Chapter 653

Confidence Intervals for One Variance using Relative Error

Introduction

This routine calculates the necessary sample size such that a sample variance estimate will achieve a specified relative distance from the true population variance at a stated confidence level when the underlying data distribution is normal.

Caution: This procedure controls the relative width of the interval as a proportion of the true population variance. For controlling the absolute width of the interval see the procedures Confidence Intervals for One Variance using Variance and Confidence Intervals for One Variance with Tolerance Probability.

Technical Details

Following the results of Desu and Raghavarao (1990) and Greenwood and Sandomire (1950), let s^2 be the variance estimate based on a sample from a normal distribution with unknown μ and unknown σ^2. Let r be the proportion of σ^2 such that s^2 is within $r\sigma^2$ of σ^2 with desired confidence. If

$$p_1 = \Pr\left(s^2 > \sigma^2 + r\sigma^2\right) = \Pr\left(s^2 > \sigma^2(1+r)\right)$$

and

$$p_2 = \Pr\left(s^2 < \sigma^2 - r\sigma^2\right) = \Pr\left(s^2 < \sigma^2(1-r)\right)$$

The confidence level for estimating σ^2 within proportion r is $1 - p_1 - p_2$.

Since $\dfrac{(n-1)s^2}{\sigma^2} \sim \chi^2_{n-1}$, it is useful to rewrite p_1 and p_2 as

$$p_1 = \Pr\left(\frac{(n-1)s^2}{\sigma^2} > (n-1)(1+r)\right)$$

and

$$p_2 = \Pr\left(\frac{(n-1)s^2}{\sigma^2} < (n-1)(1-r)\right)$$

Using the chi-square distribution, these equations can be solved for any of the unknown quantities $(n, r, p_1 + p_2)$ in terms of the others.

Confidence Level

The confidence level, $1 - \alpha$, has the following interpretation. If thousands of samples of n items are drawn from a population using simple random sampling and the variance estimates are obtained from each sample, the proportion of those estimates that are within $r\sigma^2$ of σ^2 is $1 - \alpha$.

Procedure Options

This section describes the options that are specific to this procedure. These are located on the Design tab. For more information about the options of other tabs, go to the Procedure Window chapter.

Design Tab

The Design tab contains most of the parameters and options that you will be concerned with.

Solve For

Solve For

This option specifies the parameter to be solved for from the other parameters.

One-Sided or Two-Sided Interval

Interval Type

Specify whether the interval to be used will be a two-sided confidence interval, an interval that has only an upper limit, or an interval that has only a lower limit.

In each case the limits are based on the relative error of the true population variance.

Confidence

Confidence Level

The confidence level, $1 - \alpha$, has the following interpretation. If thousands of samples of n items are drawn from a population using simple random sampling and the variance estimates are obtained from each sample, the proportion of those estimates that are within $r\sigma^2$ of σ^2 is $1 - \alpha$.

Often, the values 0.95 or 0.99 are used. You can enter single values or a range of values such as *0.90, 0.95* or *0.90 to 0.99 by 0.01*.

Sample Size

N (Sample Size)

Enter one or more values for the sample size. This is the number of individuals selected at random from the population to be in the study.

You can enter a single value or a range of values.

Precision

Relative Error

This is the distance from the true variance as a proportion of the true variance.

You can enter a single value or a list of values. The value(s) must be between 0 and 1.

Example 1 – Calculating Sample Size

Suppose a study is planned in which the researcher wishes to be 95% confident that estimated variance is within 10% of the true population variance. In addition to 10% relative error, 5%, 15%, 20% and 25% will also be considered.

The goal is to determine the necessary sample size.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Confidence Intervals for One Variance using Relative Error** procedure window by expanding **Variances**, then clicking on **One Variance**, and then clicking on **Confidence Intervals for One Variance using Relative Error**. You may then make the appropriate entries as listed below, or open **Example 1** by going to the **File** menu and choosing **Open Example Template**.

Option   **Value**

Design Tab
Solve For .... **Sample Size**
Interval Type .... **Two-Sided**
Confidence Level **0.95**
Relative Error .... **0.05 to 0.25 by 0.05**

Annotated Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results for Two-Sided Relative Error Confidence Intervals

Target Confidence Level	Actual Confidence Level	Sample Size (N)	Relative Error
0.950	0.950	3074	0.050
0.950	0.950	769	0.100
0.950	0.950	342	0.150
0.950	0.950	192	0.200
0.950	0.950	123	0.250

References
Desu, M. M. and Raghavarao, D. 1990. Sample Size Methodology. Academic Press. New York.
Greenwood, J. A. and Sandomire, M. M. 1950. 'Sample Size Required for Estimating the Standard Deviation as a
    Per Cent of its True Value', Journal of the American Statistical Association, Vol. 45, No. 250, pp. 257-260.

Confidence Intervals for One Variance using Relative Error

Report Definitions
Confidence Level is the proportion of variance estimates that will be within the relative error of the true
    variance.
Target Confidence Level is the value of the confidence level that is entered into the procedure.
Actual Confidence Level is the value of the confidence level that is obtained from the procedure.
Sample Size (N) is the size of the sample drawn from the population.
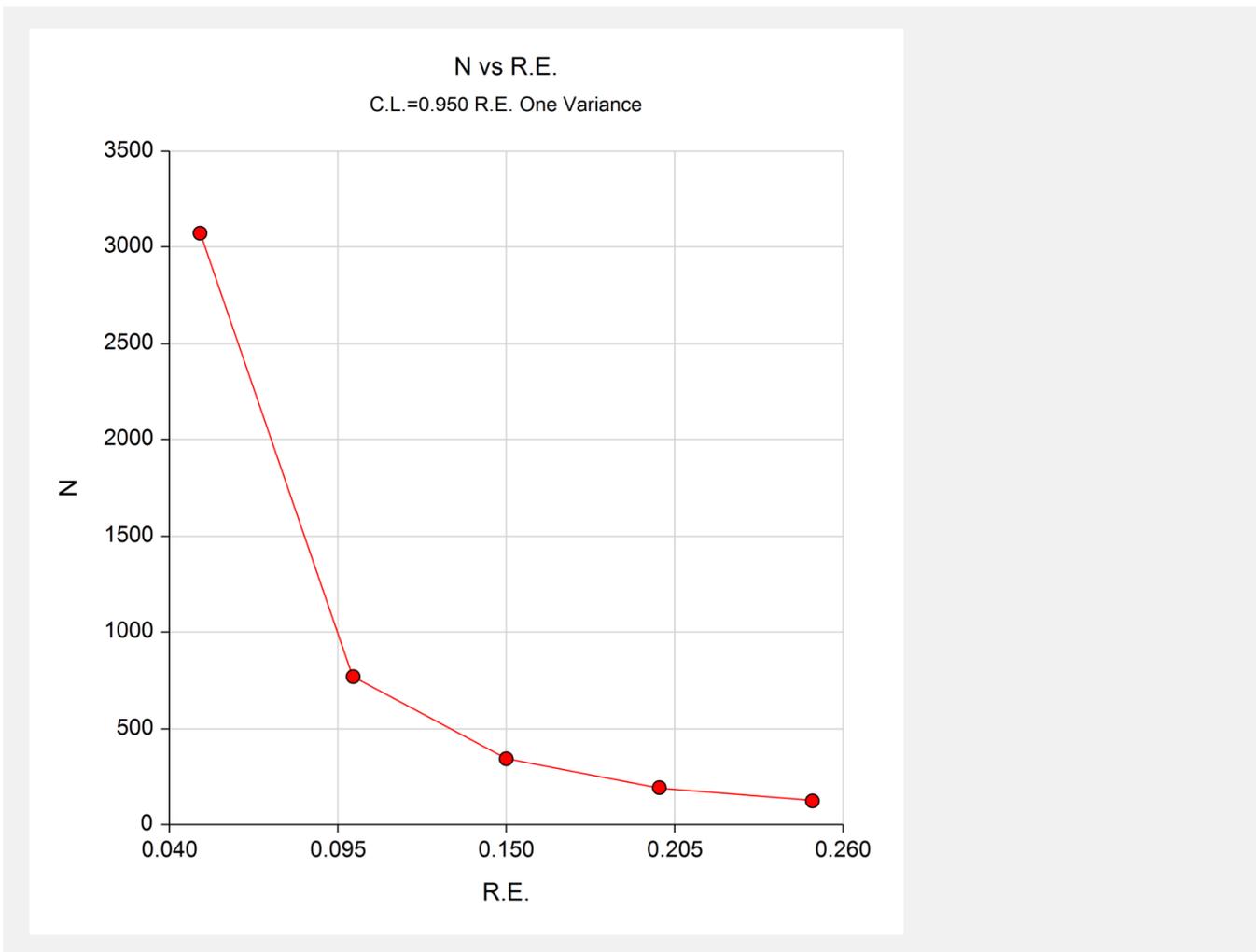Relative Error is the distance from the true variance as a proportion of the true variance.

Summary Statements
With a sample size of 3074, the probability is 0.950 (95% confidence) that the estimate of the
variance will be within 5% of the true population variance.

This report shows the calculated sample size for each of the scenarios.

Plots Section

This plot shows the sample size versus the relative error.

Example 2 – Validation using Desu and Raghavarao

Desu and Raghavarao (1990) page 6 give an example of a calculation in which the desired confidence level is 95% and the relative error is 20%. This calculation is based on a large sample approximation formula. The necessary sample size is 194.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Confidence Intervals for One Variance using Relative Error** procedure window by expanding **Variances**, then clicking on **One Variance**, and then clicking on **Confidence Intervals for One Variance using Relative Error**. You may then make the appropriate entries as listed below, or open **Example 2** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Sample Size**
Interval Type	**Two-Sided**
Confidence Level	**0.95**
Relative Error	**0.2**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Target Confidence Level	Actual Confidence Level	Sample Size (N)	Relative Error
0.950	0.950	192	0.200

PASS calculated the necessary sample size to be 192, but did not use the approximation formulas.

Using direct calculation with the non-approximate chi-square formulas with a sample size of 194, the confidence level is $1 – (0.0300164564 + 0.0188126383) = 0.951171$.

Using direct calculation with the non-approximate chi-square formulas with a sample size of 192, the confidence level is $1 – (0.0306634808 + 0.0193315700) = 0.950005$, which is closer to the prescribed confidence level. Thus, 192 is the correct value.