Chapter 217

Confidence Intervals for the Ratio of Two Proportions

Introduction

This routine calculates the group sample sizes necessary to achieve a specified interval width of the ratio of two independent proportions.

Caution: These procedures assume that the proportions obtained from future samples will be the same as the proportions that are specified. If the sample proportions are different from those specified when running these procedures, the interval width may be narrower or wider than specified.

Technical Details

A background of the comparison of two proportions is given, followed by details of the confidence interval methods available in this procedure.

Comparing Two Proportions

Suppose you have two populations from which dichotomous (binary) responses will be recorded. The probability (or risk) of obtaining the event of interest in population 1 (the treatment group) is p_1 and in population 2 (the control group) is p_2. The corresponding failure proportions are given by $q_1 = 1 - p_1$ and $q_2 = 1 - p_2$.

The assumption is made that the responses from each group follow a binomial distribution. This means that the event probability p_i is the same for all subjects within a population and that the responses from one subject to the next are independent of one another.

Random samples of *m* and *n* individuals are obtained from these two populations. The data from these samples can be displayed in a 2-by-2 contingency table as follows

	Success	Failure	Total
Population 1	*a*	*c*	*m*
Population 2	*b*	*d*	*n*
Totals	*s*	*f*	*N*

The following alternative notation is sometimes used:

	Success	Failure	Total
Population 1	x_{11}	x_{12}	n_1
Population 2	x_{21}	x_{22}	n_2
Totals	m_1	m_2	N

The binomial proportions p_1 and p_2 are estimated from these data using the formulae

$$\hat{p}_1 = \frac{a}{m} = \frac{x_{11}}{n_1} \text{ and } \hat{p}_2 = \frac{b}{n} = \frac{x_{21}}{n_2}$$

When analyzing studies such as these, you usually want to compare the two binomial probabilities p_1 and p_2. The most direct methods of comparing these quantities are to calculate their difference or their ratio. If the binomial probability is expressed in terms of odds rather than probability, another measure is the odds ratio. Mathematically, these comparison parameters are

Parameter	Computation
Difference	$\delta = p_1 - p_2$
Risk Ratio	$\phi = p_1 / p_2$
Odds Ratio	$\psi = \frac{p_1 / q_1}{p_2 / q_2} = \frac{p_1 q_2}{p_2 q_1}$

The choice of which of these measures is used might at seem arbitrary, but it is important. Not only is their interpretation different, but, for small sample sizes, the coverage probabilities may be different. This procedure focuses on the ratio. Other procedures are available in **PASS** for computing confidence intervals for the difference and odds ratio.

Ratio

The (risk) ratio $\phi = p_1 / p_2$ gives the relative change in the disease risk due to the application of the treatment. This parameter is also direct and easy to interpret. To compare this with the difference, consider a treatment that reduces the risk of disease for 0.1437 to 0.0793. Which single number is most enlightening, the fact that the absolute risk of disease has been decreased by 0.0644, or the fact that risk of disease in the treatment group is only 55.18% of that in the control group? In many cases, the percentage (risk ratio) communicates the impact of the treatment better than the absolute change.

Perhaps the biggest drawback to this parameter is that it cannot be calculated in one of the most common experimental designs: the case-control study.

Confidence Intervals for the Ratio (Relative Risk)

Many methods have been devised for computing confidence intervals for the ratio (relative risk) of two proportions $\phi = p_1 / p_2$. Six of these methods are available in the Confidence Intervals for Two Proportions [Ratios] procedure. The six confidence interval methods are

1. Score (Farrington and Manning)
2. Score (Miettinen and Nurminen)
3. Score with Correction for Skewness (Gart and Nam)

4. Logarithm (Katz)
5. Logarithm + 1/2 (Walter)
6. Fleiss

Farrington and Manning's Score

Farrington and Manning (1990) proposed a test statistic for testing whether the ratio is equal to a specified value ϕ_0. The regular MLE's $\hat{p}_1$ and $\hat{p}_2$ are used in the numerator of the score statistic while MLE's $\tilde{p}_1$ and $\tilde{p}_2$ constrained so that $\tilde{p}_1 / \tilde{p}_2 = \phi_0$ are used in the denominator. A correction factor of N/(N-1) is applied to increase the variance estimate. The significance level of the test statistic is based on the asymptotic normality of the score statistic.

Here is the formula for computing the test

$$z_{FMR} = \frac{\hat{p}_1 / \hat{p}_2 - \phi_0}{\sqrt{\left(\frac{\tilde{p}_1 \tilde{q}_1}{n_1} + \phi_0^2 \frac{\tilde{p}_2 \tilde{q}_2}{n_2} \right)}}$$

where

$$\tilde{p}_1 = \tilde{p}_2 \phi_0$$

$$\tilde{p}_2 = \frac{-B - \sqrt{B^2 - 4AC}}{2A}$$

$$A = N\phi_0$$

$$B = -\left[n_1\phi_0 + x_{11} + n_2 + x_{21}\phi_0 \right]$$

$$C = m_1$$

as in the test of Miettinen and Nurminen (1985).

Farrington and Manning (1990) proposed inverting their score test to find the confidence interval. The lower limit is found by solving

$$z_{FMR} = \left| z_{\alpha/2} \right|$$

and the upper limit is the solution of

$$z_{FMR} = -\left| z_{\alpha/2} \right|$$

Miettinen and Nurminen's Score

Miettinen and Nurminen (1985) proposed a test statistic for testing whether the ratio is equal to a specified value ϕ_0. The regular MLE's $\hat{p}_1$ and $\hat{p}_2$ are used in the numerator of the score statistic while MLE's $\tilde{p}_1$ and $\tilde{p}_2$ constrained so that $\tilde{p}_1 / \tilde{p}_2 = \phi_0$ are used in the denominator. A correction factor of N/(N-1) is applied to make the variance estimate less biased. The significance level of the test statistic is based on the asymptotic normality of the score statistic.

Here is the formula for computing the test

$$z_{MNR} = \frac{\hat{p}_1 / \hat{p}_2 - \phi_0}{\sqrt{\left(\frac{\tilde{p}_1 \tilde{q}_1}{n_1} + \phi_0^2 \frac{\tilde{p}_2 \tilde{q}_2}{n_2} \right)\left(\frac{N}{N-1} \right)}}$$

where

$$\tilde{p}_1 = \tilde{p}_2 \phi_0$$

$$\tilde{p}_2 = \frac{-B - \sqrt{B^2 - 4AC}}{2A}$$

$$A = N\phi_0$$

$$B = -\left[n_1\phi_0 + x_{11} + n_2 + x_{21}\phi_0\right]$$

$$C = m_1$$

Miettinen and Nurminen (1985) proposed inverting their score test to find the confidence interval. The lower limit is found by solving

$$z_{MNR} = \left|z_{\alpha/2}\right|$$

and the upper limit is the solution of

$$z_{MNR} = -\left|z_{\alpha/2}\right|$$

Gart and Nam's Score

Gart and Nam (1988) page 329 proposed a modification to the Farrington and Manning (1988) ratio test that corrected for skewness. Let $z_{FM}(\phi)$ stand for the Farrington and Manning ratio test statistic described above. The skewness corrected test statistic z_{GN} is the appropriate solution to the quadratic equation

$$\left(-\tilde{\varphi}\right)z_{GNR}^2 + \left(-1\right)z_{GNR} + \left(z_{FMR}(\phi) + \tilde{\varphi}\right) = 0$$

where

$$\tilde{\varphi} = \frac{1}{6\tilde{u}^{3/2}}\left(\frac{\tilde{q}_1\left(\tilde{q}_1 - \tilde{p}_1\right)}{n_1^2\tilde{p}_1^2} - \frac{\tilde{q}_2\left(\tilde{q}_2 - \tilde{p}_2\right)}{n_2^2\tilde{p}_2^2}\right)$$

$$\tilde{u} = \frac{\tilde{q}_1}{n_1\tilde{p}_1} + \frac{\tilde{q}_2}{n_2\tilde{p}_2}$$

Gart and Nam (1988) proposed inverting their score test to find the confidence interval. The lower limit is found by solving

$$z_{GNR} = \left|z_{\alpha/2}\right|$$

and the upper limit is the solution of

$$z_{GNR} = -\left|z_{\alpha/2}\right|$$

Logarithm (Katz)

This was one of the first methods proposed for computing confidence intervals for risk ratios.

For details, see Gart and Nam (1988), page 324.

$$L = \hat{\phi}\exp\left(-z\sqrt{\frac{\hat{q}_1}{n\hat{p}_1}+\frac{\hat{q}_2}{n\hat{p}_2}}\right)$$

$$U = \hat{\phi}\exp\left(z\sqrt{\frac{\hat{q}_1}{n\hat{p}_1}+\frac{\hat{q}_2}{n\hat{p}_2}}\right)$$

where

$$\hat{\phi} = \frac{\hat{p}_1}{\hat{p}_2}$$

Logarithm (Walters)

For details, see Gart and Nam (1988), page 324.

$$L = \hat{\phi}\exp\left(-z\sqrt{\hat{u}}\right)$$

$$U = \hat{\phi}\exp\left(z\sqrt{\hat{u}}\right)$$

where

$$\hat{\phi} = \exp\left(\ln\left(\frac{a+\frac{1}{2}}{m+\frac{1}{2}}\right)-\ln\left(\frac{b+\frac{1}{2}}{n+\frac{1}{2}}\right)\right)$$

$$\hat{u} = \frac{1}{a+\frac{1}{2}}-\frac{1}{m+\frac{1}{2}}+\frac{1}{b+\frac{1}{2}}-\frac{1}{n+\frac{1}{2}}$$

$$\tilde{q}_2 = 1-\tilde{p}_2$$

$$V = \left(\phi^2\left(\frac{\tilde{q}_1}{m\tilde{p}_1}+\frac{\tilde{q}_2}{n\tilde{p}_2}\right)\right)^{-1}$$

$$\tilde{p}_1 = \phi\tilde{p}_2$$

$$\tilde{q}_1 = 1-\tilde{p}_1$$

$$\tilde{q}_2 = 1-\tilde{p}_2$$

$$\tilde{\mu}_3 = v^{3/2}\left(\frac{\tilde{q}_1\left(\tilde{q}_1-\tilde{p}_1\right)}{\left(m\tilde{p}_1\right)^2}-\frac{\tilde{q}_2\left(\tilde{q}_2-\tilde{p}_2\right)}{\left(n\tilde{p}_2\right)^2}\right)$$

$$v = \left(\frac{\tilde{q}_1}{m\tilde{p}_1}+\frac{\tilde{q}_2}{n\tilde{p}_2}\right)^{-1}$$

Iterated Method of Fleiss

Fleiss (1981) presents an improved confidence interval for the odds ratio and relative risk. This method forms the confidence interval as all those value of the odds ratio which would not be rejected by a chi-square hypothesis test. Fleiss gives the following details about how to construct this confidence interval. To compute the lower limit, do the following.

1. For a trial value of ψ, compute the quantities *X, Y, W, F, U,* and *V* using the formulas

$$X = \psi(m+s) + (n-s)$$

$$Y = \sqrt{X^2 - 4ms\psi(\psi-1)}$$

$$A = \frac{X-Y}{2(\psi-1)}$$

$$B = s - A$$

$$C = m - A$$

$$D = f - m + A$$

$$W = \frac{1}{A} + \frac{1}{B} + \frac{1}{C} + \frac{1}{D}$$

$$F = \left(a - A - \tfrac{1}{2}\right)^2 W - z_{\alpha/2}^2$$

$$T = \frac{1}{2(\psi-1)^2}\left(Y - n - \frac{\psi-1}{Y}\left[X(m+s) - 2ms(2\psi-1)\right]\right)$$

$$U = \frac{1}{B^2} + \frac{1}{C^2} - \frac{1}{A^2} - \frac{1}{D^2}$$

$$V = T\left[\left(a - A - \tfrac{1}{2}\right)^2 U - 2W\left(a - A - \tfrac{1}{2}\right)\right]$$

Finally, use the updating equation below to calculate a new value for the odds ratio using the updating equation

$$\psi^{(k+1)} = \psi^{(k)} - \frac{F}{V}$$

2. Continue iterating until the value of *F* is arbitrarily close to zero.

The upper limit is found by substituting $+\tfrac{1}{2}$ for $-\tfrac{1}{2}$ in the formulas for *F* and *V*.

Confidence limits for the *relative risk* can be calculated using the expected counts *A*, *B*, *C*, and *D* from the last iteration of the above procedure. The lower limit of the relative risk

$$\phi_{lower} = \frac{A_{lower} n}{B_{lower} m}$$

$$\phi_{upper} = \frac{A_{upper} n}{B_{upper} m}$$

Confidence Level

The confidence level, $1 - \alpha$, has the following interpretation. If thousands of random samples of size n_1 and n_2 are drawn from populations 1 and 2, respectively, and a confidence interval for the true difference/ratio/odds ratio of proportions is calculated for each pair of samples, the proportion of those intervals that will include the true difference/ratio/odds ratio of proportions is $1 - \alpha$.

Procedure Options

This section describes the options that are specific to this procedure. These are located on the Design tab. For more information about the options of other tabs, go to the Procedure Window chapter.

Design Tab

The Design tab contains the parameters associated with this calculation such as the proportions or ratios, sample sizes, confidence level, and interval width.

Solve For

Solve For

This option specifies the parameter to be solved for from the other parameters.

Confidence Interval Method

Confidence Interval Formula

Specify the formula to be in used in calculation of confidence intervals.

- **Score (Farrington & Manning)**
  This formula is based on inverting Farrington and Manning's score test.
- **Score (Miettinen & Nurminen)**
  This formula is based on inverting Miettinen and Nurminen's score test.
- **Score w/ Skewness (Gart & Nam)**
  This formula is based on inverting Gart and Nam's score test, with a correction for skewness.
- **Logarithm (Katz)**
  This formula is based on the asymptotic normality of log(P1/P2).
- **Logarithm + 1/2 (Walter)**
  This formula is based on the asymptotic normality of log(P1/P2), but 1/2 is used as an adjustment.
- **Fleiss**
  This is an iterative method that was developed for the odds ratio and adapted to the proportion ratio.

One-Sided or Two-Sided Interval

Interval Type

Specify whether the interval to be used will be a two-sided confidence interval, an interval that has only an upper limit, or an interval that has only a lower limit.

Confidence

Confidence Level (1 – Alpha)

The confidence level, $1 - \alpha$, has the following interpretation. If thousands of random samples of size n_1 and n_2 are drawn from populations 1 and 2, respectively, and a confidence interval for the true difference/ratio/odds ratio of proportions is calculated for each pair of samples, the proportion of those intervals that will include the true difference/ratio/odds ratio of proportions is $1 - \alpha$.

Often, the values 0.95 or 0.99 are used. You can enter single values or a range of values such as *0.90, 0.95 or 0.90 to 0.99 by 0.01*.

Sample Size (When Solving for Sample Size)

Group Allocation

Select the option that describes the constraints on *N1* or *N2* or both.

The options are

- **Equal (N1 = N2)**

  This selection is used when you wish to have equal sample sizes in each group. Since you are solving for both sample sizes at once, no additional sample size parameters need to be entered.

- **Enter N1, solve for N2**

  Select this option when you wish to fix *N1* at some value (or values), and then solve only for *N2*. Please note that for some values of *N1*, there may not be a value of *N2* that is large enough to obtain the desired power.

- **Enter N2, solve for N1**

  Select this option when you wish to fix *N2* at some value (or values), and then solve only for *N1*. Please note that for some values of *N2*, there may not be a value of *N1* that is large enough to obtain the desired power.

- **Enter R = N2/N1, solve for N1 and N2**

  For this choice, you set a value for the ratio of *N2* to *N1*, and then PASS determines the needed *N1* and *N2*, with this ratio, to obtain the desired power. An equivalent representation of the ratio, *R*, is

  $$N2 = R * N1.$$

- **Enter percentage in Group 1, solve for N1 and N2**

  For this choice, you set a value for the percentage of the total sample size that is in Group 1, and then PASS determines the needed *N1* and *N2* with this percentage to obtain the desired power.

N1 (Sample Size, Group 1)

This option is displayed if Group Allocation = "Enter N1, solve for N2"

N1 is the number of items or individuals sampled from the Group 1 population.

N1 must be ≥ 2. You can enter a single value or a series of values.

N2 (Sample Size, Group 2)

This option is displayed if Group Allocation = "Enter N2, solve for N1"

N2 is the number of items or individuals sampled from the Group 2 population.

N2 must be ≥ 2. You can enter a single value or a series of values.

R (Group Sample Size Ratio)

This option is displayed only if Group Allocation = "Enter R = N2/N1, solve for N1 and N2."

R is the ratio of *N2* to *N1*. That is,

$$R = N2 / N1.$$

Use this value to fix the ratio of *N2* to *N1* while solving for *N1* and *N2*. Only sample size combinations with this ratio are considered.

N2 is related to *N1* by the formula:

$$N2 = [R \times N1],$$

where the value *[Y]* is the next integer ≥ *Y*.

For example, setting *R* = 2.0 results in a Group 2 sample size that is double the sample size in Group 1 (e.g., *N1* = 10 and *N2* = 20, or *N1* = 50 and *N2* = 100).

R must be greater than 0. If *R* < 1, then *N2* will be less than *N1*; if *R* > 1, then *N2* will be greater than *N1*. You can enter a single or a series of values.

Percent in Group 1

This option is displayed only if Group Allocation = "Enter percentage in Group 1, solve for N1 and N2."

Use this value to fix the percentage of the total sample size allocated to Group 1 while solving for *N1* and *N2*. Only sample size combinations with this Group 1 percentage are considered. Small variations from the specified percentage may occur due to the discrete nature of sample sizes.

The Percent in Group 1 must be greater than 0 and less than 100. You can enter a single or a series of values.

Sample Size (When Not Solving for Sample Size)

Group Allocation

Select the option that describes how individuals in the study will be allocated to Group 1 and to Group 2.

The options are

- **Equal (N1 = N2)**
  This selection is used when you wish to have equal sample sizes in each group. A single per group sample size will be entered.

- **Enter N1 and N2 individually**
  This choice permits you to enter different values for *N1* and *N2*.

- **Enter N1 and R, where N2 = R * N1**
  Choose this option to specify a value (or values) for *N1*, and obtain *N2* as a ratio (multiple) of *N1*.

- **Enter total sample size and percentage in Group 1**
  Choose this option to specify a value (or values) for the total sample size (*N*), obtain *N1* as a percentage of *N*, and then *N2* as *N* - *N1*.

Sample Size Per Group

This option is displayed only if Group Allocation = "Equal (N1 = N2)."

The Sample Size Per Group is the number of items or individuals sampled from each of the Group 1 and Group 2 populations. Since the sample sizes are the same in each group, this value is the value for *N1*, and also the value for *N2*.

The Sample Size Per Group must be ≥ 2. You can enter a single value or a series of values.

N1 (Sample Size, Group 1)

*This option is displayed if Group Allocation = "Enter N1 and N2 individually" or "Enter N1 and R, where N2 = R * N1."*

N1 is the number of items or individuals sampled from the Group 1 population.

N1 must be ≥ 2. You can enter a single value or a series of values.

N2 (Sample Size, Group 2)

This option is displayed only if Group Allocation = "Enter N1 and N2 individually."

N2 is the number of items or individuals sampled from the Group 2 population.

N2 must be ≥ 2. You can enter a single value or a series of values.

R (Group Sample Size Ratio)

*This option is displayed only if Group Allocation = "Enter N1 and R, where N2 = R * N1."*

R is the ratio of *N2* to *N1*. That is,

$$R = N2/N1$$

Use this value to obtain *N2* as a multiple (or proportion) of *N1*.

N2 is calculated from *N1* using the formula:

$$N2=[R \times N1],$$

where the value *[Y]* is the next integer ≥ *Y*.

For example, setting *R* = 2.0 results in a Group 2 sample size that is double the sample size in Group 1.

R must be greater than 0. If *R* < 1, then *N2* will be less than N1; if *R* > 1, then *N2* will be greater than *N1*. You can enter a single value or a series of values.

Total Sample Size (N)

This option is displayed only if Group Allocation = "Enter total sample size and percentage in Group 1."

This is the total sample size, or the sum of the two group sample sizes. This value, along with the percentage of the total sample size in Group 1, implicitly defines *N1* and *N2*.

The total sample size must be greater than one, but practically, must be greater than 3, since each group sample size needs to be at least 2.

You can enter a single value or a series of values.

Percent in Group 1

This option is displayed only if Group Allocation = "Enter total sample size and percentage in Group 1."

This value fixes the percentage of the total sample size allocated to Group 1. Small variations from the specified percentage may occur due to the discrete nature of sample sizes.

The Percent in Group 1 must be greater than 0 and less than 100. You can enter a single value or a series of values.

Precision

Confidence Interval Width (Two-Sided)

This is the distance from the lower confidence limit to the upper confidence limit.

You can enter a single value or a list of values. The value(s) must be greater than zero.

Distance from Ratio to Limit (One-Sided)

This is the distance from the ratio of sample proportions to the lower or upper limit of the confidence interval, depending on whether the Interval Type is set to Lower Limit or Upper Limit.

You can enter a single value or a list of values. The value(s) must be greater than zero.

Proportions (Ratio = P1/P2)

Input Type

Indicate what type of values to enter to specify the ratio. Regardless of the entry type chosen, the calculations are the same. This option is simply given for convenience in specifying the ratio.

P1/P2 (Ratio of Sample Proportions)

This option is displayed only if Input Type = "Ratios"

Enter an estimate of the ratio of sample proportion 1 to sample proportion 2. The sample size and width calculations assume that the value entered here is the ratio estimate that is obtained from the samples. If the sample ratio is different from the one specified here, the width may be narrower or wider than specified.

The value(s) must be greater than 0, and such that P1 = Ratio * P2 is between 0.0001 and 0.9999.

You can enter a range of values such as *.7 .8 .9* or *.5 to .9 by .1*.

P1 (Proportion Group 1)

This option is displayed only if Input Type = "Proportions"

Enter an estimate of the proportion for group 1. The sample size and width calculations assume that the value entered here is the proportion estimate that is obtained from the sample. If the sample proportion is different from the one specified here, the width may be narrower or wider than specified.

The value(s) must be between 0.0001 and 0.9999.

You can enter a range of values such as *.1 .2 .3* or *.1 to .5 by .1*.

P2 (Proportion Group 2)

Enter an estimate of the proportion for group 2. The sample size and width calculations assume that the value entered here is the proportion estimate that is obtained from the sample. If the sample proportion is different from the one specified here, the width may be narrower or wider than specified.

The value(s) must be between 0.0001 and 0.9999.

You can enter a range of values such as *.1 .2 .3* or *.1 to .5 by .1*.

Example 1 – Calculating Sample Size

Suppose a study is planned in which the researcher wishes to construct a two-sided 95% confidence interval for the ratio of proportions such that the width of the interval is no wider than 0.2. The confidence interval method to be used is the Logarithm (Katz) method. The confidence level is set at 0.95, but 0.99 is included for comparative purposes. The ratio estimate to be used is 1.2, and the estimate for proportion 2 is 0.6. Instead of examining only the interval width of 0.2, a series of widths from 0.1 to 0.3 will also be considered.

The goal is to determine the necessary sample size.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Confidence Intervals for the Ratio of Two Proportions** procedure window by expanding **Proportions**, then **Two Independent Proportions**, then clicking on **Confidence Interval**, and then clicking on **Confidence Intervals for the Ratio of Two Proportions**. You may then make the appropriate entries as listed below, or open **Example 1** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Sample Size**
Confidence Interval Formula	**Logarithm (Katz)**
Interval Type	**Two-Sided**
Confidence Level	**0.95 0.99**
Group Allocation	**Equal (N1 = N2)**
Confidence Interval Width (Two-Sided)	**0.10 to 0.30 by 0.05**
Input Type	**Ratios**
P1/P2 (Ratio of Sample Proportions)	**1.2**
P2	**0.6**

Annotated Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results for Two-Sided Confidence Intervals for the Ratio of Proportions
Confidence Interval Method: Logarithm (Katz)

Confidence Level	N1	N2	N	Target Width	Actual Width	P1	P2	P1/P2	Lower Limit	Upper Limit
0.950	2337	2337	4674	0.100	0.100	0.72	0.60	1.20	1.15	1.25
0.950	1040	1040	2080	0.150	0.150	0.72	0.60	1.20	1.13	1.28
0.950	586	586	1172	0.200	0.200	0.72	0.60	1.20	1.10	1.30
0.950	376	376	752	0.250	0.250	0.72	0.60	1.20	1.08	1.33
0.950	261	261	522	0.300	0.300	0.72	0.60	1.20	1.06	1.36
0.990	4037	4037	8074	0.100	0.100	0.72	0.60	1.20	1.15	1.25
0.990	1796	1796	3592	0.150	0.150	0.72	0.60	1.20	1.13	1.28
0.990	1011	1011	2022	0.200	0.200	0.72	0.60	1.20	1.10	1.30
0.990	648	648	1296	0.250	0.250	0.72	0.60	1.20	1.08	1.33
0.990	451	451	902	0.300	0.300	0.72	0.60	1.20	1.06	1.36

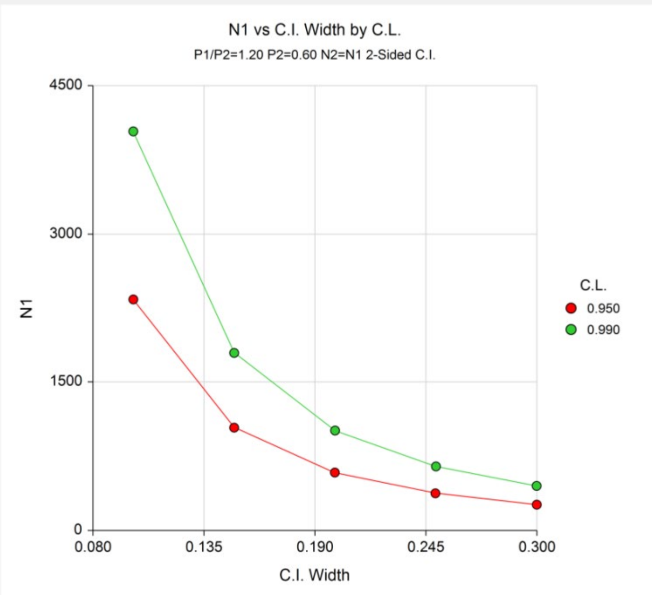
References
Gart, John J. and Nam, Jun-mo. 1988. 'Approximate Interval Estimation of the Ratio of Binomial Parameters: A Review and Corrections for Skewness.' Biometrics, Volume 44, 323-338.
Koopman, P. A. R. 1984. 'Confidence Intervals for the Ratio of Two Binomial Proportions.' Biometrics, Volume 40, Issue 2, 513-517.
Katz, D., Baptista, J., Azen, S. P., and Pike, M. C. 1978. 'Obtaining Confidence Intervals for the Risk Ratio in Cohort Studies.' Biometrics, Volume 34, 469-474.

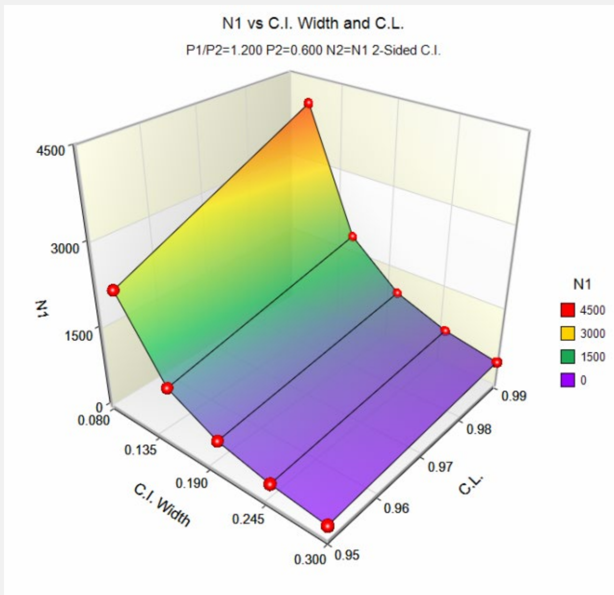
Report Definitions
Confidence level is the proportion of confidence intervals (constructed with this same confidence level, sample size, etc.) that would contain the true ratio of population proportions.
N1 and N2 are the number of items sampled from each population.
N is the total sample size, N1 + N2.
Target Width is the value of the width that is entered into the procedure.
Actual Width is the value of the width that is obtained from the procedure.
P1 and P2 are the assumed sample proportions for sample size calculations.
P1/P2 is the ratio of sample proportions at which sample size calculations are made.
Lower Limit and Upper Limit are the lower and upper limits of the confidence interval for the true ratio of proportions (Population Proportion 1 / Population Proportion 2).

Summary Statements
Group sample sizes of 2337 and 2337 produce a two-sided 95% confidence interval for the ratio of population proportions with a width that is equal to 0.100 when the estimated sample proportion 1 is 0.72, the estimated sample proportion 2 is 0.60, and the ratio of the sample proportions is 1.20.

This report shows the calculated sample sizes for each of the scenarios.

Plots Section





These plots show the group sample size versus the confidence interval width for the two confidence levels.

Example 2 – Validation using Gart and Nam (1988)

Gart and Nam (1988) page 331 give an example (Example 2) of a calculation for a confidence interval for the ratio of proportions when the confidence level is 95%, the sample proportion ratio is 2 and the sample proportion 2 is 0.3, the sample size for group 2 is 20, and the interval width is 3.437 for the Logarithm + 1/2 (Walter) method, 3.751 for the Score (Farrington and Manning) method, and 4.133 for the Score w/Skewness (Gart and Nam) method. The necessary sample size for group 1 in each case is 10.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Confidence Intervals for the Ratio of Two Proportions** procedure window by expanding **Proportions**, then **Two Independent Proportions**, then clicking on **Confidence Interval**, and then clicking on **Confidence Intervals for the Ratio of Two Proportions**. You may then make the appropriate entries as listed below, or open **Example 2(a-c)** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Sample Size**
Confidence Interval Formula	**Varies [Logarithm + 1/2 (Walter), Score (Farrington and Manning), Score w/Skewness (Gart and Nam)]**
Interval Type	**Two-Sided**
Confidence Level	**0.95**
Group Allocation	**Enter N2, solve for N1**
N2	**20**
Confidence Interval Width (Two-Sided)	**Varies (3.437, 3.751, 4.133)**
Input Type	**Ratios**
P1/P2 (Ratio of Sample Proportions)	**2**
P2	**0.3**

Output

Click the Calculate button to perform the calculations and generate the following output.

Logarithm + 1/2 (Walter)

Confidence Level	N1	N2	N	Target Width	Actual Width	P1	P2	P1/P2	Lower Limit	Upper Limit
0.950	10	20	30	3.437	3.431	0.60	0.30	2.00	0.88	4.31

PASS also calculates the necessary sample size for Group 1 to be 10.

Score (Farrington and Manning)

Confidence Level	N1	N2	N	Target Width	Actual Width	P1	P2	P1/P2	Lower Limit	Upper Limit
0.950	10	20	30	3.751	3.751	0.60	0.30	2.00	0.84	4.59

PASS also calculates the necessary sample size for Group 1 to be 10.

Score w/Skewness (Gart and Nam)

Confidence Level	N1	N2	N	Target Width	Actual Width	P1	P2	P1/P2	Lower Limit	Upper Limit
0.950	10	20	30	4.133	4.132	0.60	0.30	2.00	0.82	4.95

PASS also calculates the necessary sample size for Group 1 to be 10.

Example 3 – Validation using Katz et al (1978)

Katz et al (1978) pages 472-473 give an example of a calculation for a lower limit confidence interval for the ratio of proportions when the confidence level is 97.5%, the sample proportion ratio is 1.596078 and the sample proportion 2 is 0.153153, the sample size for group 2 is 111, and the distance from the ratio to the limit is 0.6223 for the Logarithm (Katz) method. The necessary sample size for group 1 is 225.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Confidence Intervals for the Ratio of Two Proportions** procedure window by expanding **Proportions**, then **Two Independent Proportions**, then clicking on **Confidence Interval**, and then clicking on **Confidence Intervals for the Ratio of Two Proportions**. You may then make the appropriate entries as listed below, or open **Example 3** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Sample Size**
Confidence Interval Formula	**Logarithm (Katz)**
Interval Type	**Lower Limit**
Confidence Level	**0.975**
Group Allocation	**Enter N2, solve for N1**
N2	**111**
Distance to from Ratio to Limit	**0.6223**
Input Type	**Ratios**
P1/P2 (Ratio of Sample Proportions)	**1.596078**
P2	**0.153153**

Output

Click the Calculate button to perform the calculations and generate the following output.

Logarithm (Katz)

Confidence Level	N1	N2	N	Target Dist from Ratio to Limit	Actual Dist from Ratio to Limit	P1	P2	P1/P2	Lower Limit	Upper Limit
0.975	225	111	336	0.622	0.622	0.24	0.15	1.60	0.97	Inf

PASS also calculates the necessary sample size for group 1 to be 225.