

## Chapter 818

# Confidence Intervals for Coefficient Alpha

## Introduction

*Coefficient alpha*, or *Cronbach's alpha*, is a measure of the reliability of a test consisting of  $k$  parts. The  $k$  parts usually represent  $k$  items on a questionnaire or  $k$  raters. This routine calculates the sample size needed to obtain a specified width of a confidence interval for coefficient alpha at a stated confidence level.

## Technical Details

Feldt et al. (1987) has shown that if  $CA$  is the estimated value of coefficient alpha computed from a sample of size  $N$  subjects taking a test with  $k$  items, a  $100(1 - \alpha)\%$  confidence interval for  $CA$  is given by

$$CA_L = 1 - \left[ (1 - CA) F_{\frac{\alpha}{2}, N-1, (N-1)(K-1)} \right]$$

and

$$CA_U = 1 - \left[ (1 - CA) F_{1-\frac{\alpha}{2}, N-1, (N-1)(K-1)} \right]$$

Therefore, the width of the confidence interval is  $CA_U - CA_L$ . Using the above, we can calculate the confidence interval width for any confidence level and sample size.

One-sided limits may be obtained by replacing  $\alpha/2$  by  $\alpha$ .

## Confidence Level

The confidence level,  $1 - \alpha$ , has the following interpretation. If thousands of samples of  $N$  items are drawn from a population using simple random sampling and a confidence interval is calculated for each sample, the proportion of those intervals that will include the true population correlation is  $1 - \alpha$ .

## Procedure Options

This section describes the options that are specific to this procedure. These are located on the Design tab. For more information about the options of other tabs, go to the Procedure Window chapter.

## Design Tab

The Design tab contains most of the parameters and options that you will be concerned with.

## Confidence Intervals for Coefficient Alpha

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### Solve For

#### Solve For

This option specifies the parameter to be solved for from the other parameters.

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### One-Sided or Two-Sided Interval

#### Interval Type

Specify whether the confidence interval for the population coefficient alpha is two-sided or one-sided. A one-sided interval is often called a **confidence bound** rather than a confidence interval because it only has one limit.

#### Two-Sided

The two-sided confidence interval is defined by two limits: an upper confidence limit (UCL) and a lower confidence limit (LCL).

These limits are constructed so that the designated proportion (confidence level) of such intervals will include the true value of coefficient alpha.

#### Upper One-Sided

The upper confidence interval (or bound) is defined by a limit above the estimated value of coefficient alpha. The limit is constructed so that the designated proportion (confidence level) of such limits has the true population value below them.

#### Lower One-Sided

The lower confidence interval (or bound) is defined by a limit below the estimated value of coefficient alpha. The limit is constructed so that the designated proportion (confidence level) of such limits has the true population value above them.

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### Confidence

#### Confidence Level

The confidence level,  $1 - \alpha$ , has the following interpretation. If thousands of samples of  $N$  items are drawn from a population using simple random sampling and a confidence interval is calculated for each sample, the proportion of those intervals that will include the true population correlation is  $1 - \alpha$ .

Often, the values 0.95 or 0.99 are used. You can enter single values or a range of values such as *0.90, 0.95 or 0.90 to 0.99 by 0.01*.

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### Sample Size

#### N (Number of Subjects)

Enter one or more values for N, the number of subjects who take the test. This is the number of experimental units selected at random from the population.

You can enter a single value or a range of values.

#### K (Items per Subject)

Enter one or more values for K, the number of items per subject. This is the number of questions.

You can enter a single value or a range of values greater than two.

## Confidence Intervals for Coefficient Alpha

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### Precision

#### Width of Confidence Interval

This is the distance between the lower confidence limit (*LCL*) and the upper confidence limit (*UCL*). Its calculation is  $UCL - LCL$ . It is a measure of the precision of the confidence interval. This width usually ranges between 0 and 1.

You can enter a single value or a list of values.

#### Distance from CA to Limit

This is the distance from *CA* to the lower confidence limit (*LCL*) or the upper confidence limit (*UCL*). It is calculated using  $|CA - LCL|$  or  $|UCL - CA|$ . The range is between 0 and 1.

You can enter a single value or a list of values.

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### Estimated Sample Intraclass Correlation

#### CA (Sample Coefficient Alpha)

Enter a planning estimate of the intraclass correlation. The sample size and width calculations assume that the value entered here is the actual value obtained from the sample. This is sometimes referred to as a *planning estimate*.

The range of the values of the sample coefficient alpha that can be entered is 0 to 1.

You can enter a range of values such as '0.1 0.3 0.5' or '0.1 to 0.5 by 0.2.'

## Confidence Intervals for Coefficient Alpha

## Example 1 – Calculating Sample Size

Suppose a study is planned to estimate coefficient alpha with a two-sided 95% confidence interval with a width no wider than 0.1. The researcher would like to examine values of K from 5 to 30 in steps of 5. From past studies, the researcher wants to use a planning estimate of 0.5 for the sample coefficient alpha. The goal is to determine the necessary sample size for each scenario.

### Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Confidence Intervals for Coefficient Alpha** procedure window by expanding **Correlation**, then clicking on **Coefficient (Cronbach's) Alpha**, and then clicking on **Confidence Intervals for Coefficient Alpha**. You may then make the appropriate entries as listed below, or open **Example 1** by going to the **File** menu and choosing **Open Example Template**.

| <u>Option</u>                       | <u>Value</u>            |
|-------------------------------------|-------------------------|
| <b>Design Tab</b>                   |                         |
| Solve For .....                     | <b>Sample Size</b>      |
| Interval Type .....                 | <b>Two-Sided</b>        |
| Confidence Level .....              | <b>0.95</b>             |
| K (Items per Subject) .....         | <b>5 10 15 20 25 30</b> |
| Width of Confidence Interval .....  | <b>0.1</b>              |
| CA (Sample Coefficient Alpha) ..... | <b>0.5</b>              |

### Annotated Output

Click the Calculate button to perform the calculations and generate the following output.

### Numeric Results

#### Numeric Results for Two-Sided Confidence Interval for Coefficient Alpha

| Confidence Level | Number of Subjects<br>N | Items per Subject<br>K | Width<br>UCL-LCL | Sample Coefficient Alpha<br>CA | Lower Confidence Limit<br>LCL | Upper Confidence Limit<br>UCL |
|------------------|-------------------------|------------------------|------------------|--------------------------------|-------------------------------|-------------------------------|
| 0.950            | 963                     | 5                      | 0.100            | 0.500                          | 0.448                         | 0.548                         |
| 0.950            | 855                     | 10                     | 0.100            | 0.500                          | 0.449                         | 0.549                         |
| 0.950            | 825                     | 15                     | 0.100            | 0.500                          | 0.449                         | 0.549                         |
| 0.950            | 810                     | 20                     | 0.100            | 0.500                          | 0.449                         | 0.549                         |
| 0.950            | 802                     | 25                     | 0.100            | 0.500                          | 0.449                         | 0.549                         |
| 0.950            | 796                     | 30                     | 0.100            | 0.500                          | 0.449                         | 0.549                         |

#### Report Definitions

Confidence Level is the proportion of confidence intervals (constructed with this same confidence level, sample size, etc.) that would contain the true coefficient alpha.

Number of Subjects N is the size of the random sample of subjects drawn from the population.

Items per Subject K is the number of items per subject. On a survey, this is the number of questions.

Width (UCL-LCL) is the width of the confidence interval. It is the distance from the lower limit to the upper limit.

Sample Coefficient Alpha CA is the planning estimate of the sample coefficient alpha (Cronbach's Alpha).

Lower and Upper Confidence Limits are the actual limits that would result from a dataset with these statistics.

#### References

Bonett, D. G. 2002. 'Sample Size Requirements for Testing and Estimating Coefficient Alpha.' Journal of Educational and Behavioral Statistics, Vol 27, No 4, 335-340.

Feldt, L. S., Woodruff, D. J., and Salih, F. A. 1987. 'Statistical Inference for Coefficient Alpha.' Applied Psychological Measurement, Vol 11, No. 1, 93-103.

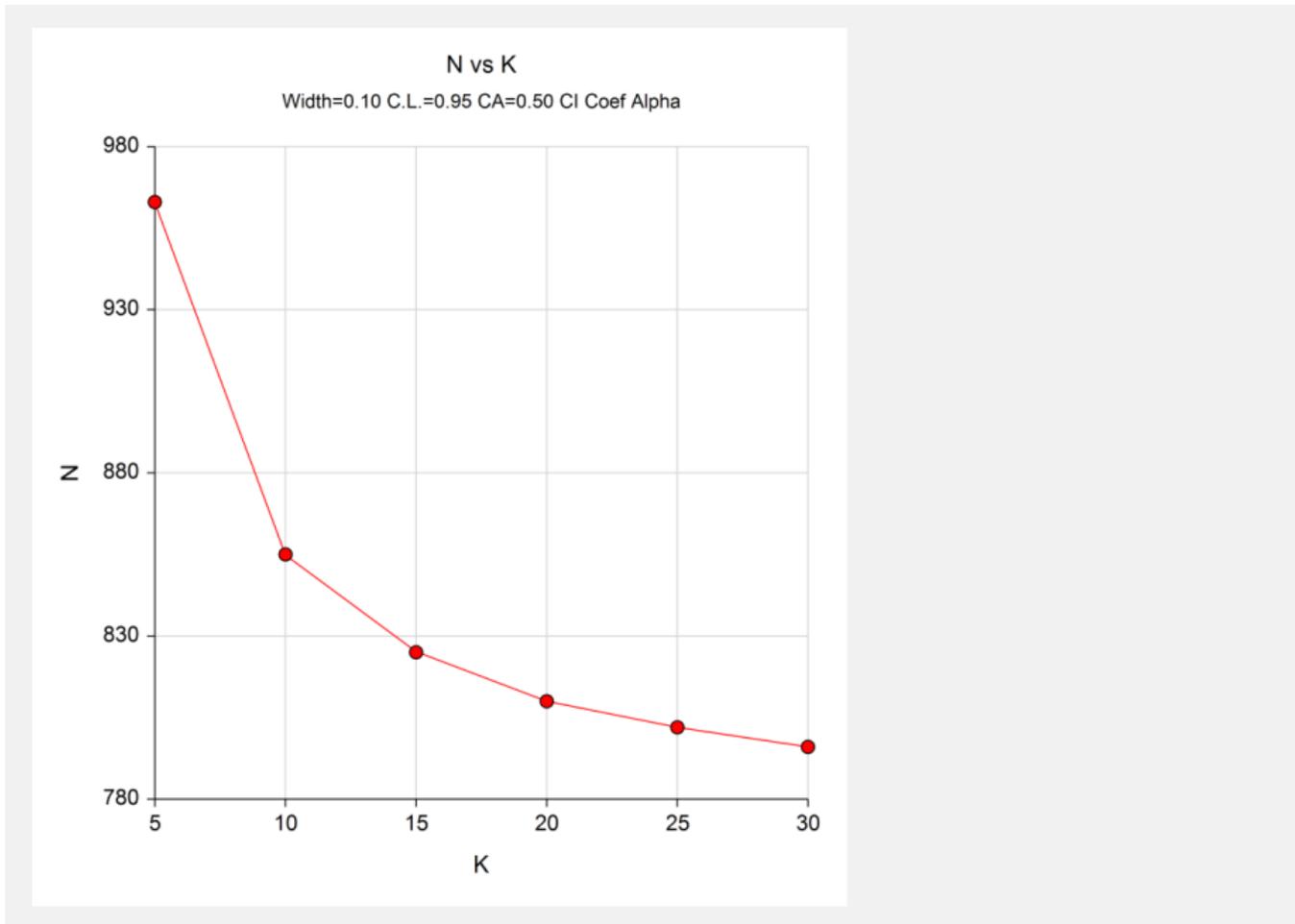
## Confidence Intervals for Coefficient Alpha

### Summary Statements

A random sample of 963 subjects who are each measured 5 times produces a two-sided 95% confidence interval with a width of 0.100 when the estimated coefficient alpha is 0.500.

This report shows the calculated sample size for each of the scenarios.

### Plots Section



This plot shows the sample size versus the value of K.

## Confidence Intervals for Coefficient Alpha

**Example 2 – Validation using Feldt**

Feldt et al. (1987), page 95, give an example calculation for  $CA = 0.79$ ,  $N = 41$ ,  $K=26$ , and confidence level = 0.90. They find that  $CAL = 0.704$  and  $CAU = 0.861$ , for a width of 0.157. We will now validate this routine using this example.

**Setup**

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Confidence Intervals for Coefficient Alpha** procedure window by expanding **Correlation**, then clicking on **Coefficient (Cronbach's) Alpha**, and then clicking on **Confidence Intervals for Coefficient Alpha**. You may then make the appropriate entries as listed below, or open **Example 2** by going to the **File** menu and choosing **Open Example Template**.

| <u>Option</u>                       | <u>Value</u>                      |
|-------------------------------------|-----------------------------------|
| <b>Design Tab</b>                   |                                   |
| Solve For .....                     | <b>CI Width or Distance to CA</b> |
| Interval Type .....                 | <b>Two-Sided</b>                  |
| Confidence Level .....              | <b>0.90</b>                       |
| N (Number of Subjects) .....        | <b>41</b>                         |
| K (Items per Subject) .....         | <b>26</b>                         |
| CA (Sample Coefficient Alpha) ..... | <b>0.79</b>                       |

**Annotated Output**

Click the Calculate button to perform the calculations and generate the following output.

| Numeric Results for Two-Sided Confidence Interval for Coefficient Alpha |                         |                        |                  |                                |                               |                               |  |
|---|-------------------------|------------------------|------------------|--------------------------------|-------------------------------|-------------------------------|--|
| Confidence Level  | Number of Subjects<br>N | Items per Subject<br>K | Width<br>UCL-LCL | Sample Coefficient Alpha<br>CA | Lower Confidence Limit<br>LCL | Upper Confidence Limit<br>UCL |  |
| 0.900   | 41                      | 26                     | 0.157            | 0.790                          | 0.705                         | 0.862                         |  |

PASS matches Feldt's results within rounding.