000	0.48827	0.05, 0.15, 0.25
0.60387*	35.00	3	105	0.05000	0.39613	0.05, 0.15, 0.25
0.67534*	40.00	3	120	0.05000	0.32466	0.05, 0.15, 0.25
0.74067*	45.00	3	135	0.05000	0.25933	0.05, 0.15, 0.25
0.78352*	50.00	3	150	0.05000	0.21648	0.05, 0.15, 0.25
0.83170*	55.00	3	165	0.05000	0.16830	0.05, 0.15, 0.25
0.86462*	60.00	3	180	0.05000	0.13538	0.05, 0.15, 0.25
0.89489*	65.00	3	195	0.05000	0.10511	0.05, 0.15, 0.25
0.91511*	70.00	3	210	0.05000	0.08489	0.05, 0.15, 0.25

* Values in this row are based on exact power calculations. Exact power was calculated for scenarios in which the largest group sample size is less than or equal to 70 and the product of all group samples sizes is less than or equal to 1000000.

This report indicates that all power values were calculated exactly based on the binomial distribution. The approximate power values calculated earlier are very close to these values.

Example 2 – Finding the Sample Size

Continuing with the last example, we will determine how large the sample size would need to be to have the power at least 0.95 with an alpha level of 0.05.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Cochran-Armitage Test for Trend in Proportions** procedure window by expanding **Proportions**, then clicking on **Trend**, and then clicking on **Cochran-Armitage Test for Trend in Proportions**. You may then make the appropriate entries as listed below, or open **Example 2** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Sample Size**
Test Type	**Z test with continuity correction**
Alternative Hypothesis (H1)	**Two-Sided**
Power	**0.95**
Alpha	**0.05**
k (Number of Groups)	**3**
Group Sample Size Pattern	**Equal**
P (Proportions)	**0.05 0.15 0.25**
Equally-Spaced X Values	**Checked**
Max Grp Sample Size for Exact Power	**20**
Max Grp Size Product for Exact Power	**1000000**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results
Test Type = Two-sided Z test with continuity correction. Correction Factor = 0.5.
H0: P1 = P2 = ... = Pk. H1: Increasing or Decreasing Trend.
Equally-Spaced X Values.

Power	Average n	k	Total N	Alpha	Beta	Proportions
0.95054	85.00	3	255	0.05000	0.04946	0.05, 0.15, 0.25

The required sample size is 85 per group or 255 subjects.

Example 3 – Calculating Power with Unequal Group Sample Sizes

Continuing with the last example, consider the impact of allowing the group sample sizes to be unequal. Suppose we have twice as many control subjects receiving no drug as subjects at the low and high dose levels. What is the power for group sample sizes of 120, 60, and 60?

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Cochran-Armitage Test for Trend in Proportions** procedure window by expanding **Proportions**, then clicking on **Trend**, and then clicking on **Cochran-Armitage Test for Trend in Proportions**. You may then make the appropriate entries as listed below, or open **Example 3** by going to the **File** menu and choosing **Open Example Template**.

Pay particular attention to how the sample size parameters were changed. The sample size multiplier, n, was set to 1 so that it is essentially ignored. The Group Sample Size Pattern contains the three sample sizes.

Option	Value
Design Tab	
Solve For	**Power**
Test Type	**Z test with continuity correction**
Alternative Hypothesis (H1)	**Two-Sided**
Alpha	**0.05**
n (Sample Size Multiplier)	**1**
k (Number of Groups)	**3**
Group Sample Size Pattern	**120 60 60**
P (Proportions)	**0.05 0.15 0.25**
Equally-Spaced X Values	**Checked**
Max Grp Sample Size for Exact Power	**20**
Max Grp Size Product for Exact Power	**1000000**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results
Test Type = Two-sided Z test with continuity correction. Correction Factor = 0.5.
H0: P1 = P2 = ... = Pk. H1: Increasing or Decreasing Trend.
Equally-Spaced X Values.

Power	Average n	k	Total N	Alpha	Beta	Proportions
0.95196	80.00	3	240	0.05000	0.04804	0.05, 0.15, 0.25

Details when Power = 0.95196 and Alpha = 0.05000

Group	Sample Size (Ni)	Percent Ni of Total N	Proportion (Pi)
1	120	50.00	0.05
2	60	25.00	0.15
3	60	25.00	0.25
ALL	240	100.00	

Group sample sizes of 120, 60, and 60 yield just over 95% power. The total sample size of 240 for 95% power for this scenario is actually less than the total of 255 from Example 2, where equal group sample sizes were used.

Example 4 – Calculating Power with Unequally-Spaced X Values

Continuing with Example 1, consider the impact of using unequally-spaced dose levels: 0, 2, and 5. Because the doses are not equally spaced, we will use the uncorrected *z* test for power calculations.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Cochran-Armitage Test for Trend in Proportions** procedure window by expanding **Proportions**, then clicking on **Trend**, and then clicking on **Cochran-Armitage Test for Trend in Proportions**. You may then make the appropriate entries as listed below, or open **Example 4** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Power**
Test Type	**Z test**
Alternative Hypothesis (H1)	**Two-Sided**
Alpha	**0.05**
n (Sample Size Multiplier)	**30 to 70 by 5**
k (Number of Groups)	**3**
Group Sample Size Pattern	**Equal**
P (Proportions)	**0.05 0.15 0.25**
Equally-Spaced X Values	**Unchecked**
X Values	**0 2 5**
Max Grp Sample Size for Exact Power	**20**
Max Grp Size Product for Exact Power	**1000000**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results
Test Type = Two-sided Z test.
H0: P1 = P2 = ... = Pk. H1: Increasing or Decreasing Trend.
X Values = 0.0, 2.0, 5.0.

Power	Average n	k	Total N	Alpha	Beta	Proportions
0.57754	30.00	3	90	0.05000	0.42246	0.05, 0.15, 0.25
0.64383	35.00	3	105	0.05000	0.35617	0.05, 0.15, 0.25
0.70190	40.00	3	120	0.05000	0.29810	0.05, 0.15, 0.25
0.75214	45.00	3	135	0.05000	0.24786	0.05, 0.15, 0.25
0.79514	50.00	3	150	0.05000	0.20486	0.05, 0.15, 0.25
0.83161	55.00	3	165	0.05000	0.16839	0.05, 0.15, 0.25
0.86229	60.00	3	180	0.05000	0.13771	0.05, 0.15, 0.25
0.88790	65.00	3	195	0.05000	0.11210	0.05, 0.15, 0.25
0.90915	70.00	3	210	0.05000	0.09085	0.05, 0.15, 0.25

The power values are quite different from those calculated with the continuity-corrected *z* test when the dose-spacing is equal. Of course, the covariate spacing you use will likely depend on more factors than the achievable power.

Example 5 – Validation of Sample Size Calculations with Approximate Power using Nam

Nam (1987) page 703 presents a table of calculated sample sizes with three equally-spaced doses and equal group sample sizes using the one-sided continuity-corrected *z* test for an increasing trend in proportions. Sample size is calculated for various proportion sets, alpha levels of 0.05 and 0.025, and power values of 0.5, 0.7, and 0.9. The table based on approximate power calculations is given below.

Alternative Proportion			Specified Nominal Power 0.50		0.70		0.90	
p_0	p_1	p_2	α = 0.025	α = 0.05	α = 0.025	α = 0.05	α = 0.025	α = 0.05
0.05	0.10	0.15	79	58	120	94	197	162
0.10	0.15	0.20	108	79	167	129	276	226
0.20	0.25	0.30	154	111	241	186	402	329
0.30	0.35	0.40	185	133	290	223	486	398
0.05	0.15	0.25	29	22	44	34	70	58
0.10	0.20	0.30	36	26	54	42	87	72
0.20	0.30	0.40	45	33	69	54	113	93
0.30	0.40	0.50	51	37	78	61	129	106
0.05	0.25	0.45	11	9	16	13	25	21
0.10	0.30	0.50	12	9	18	14	28	23
0.20	0.40	0.60	14	10	20	16	32	26
0.30	0.50	0.70	14	11	21	17	33	28

This example will replicate these results.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Cochran-Armitage Test for Trend in Proportions** procedure window by expanding **Proportions**, then clicking on **Trend**, and then clicking on **Cochran-Armitage Test for Trend in Proportions**. You may then make the appropriate entries as listed below, or open **Example 5** by going to the **File** menu and choosing **Open Example Template**. Check to see that the values have been entered into the spreadsheet by clicking the spreadsheet button to the right of P (Proportions).

Option	Value
Design Tab	
Solve For	**Sample Size**
Test Type	**Z test with continuity correction**
Alternative Hypothesis (H1)	**One-Sided (Increasing Trend)**
Power	**0.5 0.7 0.9**
Alpha	**0.025 0.05**
k (Number of Groups)	**3**
Group Sample Size Pattern	**Equal**
P (Proportions)	**=C1-C12**
Equally-Spaced X Values	**Checked**
Max Grp Sample Size for Exact Power	**0**
Max Grp Size Product for Exact Power	**1000000**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results
Test Type = One-sided Z test with continuity correction. Correction Factor = 0.5.
H0: P1 = P2 = ... = Pk. H1: P1 < P2 < ... < Pk.
Equally-Spaced X Values.

Power	Average n	k	Total N	Alpha	Beta	Proportions
0.50098	79.00	3	237	0.02500	0.49902	0.05, 0.10, 0.15
0.70301	121.00	3	363	0.02500	0.29699	0.05, 0.10, 0.15
0.90012	197.00	3	591	0.02500	0.09988	0.05, 0.10, 0.15
0.50493	59.00	3	177	0.05000	0.49507	0.05, 0.10, 0.15
0.70061	94.00	3	282	0.05000	0.29939	0.05, 0.10, 0.15
0.90150	163.00	3	489	0.05000	0.09850	0.05, 0.10, 0.15
0.50110	108.00	3	324	0.02500	0.49890	0.10, 0.15, 0.20
0.70115	167.00	3	501	0.02500	0.29885	0.10, 0.15, 0.20
0.90025	276.00	3	828	0.02500	0.09975	0.10, 0.15, 0.20
0.50156	79.00	3	237	0.05000	0.49844	0.10, 0.15, 0.20
0.70244	130.00	3	390	0.05000	0.29756	0.10, 0.15, 0.20
0.90073	227.00	3	681	0.05000	0.09927	0.10, 0.15, 0.20
0.50029	154.00	3	462	0.02500	0.49971	0.20, 0.25, 0.30
0.70057	241.00	3	723	0.02500	0.29943	0.20, 0.25, 0.30
0.90008	402.00	3	1206	0.02500	0.09992	0.20, 0.25, 0.30
0.50249	112.00	3	336	0.05000	0.49751	0.20, 0.25, 0.30
0.70052	186.00	3	558	0.05000	0.29948	0.20, 0.25, 0.30
0.90065	330.00	3	990	0.05000	0.09935	0.20, 0.25, 0.30
0.50078	185.00	3	555	0.02500	0.49922	0.30, 0.35, 0.40
0.70141	291.00	3	873	0.02500	0.29859	0.30, 0.35, 0.40
0.90003	486.00	3	1458	0.02500	0.09997	0.30, 0.35, 0.40
0.50023	133.00	3	399	0.05000	0.49977	0.30, 0.35, 0.40

Numeric Results
Test Type = One-sided Z test with continuity correction. Correction Factor = 0.5.
H0: P1 = P2 = ... = Pk. H1: P1 < P2 < ... < Pk.
Equally-Spaced X Values.

Power	Average n	k	Total N	Alpha	Beta	Proportions
0.70102	224.00	3	672	0.05000	0.29898	0.30, 0.35, 0.40
0.90019	398.00	3	1194	0.05000	0.09981	0.30, 0.35, 0.40
0.51187	30.00	3	90	0.02500	0.48813	0.05, 0.15, 0.25
0.70523	44.00	3	132	0.02500	0.29477	0.05, 0.15, 0.25
0.90093	70.00	3	210	0.02500	0.09907	0.05, 0.15, 0.25
0.50072	22.00	3	66	0.05000	0.49928	0.05, 0.15, 0.25
0.70988	35.00	3	105	0.05000	0.29012	0.05, 0.15, 0.25
0.90203	58.00	3	174	0.05000	0.09797	0.05, 0.15, 0.25
0.50579	36.00	3	108	0.02500	0.49421	0.10, 0.20, 0.30
0.70361	54.00	3	162	0.02500	0.29639	0.10, 0.20, 0.30
0.90039	87.00	3	261	0.02500	0.09961	0.10, 0.20, 0.30
0.50913	27.00	3	81	0.05000	0.49087	0.10, 0.20, 0.30
0.70083	42.00	3	126	0.05000	0.29917	0.10, 0.20, 0.30
0.90182	72.00	3	216	0.05000	0.09818	0.10, 0.20, 0.30
0.50791	46.00	3	138	0.02500	0.49209	0.20, 0.30, 0.40
0.70640	70.00	3	210	0.02500	0.29360	0.20, 0.30, 0.40
0.90216	114.00	3	342	0.02500	0.09784	0.20, 0.30, 0.40
0.50912	34.00	3	102	0.05000	0.49088	0.20, 0.30, 0.40
0.70220	54.00	3	162	0.05000	0.29780	0.20, 0.30, 0.40
0.90010	93.00	3	279	0.05000	0.09990	0.20, 0.30, 0.40
0.50022	51.00	3	153	0.02500	0.49978	0.30, 0.40, 0.50
0.70399	79.00	3	237	0.02500	0.29601	0.30, 0.40, 0.50
0.90012	129.00	3	387	0.02500	0.09988	0.30, 0.40, 0.50
0.50717	38.00	3	114	0.05000	0.49283	0.30, 0.40, 0.50
0.70142	61.00	3	183	0.05000	0.29858	0.30, 0.40, 0.50
0.90046	106.00	3	318	0.05000	0.09954	0.30, 0.40, 0.50
0.53014	12.00	3	36	0.02500	0.46986	0.05, 0.25, 0.45
0.72689	17.00	3	51	0.02500	0.27311	0.05, 0.25, 0.45
0.90119	25.00	3	75	0.02500	0.09881	0.05, 0.25, 0.45
0.51914	9.00	3	27	0.05000	0.48086	0.05, 0.25, 0.45
0.70730	13.00	3	39	0.05000	0.29270	0.05, 0.25, 0.45
0.90649	21.00	3	63	0.05000	0.09351	0.05, 0.25, 0.45
0.52280	13.00	3	39	0.02500	0.47720	0.10, 0.30, 0.50
0.70192	18.00	3	54	0.02500	0.29808	0.10, 0.30, 0.50
0.90139	28.00	3	84	0.02500	0.09861	0.10, 0.30, 0.50
0.52790	10.00	3	30	0.05000	0.47210	0.10, 0.30, 0.50
0.72817	15.00	3	45	0.05000	0.27183	0.10, 0.30, 0.50
0.90025	23.00	3	69	0.05000	0.09975	0.10, 0.30, 0.50
0.50324	14.00	3	42	0.02500	0.49676	0.20, 0.40, 0.60
0.71861	21.00	3	63	0.02500	0.28139	0.20, 0.40, 0.60
0.90163	32.00	3	96	0.02500	0.09837	0.20, 0.40, 0.60
0.52289	11.00	3	33	0.05000	0.47711	0.20, 0.40, 0.60
0.70206	16.00	3	48	0.05000	0.29794	0.20, 0.40, 0.60
0.90863	27.00	3	81	0.05000	0.09137	0.20, 0.40, 0.60
0.52104	15.00	3	45	0.02500	0.47896	0.30, 0.50, 0.70
0.72308	22.00	3	66	0.02500	0.27692	0.30, 0.50, 0.70
0.90793	34.00	3	102	0.02500	0.09207	0.30, 0.50, 0.70
0.50788	11.00	3	33	0.05000	0.49212	0.30, 0.50, 0.70
0.71329	17.00	3	51	0.05000	0.28671	0.30, 0.50, 0.70
0.90750	28.00	3	84	0.05000	0.09250	0.30, 0.50, 0.70

The sample sizes calculated by ***PASS*** match those of Nam (1987). In many cases, ***PASS*** reports a sample size that is one greater than that reported Nam (1987). This difference is due to rounding. Nam (1987) rounds some power values up when they are actually slightly lower than the nominal value. ***PASS*** does not round power values up when computing the sample size. All sample sizes result in at least the nominal power.

Example 6 – Validation of Exact Power Calculations using Nam

Nam (1987) page 703 presents a table of calculated sample sizes with three equally-spaced doses and equal group sample sizes using the one-sided continuity-corrected *z* test for an increasing trend in proportions. Sample size is calculated for various proportion sets, alpha levels of 0.05 and 0.025, and power values of 0.5, 0.7, and 0.9. The table of calculated sample sizes is given in Example 5. Nam (1987) further calculates the exact power for scenarios in which the resulting sample size is less than or equal to 50. The results are given below.

Alternative Proportion			Specified Nominal Power 0.50		0.70		0.90	
p_0	p_1	p_2	α = 0.025	α = 0.05	α = 0.025	α = 0.05	α = 0.025	α = 0.05
0.05	0.15	0.25	29 (0.51)	22 (0.51)	44 (0.73)	34 (0.71)		
0.10	0.20	0.30	36 (0.52)	26 (0.50)		42 (0.71)		
0.20	0.30	0.40	45 (0.50)	33 (0.51)				
0.30	0.40	0.50		37 (0.49)				
0.05	0.25	0.45	11 (0.50)	9 (0.57)	16 (0.71)	13 (0.71)	25 (0.92)	21 (0.91)
0.10	0.30	0.50	12 (0.50)	9 (0.54)	18 (0.72)	14 (0.71)	28 (0.91)	23 (0.91)
0.20	0.40	0.60	**14 (0.53)**	10 (0.47)	20 (0.71)	16 (0.69)	32 (0.90)	26 (0.89)
0.30	0.50	0.70	**14 (0.53)**	11 (0.50)	21 (0.69)	17 (0.69)	33 (0.90)	28 (0.91)

This example will replicate the results in bold type.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Cochran-Armitage Test for Trend in Proportions** procedure window by expanding **Proportions**, then clicking on **Trend**, and then clicking on **Cochran-Armitage Test for Trend in Proportions**. You may then make the appropriate entries as listed below, or open **Example 6** by going to the **File** menu and choosing **Open Example Template**. Check to see that the values have been entered into the spreadsheet by clicking the spreadsheet button to the right of P (Proportions).

Option	Value
Design Tab	
Solve For	**Power**
Test Type	**Z test with continuity correction**
Alternative Hypothesis (H1)	**One-Sided (Increasing Trend)**
Alpha	**0.025**
n (Sample Size Multiplier)	**14**
k (Number of Groups)	**3**
Group Sample Size Pattern	**Equal**
P (Proportions)	**=C11-C12**
Equally-Spaced X Values	**Checked**
Max Grp Sample Size for Exact Power	**50**
Max Grp Size Product for Exact Power	**1000000**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results
Test Type = One-sided Z test with continuity correction. Correction Factor = 0.5.
H0: P1 = P2 = ... = Pk. H1: P1 < P2 < ... < Pk.
Equally-Spaced X Values.

Power	Average n	k	Total N	Alpha	Beta	Proportions
0.53000*	14.00	3	42	0.02500	0.47000	0.20, 0.40, 0.60
0.52761*	14.00	3	42	0.02500	0.47239	0.30, 0.50, 0.70

* Values in this row are based on exact power calculations. Exact power was calculated for scenarios in which the largest group sample size is less than or equal to 50 and the product of all group samples sizes is less than or equal to 1000000.

The exact power values calculated by PASS match those calculated in Nam (1987) exactly if you round to two decimal places. Group sample sizes of *14* results in power of *0.53* for both scenarios. If you replicate the other scenarios in the table, you will find that the PASS results for exact power match Nam (1987) after rounding to two decimal places.