Chapter 656

Confidence Intervals for the Ratio of Two Variances using Variances

Introduction

This routine calculates the group sample sizes necessary to achieve a specified interval width or distance from the variance ratio to the confidence limit at a stated confidence level for a confidence interval about the variance ratio when the underlying data distribution is normal.

Caution: This procedure assumes that the variances of the future samples will be the same as the variances that are specified. The Confidence Intervals for the Ratio of Two Variances using Relative Error controls the width or distance from the variance ratio to the limits by controlling the width or distance as a percent of the true variance ratio.

Technical Details

For a ratio of two variances from normal distributions, a two-sided, 100(1 – α)% confidence interval is calculated by

$$\left[\frac{s_1^2}{s_2^2}\frac{1}{F_{\alpha/2,n_1-1,n_2-1}}, \frac{s_1^2}{s_2^2}F_{\alpha/2,n_2-1,n_1-1}\right]$$

A one-sided 100(1 – α)% upper confidence limit is calculated by

$$\frac{s_1^2}{s_2^2}F_{\alpha,n_2-1,n_1-1}$$

Similarly, the one-sided 100(1 – α)% lower confidence limit is

$$\frac{s_1^2}{s_2^2}\frac{1}{F_{\alpha,n_1-1,n_2-1}}$$

For two-sided intervals, the distance from the variance ratio to each of the limits is different. Thus, instead of specifying the distance to the limits we specify the width of the interval, *W*.

The basic equation for determining sample size for a two-sided interval when W has been specified is

$$W = \frac{s_1^2}{s_2^2} F_{\alpha/2,n_2-1,n_1-1} - \frac{s_1^2}{s_2^2} \frac{1}{F_{\alpha/2,n_1-1,n_2-1}}$$

For one-sided intervals, the distance from the variance ratio to limit, D, is specified.

The basic equation for determining sample size for a one-sided upper limit when D has been specified is

$$D = \frac{s_1^2}{s_2^2} F_{\alpha,n_2-1,n_1-1} - \frac{s_1^2}{s_2^2}$$

The basic equation for determining sample size for a one-sided lower limit when D has been specified is

$$D = \frac{s_1^2}{s_2^2} - \frac{s_1^2}{s_2^2} \frac{1}{F_{\alpha,n_1-1,n_2-1}}$$

These equations can be solved for any of the unknown quantities in terms of the others.

Confidence Level

The confidence level, $1 - \alpha$, has the following interpretation. If thousands of random samples of size n_1 and n_2 are drawn from populations 1 and 2, respectively, and a confidence interval for the variance ratio is calculated for each pair of samples, the proportion of those intervals that will include the true variance ratio is $1 - \alpha$.

Procedure Options

This section describes the options that are specific to this procedure. These are located on the Design tab. For more information about the options of other tabs, go to the Procedure Window chapter.

Design Tab

The Design tab contains most of the parameters and options that you will be concerned with.

Solve For

Solve For

This option specifies the parameter to be solved for from the other parameters.

One-Sided or Two-Sided Interval

Interval Type

Specify whether the interval to be used will be a two-sided confidence interval, an interval that has only an upper limit, or an interval that has only a lower limit.

Confidence

Confidence Level

The confidence level, $1 - \alpha$, has the following interpretation. If thousands of random samples of size n_1 and n_2 are drawn from populations 1 and 2, respectively, and a confidence interval for the variance ratio is calculated for each pair of samples, the proportion of those intervals that will include the true variance ratio is $1 - \alpha$.

Often, the values 0.95 or 0.99 are used. You can enter single values or a range of values such as *0.90, 0.95* or *0.90 to 0.99 by 0.01*.

Sample Size (When Solving for Sample Size)

Group Allocation

Select the option that describes the constraints on *N1* or *N2* or both.

The options are

- **Equal (N1 = N2)**

  This selection is used when you wish to have equal sample sizes in each group. Since you are solving for both sample sizes at once, no additional sample size parameters need to be entered.

- **Enter N1, solve for N2**

  Select this option when you wish to fix *N1* at some value (or values), and then solve only for *N2*. Please note that for some values of *N1*, there may not be a value of *N2* that is large enough to obtain the desired power.

- **Enter N2, solve for N1**

  Select this option when you wish to fix *N2* at some value (or values), and then solve only for *N1*. Please note that for some values of *N2*, there may not be a value of *N1* that is large enough to obtain the desired power.

- **Enter R = N2/N1, solve for N1 and N2**

  For this choice, you set a value for the ratio of *N2* to *N1*, and then PASS determines the needed *N1* and *N2*, with this ratio, to obtain the desired power. An equivalent representation of the ratio, *R*, is

  $$N2 = R * N1.$$

- **Enter percentage in Group 1, solve for N1 and N2**

  For this choice, you set a value for the percentage of the total sample size that is in Group 1, and then PASS determines the needed *N1* and *N2* with this percentage to obtain the desired power.

N1 (Sample Size, Group 1)

This option is displayed if Group Allocation = "Enter N1, solve for N2"

N1 is the number of items or individuals sampled from the Group 1 population.

N1 must be ≥ 2. You can enter a single value or a series of values.

N2 (Sample Size, Group 2)

This option is displayed if Group Allocation = "Enter N2, solve for N1"

N2 is the number of items or individuals sampled from the Group 2 population.

N2 must be ≥ 2. You can enter a single value or a series of values.

R (Group Sample Size Ratio)

This option is displayed only if Group Allocation = "Enter R = N2/N1, solve for N1 and N2."

R is the ratio of *N2* to *N1*. That is,

$$R = N2 / N1.$$

Use this value to fix the ratio of *N2* to *N1* while solving for *N1* and *N2*. Only sample size combinations with this ratio are considered.

N2 is related to *N1* by the formula:

$$N2 = [R \times N1],$$

where the value *[Y]* is the next integer $\geq Y$.

For example, setting $R = 2.0$ results in a Group 2 sample size that is double the sample size in Group 1 (e.g., $N1 = 10$ and $N2 = 20$, or $N1 = 50$ and $N2 = 100$).

R must be greater than 0. If $R < 1$, then *N2* will be less than *N1*; if $R > 1$, then *N2* will be greater than *N1*. You can enter a single or a series of values.

Percent in Group 1

This option is displayed only if Group Allocation = "Enter percentage in Group 1, solve for N1 and N2."

Use this value to fix the percentage of the total sample size allocated to Group 1 while solving for *N1* and *N2*. Only sample size combinations with this Group 1 percentage are considered. Small variations from the specified percentage may occur due to the discrete nature of sample sizes.

The Percent in Group 1 must be greater than 0 and less than 100. You can enter a single or a series of values.

Sample Size (When Not Solving for Sample Size)

Group Allocation

Select the option that describes how individuals in the study will be allocated to Group 1 and to Group 2.

The options are

- **Equal (N1 = N2)**

  This selection is used when you wish to have equal sample sizes in each group. A single per group sample size will be entered.

- **Enter N1 and N2 individually**

  This choice permits you to enter different values for *N1* and *N2*.

- **Enter N1 and R, where N2 = R * N1**

  Choose this option to specify a value (or values) for *N1*, and obtain *N2* as a ratio (multiple) of *N1*.

- **Enter total sample size and percentage in Group 1**

  Choose this option to specify a value (or values) for the total sample size (*N*), obtain *N1* as a percentage of *N*, and then *N2* as *N - N1*.

Sample Size Per Group

This option is displayed only if Group Allocation = "Equal (N1 = N2)."

The Sample Size Per Group is the number of items or individuals sampled from each of the Group 1 and Group 2 populations. Since the sample sizes are the same in each group, this value is the value for *N1*, and also the value for *N2*.

The Sample Size Per Group must be ≥ 2. You can enter a single value or a series of values.

N1 (Sample Size, Group 1)

*This option is displayed if Group Allocation = "Enter N1 and N2 individually" or "Enter N1 and R, where N2 = R * N1."*

N1 is the number of items or individuals sampled from the Group 1 population.

N1 must be ≥ 2. You can enter a single value or a series of values.

N2 (Sample Size, Group 2)

This option is displayed only if Group Allocation = "Enter N1 and N2 individually."

N2 is the number of items or individuals sampled from the Group 2 population.

N2 must be ≥ 2. You can enter a single value or a series of values.

R (Group Sample Size Ratio)

*This option is displayed only if Group Allocation = "Enter N1 and R, where N2 = R * N1."*

R is the ratio of *N2* to *N1*. That is,

$$R = N2/N1$$

Use this value to obtain *N2* as a multiple (or proportion) of *N1*.

N2 is calculated from *N1* using the formula:

$$N2=[R \times N1],$$

where the value *[Y]* is the next integer ≥ *Y*.

For example, setting *R* = 2.0 results in a Group 2 sample size that is double the sample size in Group 1.

R must be greater than 0. If *R* < 1, then *N2* will be less than N1; if *R* > 1, then *N2* will be greater than *N1*. You can enter a single value or a series of values.

Total Sample Size (N)

This option is displayed only if Group Allocation = "Enter total sample size and percentage in Group 1."

This is the total sample size, or the sum of the two group sample sizes. This value, along with the percentage of the total sample size in Group 1, implicitly defines *N1* and *N2*.

The total sample size must be greater than one, but practically, must be greater than 3, since each group sample size needs to be at least 2.

You can enter a single value or a series of values.

Percent in Group 1

This option is displayed only if Group Allocation = "Enter total sample size and percentage in Group 1."

This value fixes the percentage of the total sample size allocated to Group 1. Small variations from the specified percentage may occur due to the discrete nature of sample sizes.

The Percent in Group 1 must be greater than 0 and less than 100. You can enter a single value or a series of values.

Precision

Confidence Interval Width (Two-Sided)

This is the distance from the lower confidence limit to the upper confidence limit. The distance from the variance ratio to the lower and upper limits is not equal.

You can enter a single value or a list of values. The value(s) must be greater than zero.

Distance from Ratio to Limit (One-Sided)

This is the distance from the variance ratio to the lower or upper limit of the confidence interval, depending on whether the Interval Type is set to Lower Limit or Upper Limit.

You can enter a single value or a list of values. The value(s) must be greater than zero.

Variances for Variance Ratio (V1/V2)

V1 (Variance Group 1)

Enter an estimate of the variance for group 1 (must be positive). The sample size and width calculations assume that the value entered here is the variance estimate that is obtained from the sample. If the sample variance is different from the one specified here, the width may be narrower or wider than specified.

For confidence intervals with widths that are specified in terms of a percentage of relative error, see the procedure 'Confidence Intervals for the Ratio of Two Variances using Relative Error'.

You can enter a range of values such as *1 2 3* or *1 to 10 by 1*.

V2 (Variance Group 2)

Enter an estimate of the variance for group 2 (must be positive). The sample size and width calculations assume that the value entered here is the variance estimate that is obtained from the sample. If the sample variance is different from the one specified here, the width may be narrower or wider than specified.

For confidence intervals with widths that are specified in terms of a percentage of relative error, see the procedure 'Confidence Intervals for the Ratio of Two Variances using Relative Error'.

You can enter a range of values such as *1 2 3* or *1 to 10 by 1*.

Example 1 – Calculating Sample Size

Suppose a study is planned in which the researcher wishes to construct a two-sided 95% confidence interval for the variance ratio such that the width of the interval is no wider than 0.5. The confidence level is set at 0.95, but 0.99 is included for comparative purposes. The variance estimates to be used are 5 for Group 1, and 10 for Group 2. Instead of examining only the interval width of 0.5, a series of widths from 0.2 to 0.8 will also be considered.

The goal is to determine the necessary sample size.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Confidence Intervals for the Ratio of Two Variances using Variances** procedure window by expanding **Variances**, then clicking on **Two Variances**, and then clicking on **Confidence Intervals for the Ratio of Two Variances using Variances**. You may then make the appropriate entries as listed below, or open **Example 1** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Sample Size**
Interval Type	**Two-Sided**
Confidence Level	**0.95 0.99**
Group Allocation	**Equal (N1 = N2)**
Confidence Interval Width (Two-Sided)	**0.2 to 0.8 by 0.1**
V1	**5**
V2	**10**

Annotated Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results for Two-Sided Confidence Intervals for the Variance Ratio

Confidence Level	N1	N2	N	Target Width	Actual Width	V1	V2	Lower Limit	Upper Limit
0.950	392	392	784	0.200	0.200	5.00	10.00	0.41	0.61
0.950	178	178	356	0.300	0.300	5.00	10.00	0.37	0.67
0.950	104	104	208	0.400	0.398	5.00	10.00	0.34	0.74
0.950	69	69	138	0.500	0.498	5.00	10.00	0.31	0.81
0.950	50	50	100	0.600	0.597	5.00	10.00	0.28	0.88
0.950	39	39	78	0.700	0.691	5.00	10.00	0.26	0.95
0.950	31	31	62	0.800	0.796	5.00	10.00	0.24	1.04
0.990	675	675	1350	0.200	0.200	5.00	10.00	0.41	0.61
0.990	307	307	614	0.300	0.300	5.00	10.00	0.37	0.67
0.990	178	178	356	0.400	0.399	5.00	10.00	0.34	0.74
0.990	118	118	236	0.500	0.498	5.00	10.00	0.31	0.81
0.990	85	85	170	0.600	0.598	5.00	10.00	0.28	0.88
0.990	65	65	130	0.700	0.699	5.00	10.00	0.26	0.96
0.990	53	53	106	0.800	0.791	5.00	10.00	0.24	1.03

References

Ostle, B. and Malone, L.C. 1988. Statistics in Research. Iowa State University Press. Ames, Iowa.
Zar, Jerrold H. 1984. Biostatistical Analysis (Second Edition). Prentice-Hall. Englewood Cliffs, New Jersey.

Report Definitions

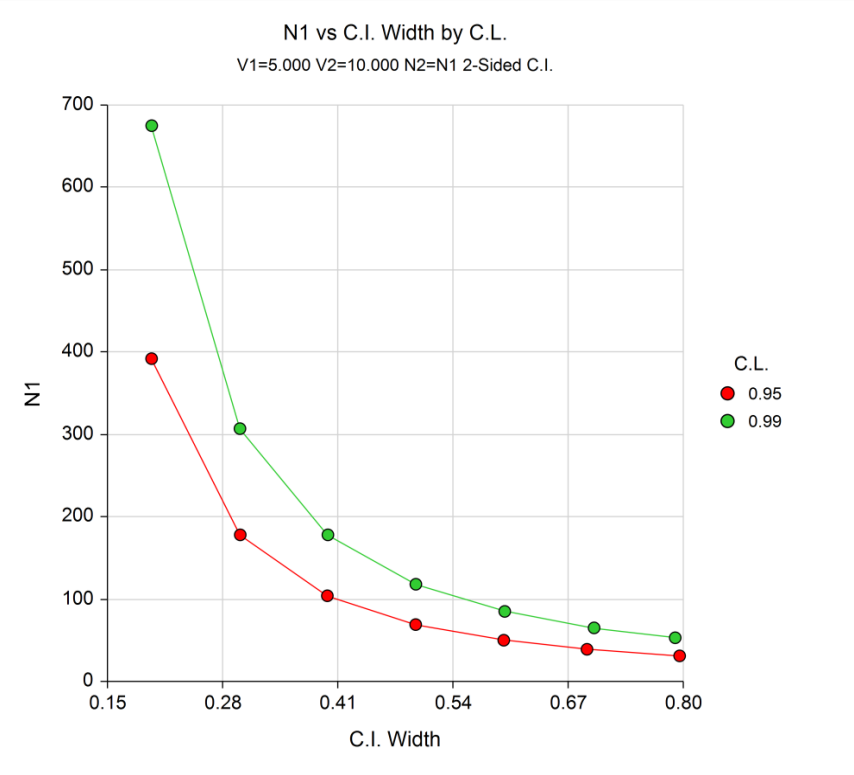
Confidence level is the proportion of confidence intervals (constructed with this same confidence level, sample size, etc.) that would contain the true ratio in population proportions.
N1 and N2 are the number of items sampled from each population.
N is the total sample size, N1 + N2.
Target Width is the value of the width that is entered into the procedure.
Actual Width is the value of the width that is obtained from the procedure.
V1 is the assumed sample variances for Group 1, where the ratio is V1 / V2.
V2 is the assumed sample variances for Group 2.
Lower Limit and Upper Limit are the lower and upper limits of the confidence interval for the true variance ratio (Population Variance 1 / Population Variance 2).

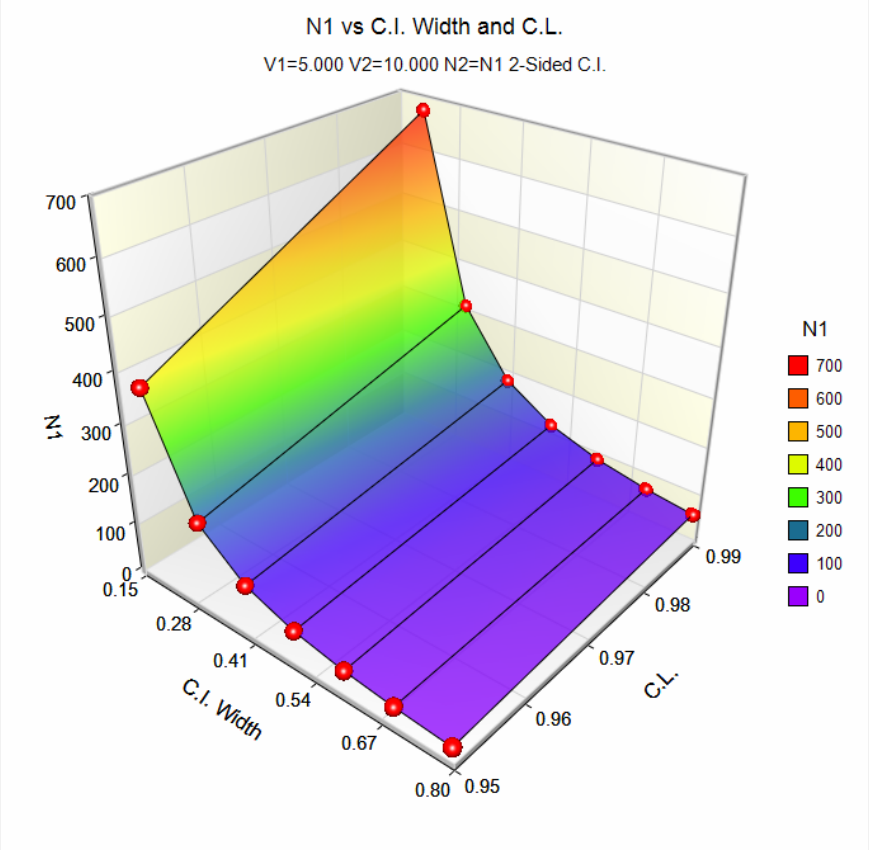
Summary Statements

Group sample sizes of 392 and 392 produce a two-sided 95% confidence interval with a width that is equal to 0.200 when the estimated numerator variance is 5.00 and the estimated denominator variance is 10.00.

This report shows the calculated sample size for each of the scenarios.

Plots Section





These plots show the group sample size versus the confidence interval width for the two confidence levels.

Example 2 – Validation using Sachs

Sachs (1984) page 261 gives an example of a calculation for a confidence interval for the variance ratio when the confidence level is 90%, the variances are 8 and 3, and the interval width is 4.56. The necessary sample size is 20 per group.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Confidence Intervals for the Ratio of Two Variances using Variances** procedure window by expanding **Variances**, then clicking on **Two Variances**, and then clicking on **Confidence Intervals for the Ratio of Two Variances using Variances**. You may then make the appropriate entries as listed below, or open **Example 2** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Sample Size**
Interval Type	**Two-Sided**
Confidence Level	**0.90**
Group Allocation	**Equal (N1 = N2)**
Confidence Interval Width (Two-Sided)	**4.56**
V1	**8**
V2	**3**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Confidence Level	N1	N2	N	Target Width	Actual Width	V1	V2	Lower Limit	Upper Limit
0.900	20	20	40	4.560	4.552	8.00	3.00	1.23	5.78

PASS also calculated the necessary sample size to be 20 per group.