

## Chapter 651

# Confidence Intervals for One Variance using Variance

---

## Introduction

This routine calculates the sample size necessary to achieve a specified interval width or distance from the variance to the confidence limit at a stated confidence level for a confidence interval about the variance when the underlying data distribution is normal.

Caution: This procedure assumes that the variance of the future sample will be the same as the variance that is specified. If the variance to be used in the procedure is estimated from a previous sample or represents the population variance, the Confidence Intervals for One Variance with Tolerance Probability procedure should be considered. That procedure controls the probability that the width or the distance from the variance to the confidence limits will be less than or equal to the value specified. The Confidence Intervals for One Variance using Relative Error controls the width or distance from the variance to the limits by controlling the width or distance as a percent of the true variance.

---

## Technical Details

For a single variance from a normal distribution with unknown mean, a two-sided,  $100(1 - \alpha)\%$  confidence interval is calculated by

$$\left[ \frac{(n-1)s^2}{\chi^2_{1-\alpha/2, n-1}}, \frac{(n-1)s^2}{\chi^2_{\alpha/2, n-1}} \right]$$

A one-sided  $100(1 - \alpha)\%$  upper confidence limit is calculated by

$$\frac{(n-1)s^2}{\chi^2_{\alpha, n-1}}$$

Similarly, the one-sided  $100(1 - \alpha)\%$  lower confidence limit is

$$\frac{(n-1)s^2}{\chi^2_{1-\alpha, n-1}}$$

For two-sided intervals, the distance from the variance to each of the limits is different. Thus, instead of specifying the distance to the limits we specify the width of the interval,  $W$ .

## Confidence Intervals for One Variance using Variance

The basic equation for determining sample size for a two-sided interval when  $W$  has been specified is

$$W = \frac{(n-1)s^2}{\chi_{\alpha/2, n-1}^2} - \frac{(n-1)s^2}{\chi_{1-\alpha/2, n-1}^2}$$

For one-sided intervals, the distance from the variance to limits,  $D$ , is specified.

The basic equation for determining sample size for a one-sided upper limit when  $D$  has been specified is

$$D = \frac{(n-1)s^2}{\chi_{\alpha, n-1}^2} - s^2$$

The basic equation for determining sample size for a one-sided lower limit when  $D$  has been specified is

$$D = s^2 - \frac{(n-1)s^2}{\chi_{1-\alpha, n-1}^2}$$

These equations can be solved for any of the unknown quantities in terms of the others.

## Confidence Level

The confidence level,  $1 - \alpha$ , has the following interpretation. If thousands of samples of  $n$  items are drawn from a population using simple random sampling and a confidence interval is calculated for each sample, the proportion of those intervals that will include the true population variance is  $1 - \alpha$ .

## Procedure Options

This section describes the options that are specific to this procedure. These are located on the Design tab. For more information about the options of other tabs, go to the Procedure Window chapter.

## Design Tab

The Design tab contains most of the parameters and options that you will be concerned with.

### Solve For

#### Solve For

This option specifies the parameter to be solved for from the other parameters.

### One-Sided or Two-Sided Interval

#### Interval Type

Specify whether the interval to be used will be a two-sided confidence interval, an interval that has only an upper limit, or an interval that has only a lower limit.

## Confidence Intervals for One Variance using Variance

---

### Confidence

#### Confidence Level

The confidence level,  $1 - \alpha$ , has the following interpretation. If thousands of samples of  $n$  items are drawn from a population using simple random sampling and a confidence interval is calculated for each sample, the proportion of those intervals that will include the true population variance is  $1 - \alpha$ .

Often, the values 0.95 or 0.99 are used. You can enter single values or a range of values such as *0.90, 0.95* or *0.90 to 0.99 by 0.01*.

---

### Sample Size

#### N (Sample Size)

Enter one or more values for the sample size. This is the number of individuals selected at random from the population to be in the study.

You can enter a single value or a range of values.

---

### Precision

#### Confidence Interval Width (Two-Sided)

This is the distance from the lower confidence limit to the upper confidence limit. The distance from the variance to the lower and upper limits is not equal.

You can enter a single value or a list of values. The value(s) must be greater than zero.

#### Distance from Var to Limit (One-Sided)

This is the distance from the variance to the lower or upper limit of the confidence interval, depending on whether the Interval Type is set to Lower Limit or Upper Limit.

You can enter a single value or a list of values. The value(s) must be greater than zero.

---

### Variance

#### Variance

Enter an estimate of the variance (must be positive). The sample size and width calculations assume that the value entered here is the variance estimate that is obtained from the sample. If the sample variance is different from the one specified here, the width may be narrower or wider than specified.

For controlling the probability that the width is less than the value specified, see the procedure 'Confidence Intervals for One Variance with Tolerance Probability'.

For confidence intervals with widths that are specified in terms of a percentage of relative error, see the procedure 'Confidence Intervals for One Variance using Relative Error'.

You can enter a range of values such as *1 2 3* or *1 to 10 by 1*.

Press the Standard Deviation Estimator button to load the Standard Deviation Estimator window.

## Confidence Intervals for One Variance using Variance

**Example 1 – Calculating Sample Size**

Suppose a study is planned in which the researcher wishes to construct a two-sided 95% confidence interval for the variance such that the width of the interval is no wider than 40 units. The confidence level is set at 0.95, but 0.99 is included for comparative purposes. The variance estimate, based on the range of data values, is 24. Instead of examining only the interval width of 40, a series of widths from 30 to 50 will also be considered.

The goal is to determine the necessary sample size.

**Setup**

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Confidence Intervals for One Variance using Variance** procedure window by expanding **Variations**, then clicking on **One Variance**, and then clicking on **Confidence Intervals for One Variance using Variance**. You may then make the appropriate entries as listed below, or open **Example 1** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
<b>Design Tab</b>	
Solve For .....	<b>Sample Size</b>
Interval Type .....	<b>Two-Sided</b>
Confidence Level .....	<b>0.95 0.99</b>
Confidence Interval Width (Two-Sided) ..	<b>30 to 50 by 5</b>
Variance .....	<b>24</b>

**Annotated Output**

Click the Calculate button to perform the calculations and generate the following output.

**Numeric Results****Numeric Results for Two-Sided Confidence Intervals**

Confidence Level	Sample Size (N)	Target Width	Actual Width	Variance	Lower Limit	Upper Limit
0.950	28	30.000	29.463	24.000	15.002	44.465
0.990	46	30.000	29.663	24.000	14.761	44.424
0.950	22	35.000	34.808	24.000	14.206	49.013
0.990	36	35.000	34.924	24.000	13.936	48.860
0.950	19	40.000	38.783	24.000	13.703	52.486
0.990	30	40.000	39.745	24.000	13.299	53.044
0.950	16	45.000	44.392	24.000	13.096	57.488
0.990	26	45.000	44.251	24.000	12.786	57.036
0.950	14	50.000	49.678	24.000	12.613	62.291
0.990	23	50.000	48.754	24.000	12.338	61.092

**References**

Zar, Jerrold H. 1984. Biostatistical Analysis. Second Edition. Prentice-Hall. Englewood Cliffs, New Jersey.

**Report Definitions**

Confidence level is the proportion of confidence intervals (constructed with this same confidence level, sample size, etc.) that would contain the population variance.

N is the size of the sample drawn from the population.

Width is the distance from the lower limit to the upper limit.

Target Width is the value of the width that is entered into the procedure.

Actual Width is the value of the width that is obtained from the procedure.

Variance is the assumed sample variance.

Lower Limit is the lower limit of the confidence interval.

Upper Limit is the upper limit of the confidence interval.

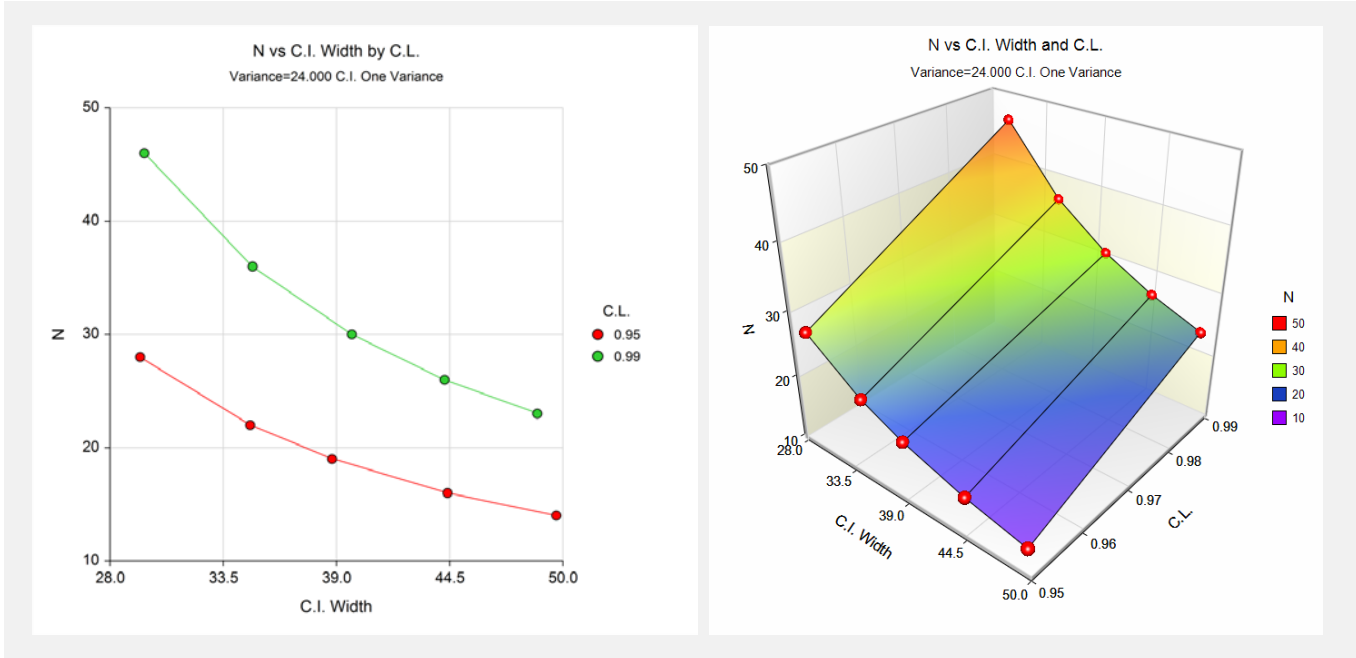
## Confidence Intervals for One Variance using Variance

### Summary Statements

A sample size of 28 produces a two-sided 95% confidence interval with a width equal to 29.463 when the sample variance is 24.000.

This report shows the calculated sample size for each of the scenarios.

### Plots Section



These plots show the sample size vs. the confidence interval width for the two confidence levels.

## Confidence Intervals for One Variance using Variance

**Example 2 – Validation using Zar**

Zar (1984) page 115 give an example of a calculation for a confidence interval on the variance when the confidence level is 95%, the variance is 18.0388, and the interval width is 23.91244. The necessary sample size is 25.

**Setup**

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Confidence Intervals for One Variance using Variance** procedure window by expanding **Variances**, then clicking on **One Variance**, and then clicking on **Confidence Intervals for One Variance using Variance**. You may then make the appropriate entries as listed below, or open **Example 2** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
<b>Design Tab</b>	
Find (Solve For) .....	<b>Sample Size</b>
Interval Type .....	<b>Two-Sided</b>
Confidence Level .....	<b>0.95</b>
Confidence Interval Width (Two-Sided) ..	<b>23.91244</b>
Variance .....	<b>18.0388</b>

**Output**

Click the Calculate button to perform the calculations and generate the following output.

**Numeric Results**

Confidence Level	Sample Size (N)	Target Width	Actual Width	Variance	Lower Limit	Upper Limit
0.950	25	23.912	23.912	18.039	10.998	34.911

PASS also calculated the necessary sample size to be 25.