Chapter 296

Confidence Intervals for Cp

Introduction

This routine calculates the sample size needed to obtain a specified width of a Cp confidence interval at a stated confidence level.

Cp is a process capability index used to approximate the *potential* of a process to produce non-defective items. Cp measures the defective rate of a process by comparing its distribution to specification limits. The comparison is based on the process mean and standard deviation. Cp requires the assumption that the measurements are normally distributed and that the process mean is centered between the specification limits.

The formula for the calculation of Cp is

$$Cp = (USL - LSL) / 6\sigma$$

where USL and LSL are the upper and lower specification limits, respectively. Note that in the normal distribution, approximately 99.7% of the data values are within three standard deviations of the mean, so if Cp is a little greater than one, it can be assumed that the data values are all within the specification limits. Thus a process with a Cp of 2.0 is considered excellent, while one with a Cp of 1.33 is considered adequate.

Technical Details

This procedure is based on the results of Mathews (2010). Since Cp is a simple transformation of the variance, the confidence interval of the variance can be used to obtain the following two-sided, $100(1 - \alpha)$ % confidence interval for Cp.

$$P\left(\widehat{Cp} \sqrt{\frac{\chi^2_{n-1,\alpha/2}}{n-1}} \leq Cp \leq \widehat{Cp} \sqrt{\frac{\chi^2_{n-1,1-\alpha/2}}{n-1}} \right) = 1 - \alpha$$

where $\widehat{Cp}$ is the estimated value of Cp, n is the sample size, and $\chi^2_{df,\varphi}$ is the specific value of the chi-square random variate with df degrees of freedom that has probability φ to the left.

One-sided limits may be obtained by replacing $\alpha/2$ by α.

For two-sided intervals, the distance from the sample Cp to each of the limits may be different. So, instead of specifying the distance to the limits (the half width), we specify the width of the entire interval.

Confidence Interval Width

The confidence interval width, confidence level, and sample size are related in the equation

$$Width = \widehat{Cp}\left(\sqrt{\frac{\chi^2_{n-1,1-\alpha/2}}{n-1}} - \sqrt{\frac{\chi^2_{n-1,\alpha/2}}{n-1}}\right)$$

This equation can be used to find n, α, or the width.

Confidence Level

The confidence level, $1 - \alpha$, has the following interpretation. If thousands of samples of n items are drawn from a population using simple random sampling and a confidence interval is calculated for each sample, the proportion of those intervals that will include the true population parameter is $1 - \alpha$.

Procedure Options

This section describes the options that are specific to this procedure. These are located on the Design tab. For more information about the options of other tabs, go to the Procedure Window chapter.

Design Tab

The Design tab contains most of the parameters and options that you will be concerned with.

Solve For

This option specifies the parameter to be solved for from the other parameters.

One-Sided or Two-Sided Interval

Interval Type

Specify whether the confidence interval for the population correlation is two-sided or one-sided. A one-sided interval is often called a **confidence bound** rather than a confidence interval because it only has one limit.

Two-Sided

The two-sided confidence interval is defined by two limits: an upper confidence limit (UCL) and a lower confidence limit (LCL).

These limits are constructed so that the designated proportion (confidence level) of such intervals will include the true population value.

Upper One-Sided

The upper confidence interval (or bound) is defined by a limit above the estimated parameter value. The limit is constructed so that the designated proportion (confidence level) of such limits has the true population value below it.

Lower One-Sided

The lower confidence interval (or bound) is defined by a limit below the estimated parameter value. The limit is constructed so that the designated proportion (confidence level) of such limits has the true population value above it.

Confidence

Confidence Level

The confidence level, $1 - \alpha$, has the following interpretation. If thousands of samples of *n* items are drawn from a population using simple random sampling and a confidence interval is calculated for each sample, the proportion of those intervals that will include the true population Cp is $1 - \alpha$.

Often, the values 0.95 or 0.99 are used. You can enter single values or a range of values such as *0.90, 0.95* or *0.90 to 0.99 by 0.01*.

Sample Size

N (Sample Size)

Enter one or more values for the sample size. This is the number of individuals selected at random from the population to be in the study.

You can enter a single value or a range of values.

Precision

Width of Confidence Interval (Two-Sided)

This is the distance from the lower confidence limit to the upper confidence limit. You can enter a single value or a list of values.

Distance from Cp to Limit (One-Sided)

This is the distance from the sample Cp to the lower or upper limit of the confidence interval, depending on the Interval Type.

You can enter a single value or a list of values.

Planning Estimate of Cp

Cp

Enter an estimate of the sample Cp. This value can be obtained from prior studies, expert opinion, or as a reasonable guess. The sample size and width calculations assume that the value entered here is the actual Cp estimate obtained from the sample. The accuracy of your results will depend on the accuracy of this estimate.

Cp can be any number. Often, Cp is between one and three in stable processes. You can enter a single value, a list of values, or a series of values. Common values are 1, 1.5, and 2.

Example 1 – Calculating Sample Size

Suppose a study is planned in which the researcher wishes to construct a two-sided 95% confidence interval for Cp such that the width of the interval is no wider than 0.10. The researcher would like to examine Cp values of 1.0, 1.5, 2.0, and 3.0 to determine the effect of the Cp estimate on necessary sample size.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Confidence Intervals for Cp** procedure window by clicking on **Quality Control** and then clicking on **Confidence Intervals for Cp**. You may then make the appropriate entries as listed below, or open **Example 1** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Sample Size**
Interval Type	**Two-Sided**
Confidence Level (1 – Alpha)	**0.95**
Confidence Interval Width (Two-Sided)..	**0.10**
Cp	**1 1.5 2 3**

Annotated Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results for Two-Sided Confidence Intervals for Cp

Confidence Level	Sample Size N	Target Width	Actual Width	Cp	C.I. Lower Limit	C.I. Upper Limit
0.950	769	0.100	0.100	1.000	0.950	1.050
0.950	1730	0.100	0.100	1.500	1.450	1.550
0.950	3074	0.100	0.100	2.000	1.950	2.050
0.950	6916	0.100	0.100	3.000	2.950	3.050

Report Definitions
Confidence Level is the proportion of confidence intervals (constructed with this same confidence level, sample size, etc.) that would contain the true value of Cp.
Sample Size N is the size of the sample drawn from the population.
Target Width is the width that was requested.
Actual Width is the calculated width. This is slightly different from the Target Width because N is an integer.
Cp is equal to (USL - LSL) / 6σ, where USL and LSL are the upper and lower specification limits and σ is the process standard deviation.
C.I. Lower Limit and C.I. Upper Limit is the confidence interval of Cp.

References
Kotz, S. and Johnson, N. 1993. Process Capability Indices. Chapman & Hall.
Mathews, Paul. 2010. Sample Size Calculations: Practical Methods for Engineers and Scientists. Mathews Malnar and Bailey, Inc.
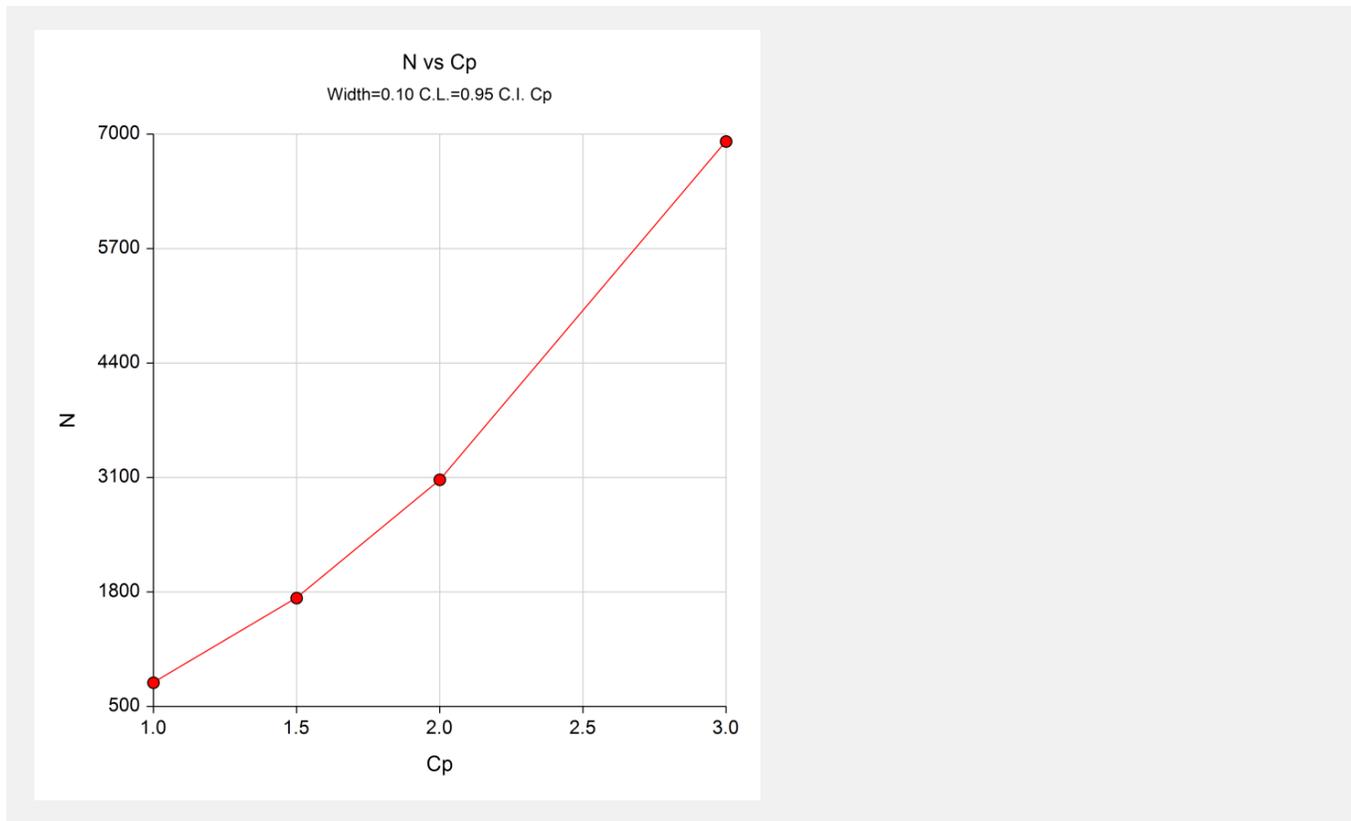
Summary Statements
A sample size of 769 produces a two-sided 95% confidence interval with a width equal to 0.100 when the estimate of Cp is 1.000.

This report shows the calculated sample size for each of the scenarios.

Plots Section

This plot shows the sample size versus Cp.

Example 2 – Validation using Mathews

Mathews (2010), page 230, gives an example of a sample size calculation. In this example the value of Cp is 1.0, the confidence level is 90%, and the width is 0.20. The resulting sample size is 136. Note that Mathews uses a normal approximation to the chi-square distribution which may make his results a little different than ours.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Confidence Intervals for Cp** procedure window by clicking on **Quality Control** and then clicking on **Confidence Intervals for Cp**. You may then make the appropriate entries as listed below, or open **Example 2** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Sample Size**
Interval Type	**Two-Sided**
Confidence Level (1 – Alpha)	**0.90**
Confidence Interval Width (Two-Sided)..	**0.20**
Cp	**1**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Confidence Level	Sample Size N	Target Width	Actual Width	Cp	C.I. Lower Limit	C.I. Upper Limit
0.900	137	0.200	0.199	1.000	0.900	1.099

PASS calculates the sample size to be 137 which is only slightly different than the 136 found by Mathews. Remember that PASS uses exact calculation of the chi-square distribution and guarantees that the width requirement is met.