

Chapter 295

Control Charts for Variability (Simulation)

Introduction

This procedure allows you to study the run length distribution of R , S , and S with probability limits process control charts using simulation. This procedure can also be used to study charts with two or more observations at each sample. The in-control standard deviation can be input directly or a specified number of in-control preliminary samples can be simulated based on a user-determined in-control distribution. This procedure can also be used to determine the necessary sample size to obtain a given run length.

Simulation Details

If the in-control standard deviation is assumed to be known (standard), the steps to the simulation process are as follows (assume a sample consists of n observations).

1. An out-of-control sample of size n is generated from a normal distribution with the specified out-of-control standard deviation.
2. The standard deviation of the sample is calculated and compared to the control limits.
3. If the sample results in an out-of-control signal, the sample number is recorded as the run length for that simulation. If the sample does not result in an out-of-control signal, return to Step 1.
4. Steps 1 through 3 are repeated until the number of simulations (N_{sim}) is reached. The result is N_{sim} run lengths.
5. The average or median or specified percentile of the run length distribution is reported.

If the in-control standard deviation is to be simulated based on in-control preliminary samples (N_{Prelim}), the steps to the simulation process are as follows (assume a sample consists of n observations).

1. N_{Prelim} in-control samples of size n are generated from a normal distribution with specified standard deviation of the in-control distribution.
2. The in-control standard deviation is calculated based on the N_{Prelim} simulated in-control samples.
3. An out-of-control sample of size n is generated from a normal distribution with the specified out-of-control standard deviation.
4. The standard deviation of the sample is calculated and compared to the control limits.
5. If the sample results in an out-of-control signal, the sample number is recorded as the run length for that simulation. If the sample does not result in an out-of-control signal, return to Step 3.
6. Steps 1 through 5 are repeated until the number of simulations (N_{sim}) is reached. The result is N_{sim} run lengths.
7. The average or median or specified percentile of the run length distribution is reported.

Formulas for Constructing Control Charts

The formula used for each set of control chart limits depends on whether the standard deviation is assumed to be a known, standard value, or estimated from in-control preliminary samples.

R Chart Limits

If the standard deviation is a known, standard value, the lower and upper control limits for the R chart are calculated (see Montgomery, 1991) using the formulae

$$LCL = d_2\sigma - z_{1-\alpha/2}d_3\sigma$$

$$UCL = d_2\sigma + z_{1-\alpha/2}d_3\sigma$$

where $d_2 = R/\sigma$ and $d_3 = \sigma_R/\sigma$.

If the limits are to be created from in-control preliminary samples, the lower and upper control limits for the R chart are calculated using the formulae

$$LCL = \bar{R} - z_{1-\alpha/2}d_3\frac{\bar{R}}{d_2}$$

$$UCL = \bar{R} + z_{1-\alpha/2}d_3\frac{\bar{R}}{d_2}$$

The statistic that is compared to the limits at each subgroup is the subgroup range

$$R_i = x_{(n)} - x_{(1)}$$

S Chart Limits

If the standard deviation is a known, standard value, the lower and upper control limits for the S chart are calculated (see Montgomery, 1991) using the formulae

$$LCL = c_4\sigma - z_{1-\alpha/2}\sigma\sqrt{1-c_4^2}$$

$$UCL = c_4\sigma + z_{1-\alpha/2}\sigma\sqrt{1-c_4^2}$$

where

$$c_4 = \sqrt{\frac{2}{n-1}} \frac{\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)}$$

If the limits are to be created from in-control preliminary samples, the lower and upper control limits for the S chart are calculated using the formulae

$$LCL = \bar{S} - z_{1-\alpha/2}\frac{\bar{S}}{c_4}\sqrt{1-c_4^2}$$

$$UCL = \bar{S} + z_{1-\alpha/2}\frac{\bar{S}}{c_4}\sqrt{1-c_4^2}$$

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The statistic that is compared to the limits at each subgroup is the subgroup standard deviation

$$s_i = \sqrt{\frac{\sum_{j=1}^n (x_{ij} - \bar{x}_i)^2}{n-1}}$$

S Chart with Probability Limits

If the standard deviation is a known, standard value, the lower and upper control limits for the S chart with probability limits are calculated (see Ryan, 1989) using the formulae

$$LCL = \sigma \sqrt{\frac{\chi_{\alpha/2}^2}{n-1}}$$

$$UCL = \sigma \sqrt{\frac{\chi_{1-\alpha/2}^2}{n-1}}$$

If the limits are to be created from in-control preliminary samples, the lower and upper control limits for the S chart with probability limits are calculated using the formulae

$$LCL = \frac{\bar{S}}{c_4} \sqrt{\frac{\chi_{\alpha/2}^2}{n-1}}$$

$$UCL = \frac{\bar{S}}{c_4} \sqrt{\frac{\chi_{1-\alpha/2}^2}{n-1}}$$

The statistic that is compared to the limits at each subgroup is the same as that of the previous S Chart, namely, the subgroup standard deviation

$$s_i = \sqrt{\frac{\sum_{j=1}^n (x_{ij} - \bar{x}_i)^2}{n-1}}$$

Procedure Options

This section describes the options that are specific to this procedure. These are located on the Design tab. For more information about the options of other tabs, go to the Procedure Window chapter.

Design Tab

Solve For

Solve For

Solve for either the *run length distribution* or the *sample size*. This is the parameter displayed on the vertical axis of the plot.

Note: The search for the sample size may take several minutes to complete. You may find it quicker and more informative to solve for run length distribution for a range of sample sizes.

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Simulations

Simulations

Specify the number of Monte Carlo iterations. Each simulation consists of the full process required to obtain a run length. That is, the number of simulations here is the number of run lengths that are used to form the run length distribution.

A 2007 article in *Communication in Statistics-Simulation and Computation* suggests that 5000 to 10000 simulations is usually sufficient. For estimating percentiles near 0 or 1, more simulations may be needed.

For searches for N, you may wish to use fewer simulations to determine the ballpark for N and then larger numbers of simulations for fine-tuning.

Maximum Run Length

For each simulation, if the run length reaches the maximum run length, the maximum run length is used for that simulation. This keeps simulations from going on indefinitely when run lengths are extremely long.

For example, if the average run length is close to the maximum run length, the maximum run length should be increased.

Run Length

Run Length Summary 1 and 2

Select the first and second run length distribution summary value to be reported.

If a value between 0 and 1 (not inclusive) is entered, the corresponding distribution percentile is reported.

For example, if 0.40 is entered, the run length such that 40% of simulated run lengths are shorter is reported.

Percentile or Average

When solving for N, the value entered for "Percentile or Average" indicates whether the search is based on the average run length or some percentile, such as the median.

If a value between 0 and 1 (not inclusive) is entered, the corresponding distribution percentile is used in the search for N.

For example, if ARL (Average Run Length) is entered, and "Run Length" is 10, the program will search for the smallest N for which the average run length is 10. If 0.40 is entered, and "Run Length" is 10, the program will search for the smallest N for which 40% of simulated run lengths are shorter than 10.

Run Length

When solving for N, this is run length that is searched for based on the specified "Percentile or Average".

For example, if ARL (Average Run Length) is entered, and "Run Length" is 10, the program will search for the smallest N for which the average run length is 10. If 0.40 is entered, and "Run Length" is 10, the program will search for the smallest N for which 40% of simulated run lengths are shorter than 10.

Sample Size

N (Sample Size)

Enter a value for the sample size, N. This is the number of observations in each sample. This sample size is also used when preliminary samples are used.

When a sample size of one is used, the preliminary sample standard deviation is estimated with all the preliminary data jointly.

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You may enter a range such as 2 to 10 by 2 or a list of values separated by commas or blanks.

You might try entering the same number two or three times to get an idea of the variability in your results. For example, you could enter "10 10 10".

Distributions, Tests Tab

In-Control and Out-of-Control Distributions

In-Control Distributions Specified By

Specify whether a standard known standard deviation will be used for the standard deviation for control limits, or if the standard deviation for control limits will be based on simulated preliminary samples.

In-Control and Out-of-Control Distributions - Known

In-Control Standard Deviation

This is an assumed known standard deviation from which the boundaries are calculated.

In-Control and Out-of-Control Distributions – Preliminary Samples

In-Control Distribution Standard Deviation

This is the standard deviation of the normal distribution that is used to simulate preliminary samples.

You can enter a single value or a list of values using the syntax *1 2 3* or *1 to 5 by 1*.

Number of Preliminary Samples

For each run length simulation, this is the number of samples that are simulated to determine the range or standard deviation for calculating the limits of the control chart.

For example, if a control chart uses 50 samples with $N = 5$ to establish the in-control standard deviation, 50 should be entered here. The program will generate 50 samples of size 5 each, obtain standard deviations (or ranges) for each sample and use those to produce a standard deviation (or range) estimate as a basis for the simulated control chart.

Commonly recommended numbers of in-control preliminary samples are 30, 50, and 60.

Varying this number allows one to see the effect on the distribution of run lengths.

In-Control and Out-of-Control Distributions – Out-of-Control

Out-of-Control Standard Deviation

This is the standard deviation of the out-of-control distribution. Out-of-control samples are simulated based on a normal distribution with this standard deviation.

You can enter a single value or a list of values using the syntax *1 2 3* or *1 to 5 by 1*.

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Tests

Include Tests

Specify whether to include this test in the Reports and Plots. The specific options for each test are specified below.

Tests – Z or Probability Options

Specify Probability Using

For R and S charts, specify whether the Z-multiplier will be input directly or calculated based on the probability.

The Z-multiplier is the value of Z in the chart formula for the limits:

$Rbar \pm Z * SD_R$ for two-sided R charts, and

$Sbar \pm Z * SD_S$ for two-sided S charts.

For S with Probability Limits, if a Z-multiplier is specified, the probability corresponding to the Z-multiplier is used to create the limits.

Z-Multiplier

The Z-multiplier is the value of Z in the chart formula for the limits:

$Rbar \pm Z * SD_R$ for two-sided R charts, and

$Sbar \pm Z * SD_S$ for two-sided S charts.

For S with Probability Limits, the probability corresponding to the Z-multiplier is used to create the limits.

You can enter a list of values using the syntax *2 3 4* or *2 to 5 by 0.5*.

Probability

For R and S charts, the probability is used to determine the Z-multiplier for the limits:

$Rbar \pm Z * SD_R$ for two-sided R charts, and

$Sbar \pm Z * SD_S$ for two-sided S charts.

For S with Probability Limits, the probability is used directly to create the limits using the chi-square distribution.

For two-sided tests, the probability is divided by two (for each side).

You can enter a list of values using the syntax *0.001 0.002 0.003* or *0.001 to 0.005 by 0.001*.

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Example 1 – Run Length Distribution

A researcher wishes to examine the run length distribution for a process monitored by an R chart. Ten observations are to make up the sample examined at each hour. The in-control standard deviation is known to be 3.8. The researcher would like see the run length distribution if the out-of-control standard deviation is 4.6. A probability of 0.00135 is to be used in the control chart for the boundaries.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Control Charts for Variability (Simulation)** procedure window by clicking on **Quality Control**, and then clicking on **Control Charts for Variability (Simulation)**. You may then make the appropriate entries as listed below, or open **Example 1** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
Design Tab	
Solve For	Run Length Distribution
Simulations	5000
Maximum Run Length.....	5000
Run Length Summary 1.....	ARL (Average Run Length)
Run Length Summary 2.....	MRL (Median Run Length)
N (Sample Size).....	10
Distributions, Tests Tab	
In-Control Distributions Specified By	Standard Deviation Directly
In-Control Standard Deviation	3.8
Out-of-Control Standard Deviation	4.6
R	Checked
All Other Tests	Unchecked
Two-Sided or Upper Limit Only	Upper Limit Only
Specify Probability Using.....	Probability
Probability	0.00135

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results for Control Charts for Process Variability
 Control limits were determined by specifying a standard in-control standard deviation directly.
 Out-of-Control Distribution: Normal with Out-of-Control SD

Chart Type	Average Run Length	Median Run Length	N	In-Control SD	One-Sided Prob	Z	LCL	UCL	Out-of-Control SD
R	22.0	15.0	10	3.8000	0.00135	3.205	0.0000	20.7821	4.6000

Notes

Simulations: 5000. Run Time: 4.0 seconds.

References

Ryan, T.P. 1989. Statistical Methods for Quality Improvement. Wiley. New York.
 Montgomery, D.C. 1991. Introduction to Statistical Quality Control. Wiley. New York.

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Report Definitions

Average Run Length is the mean of the run lengths across all simulations.
 Median Run Length is the median of the run lengths across all simulations.
 N is the number of units measured in each sample.
 In-control SD is the assumed known standard value of the standard deviation.
 One-Sided Prob is the probability of a sample range or standard deviation outside the control limits.
 Z is the Z-multiplier which corresponds to the One-sided probability of a single sample range or standard deviation outside the control limits.
 In-control SD is the assumed known standard deviation that is used in the calculation of limits.
 LCL and UCL are the lower and upper control chart limits, respectively.
 Out-of-Control SD is the standard deviation of the normal distribution from which out-of-control samples are drawn.

Summary Statements

For an R control chart with lower and upper control limits of 0.0000 and 20.7821, respectively, samples of size 10 from a normal distribution with standard deviation 4.6000 have an average run length of 22.0 and a median run length of 15.0. These results are based on 5000 Monte Carlo samples.

Individual Summaries

Control limits were determined by specifying a standard in-control standard deviation directly.
Out-of-Control Distribution: Normal with Out-of-Control SD

Chart Type	Average Run Length	Median Run Length	N	In-Control SD	One-Sided Prob	Z	LCL	UCL	Out-of-Control SD
R	22.0	15.0	10	3.8000	0.00135	3.205	0.0000	20.7821	4.6000

Average Run Length 95% CI: (21.4, 22.6)

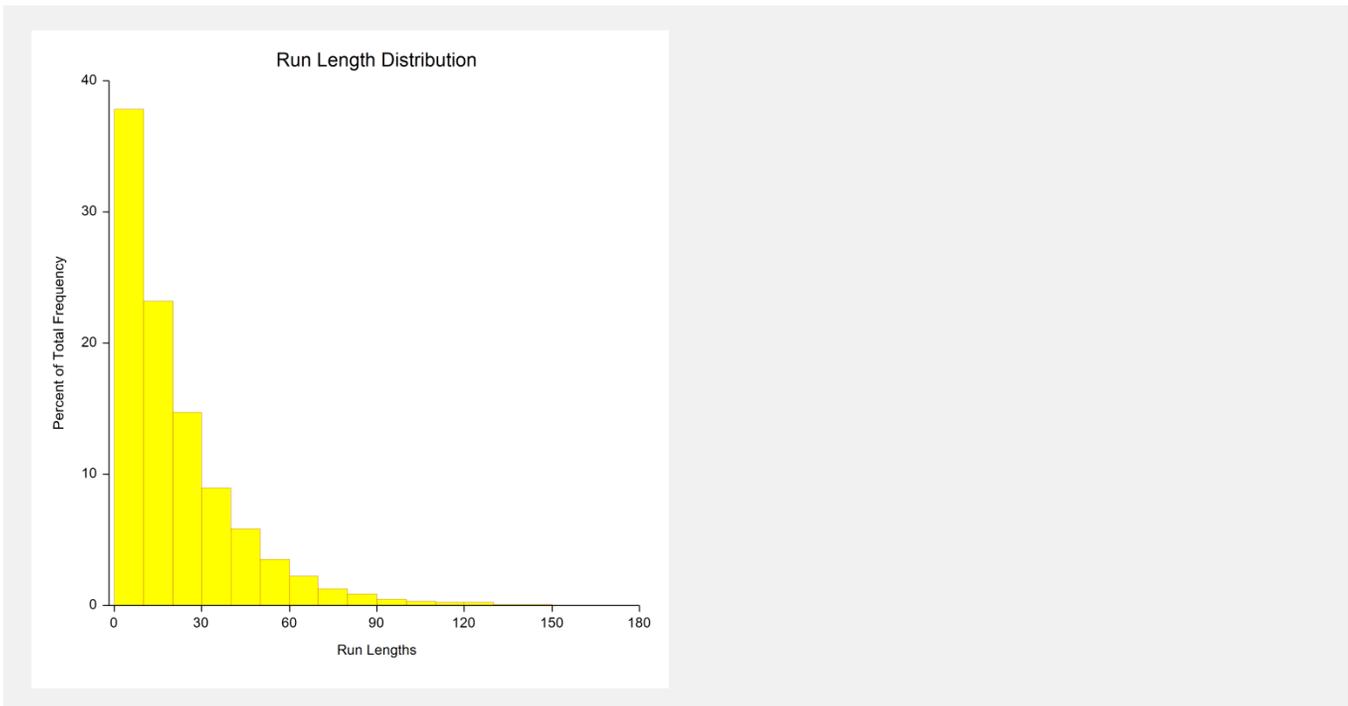
Median Run Length 95% CI: (15.0, 16.0)

Average Run Length and Percentiles

Avg	1%	5%	10%	25%	50%	75%	90%	95%	99%
22.0	1.0	2.0	3.0	7.0	15.0	30.0	50.0	66.0	103.0

The results will vary slightly because they are based on simulation.

Plots Section



This plot shows the distribution of run lengths of 5000 simulated runs.

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Example 2 – Comparing Tests

Continuing with the Example 1 parameters, the researchers would like to compare the various control chart tests available.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Control Charts for Means (Simulation)** procedure window by clicking on **Quality Control**, and then clicking on **Control Charts for Means (Simulation)**. You may then make the appropriate entries as listed below, or open **Example 2** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
Design Tab	
Solve For	Run Length Distribution
Simulations	5000
Maximum Run Length.....	5000
Run Length Summary 1.....	ARL (Average Run Length)
Run Length Summary 2.....	MRL (Median Run Length)
N (Sample Size).....	10
Distributions, Tests Tab	
In-Control Distributions Specified By	Standard Deviation Directly
In-Control Standard Deviation	3.8
Out-of-Control Standard Deviation	4.6
All Tests	Checked
Two-Sided or Upper Limit Only	Upper Limit Only
Specify Probability Using.....	Probability
Probability	0.00135

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results for Control Charts for Process Variability
 Control limits were determined by specifying a standard in-control standard deviation directly.
 Out-of-Control Distribution: Normal with Out-of-Control SD

Chart Type	Average Run Length	Median Run Length	N	In-Control SD	One-Sided Prob	Z	LCL	UCL	Out-of-Control SD
R	22.2	16.0	10	3.8000	0.00135	3.205	0.0000	20.7821	4.6000
S	21.5	15.0	10	3.8000	0.00135	3.205	0.0000	6.3436	4.6000
S Prob	33.7	24.0	10	3.8000	0.00135		0.0000	6.5931	4.6000

The R and S control chart tests show the process is out-of-control a bit sooner than the S test with probability limits.

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Example 3 – Validation Using Ryan

Ryan (1989), page 93, indicates that in-control processes with a sample size of 4 at each look will have a false signal on average after 222 samples for the S test, while the S with Probability Limits test will only give a false positive after 741 (1/0.00135) samples.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Control Charts for Means (Simulation)** procedure window by clicking on **Quality Control**, and then clicking on **Control Charts for Means (Simulation)**. You may then make the appropriate entries as listed below, or open **Example 3** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
Design Tab	
Solve For	Run Length Distribution
Simulations	10000
Maximum Run Length.....	5000
Run Length Summary 1.....	ARL (Average Run Length)
Run Length Summary 2.....	MRL (Median Run Length)
N (Sample Size).....	4
Distributions, Tests Tab	
In-Control Distributions Specified By	Standard Deviation Directly
In-Control Standard Deviation	3.8
Out-of-Control Standard Deviation	3.8
R	Unchecked
S.....	Checked
S with Probability Limits.....	Checked
Two-Sided or Upper Limit Only	Upper Limit Only
Specify Probability Using.....	Probability
Probability	0.00135

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results for Control Charts for Process Variability
Control limits were determined by specifying a standard in-control standard deviation directly.
Out-of-Control Distribution: Normal with Out-of-Control SD

Chart Type	Average Run Length	Median Run Length	N	In-Control SD	One-Sided Prob	Z	LCL	UCL	Out-of-Control SD
S	221.7	152.0	4	3.8000	0.00135	3.205	0.0000	7.9334	3.8000
S Prob	737.3	501.0	4	3.8000	0.00135		0.0000	8.6738	3.8000

The average run lengths are very close to those presented in Ryan (1989).