Chapter 389

GEE Tests for the TAD of Multiple Groups in a Repeated Measures Design (Binary Outcome)

Introduction

This module calculates the power for testing for differences among the time-averaged responses (TAD) of two or more groups from correlated binary data that are analyzed using the GEE method. Such data can occur in two design types: clustered and longitudinal.

GEE is different from mixed models in that it does not require the full specification of the joint distribution of the repeated measurements, as long as the marginal mean model is correctly specified. Estimation consistency is achieved even if the correlation matrix is incorrect. Also, the correlation matrix of the responses is specified directly, rather than using an intermediate, random effects model as is the case in MM. For clustered designs, GEE often uses a compound symmetric (CS) correlation structure. For longitudinal data, an autoregressive (AR(1)) correlation structure is often used.

Time-averaged response analysis is often used when the outcome to be measured varies with time. For example, suppose that you want to compare three treatment groups based on the means of a certain outcome such as whether the pressure is over 140. It is known that a person's blood pressure depends on several instantaneous factors such as amount of sleep, excitement level, mood, exercise, etc. If only a single measurement is taken from each patient then the comparison of group values may be insensitive because of the large degree of variation in blood pressure levels within a patient. The precision of the experiment is increased by taking multiple measurements from each individual and comparing the time-averaged responses among the groups. Care must be taken in the analysis because of the correlation that is introduced when several measurements are taken from the same individual. The correlation structure may take on several forms depending on the nature of the experiment and the subjects involved.

Missing Values

This procedure allows you to specify various patterns of incomplete (or missing) data. Subjects may miss some appointments, but attend others. This phenomenon of incomplete data can be accounted for in the sample size calculation which can greatly reduce the overall sample size from that calculated by just omitting subjects with incomplete observations.
Technical Details

Theory and Notation

Technical details are given in Wang, Zhang, and Ahn (2018).

Suppose we have \( n_k \) \((k = 1, \ldots, G)\) subjects in each of \( G \) groups for a total of \( N \) subjects, each measured on \( M \) occasions at times \( t_j \) \((j = 1, \ldots, M)\). For convenience, we normalize these time points to the proportion of total time so that \( t_1 = 0 \) and \( t_M = 1 \).

Let \( y_{ki} \) be the binary response of subject \( i \) in group \( k \) at time \( t_j \). The response is modeled by the logistic model

\[ Y_{ki} \sim \text{Bernoulli}(P_k) \]

The mean of \( Y_{ki} \) is modeled by

\[ \logit(P_k) = \log\left( \frac{P_k}{1 - P_k} \right) = \beta_k \]

This implies that

\[ E(Y_{ki}) = P_k = \frac{e^{\beta_k}}{1 + e^{\beta_k}} \]

The GEE estimator of \( \beta_k \) is \( b_k \), given by

\[ b_k = \log\left( \frac{\sum_{i=1}^{n_k} \sum_{j=1}^{M} Y_{ki}/(n_k M)}{1 - \sum_{i=1}^{n_k} \sum_{j=1}^{M} Y_{ki}/(n_k M)} \right) \]

In this procedure, the primary interest is to test that the contrast based on the coefficients \( C = c_1, \ldots, c_G \) is zero, that is, that \( H_0: \sum_{k=1}^{G} \beta_k c_k = 0 \) against the alternative that it is non-zero.

GEE is used to estimate the \( \beta_i \)'s and test this hypothesis. The test statistic is

\[ Z = \frac{C'b}{\sqrt{\text{Var}(C'b)}} \]

\( H_0 \) is rejected with a type I error \( \alpha \) if \( |Z| > z_{1-\alpha/2} \) where \( z_{1-\alpha/2} \) is the \( 100(1 - \alpha/2) \)th percentile of a standard normal distribution.
Correlation Patterns

In a longitudinal design with $N$ subjects, each measured $m$ times, observations from a single subject are correlated, and a pattern of those correlations through time needs to be specified. Several choices are available.

**Compound Symmetry**
A compound symmetry covariance model assumes that all covariances are equal, and all variances on the diagonal are equal. That is

$$
[p_{jj'}] = \begin{bmatrix}
1 & \rho & \rho & \rho & \cdots & \rho \\
\rho & 1 & \rho & \rho & \cdots & \rho \\
\rho & \rho & 1 & \rho & \cdots & \rho \\
\rho & \rho & \rho & 1 & \cdots & \rho \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\rho & \rho & \rho & \rho & \cdots & 1
\end{bmatrix}_{M \times M}
$$

where $\rho$ is the baseline correlation.

**Banded(1)**
A Banded(1) (banded order 1) correlation model assumes that all variances on the diagonal are equal, covariances for observations one time period apart are equal to $\rho$, and covariances for measurements greater than one time period apart are equal to zero. That is

$$
[p_{jj'}] = \begin{bmatrix}
1 & \rho & 0 & 0 & \cdots & 0 \\
\rho & 1 & \rho & 0 & \cdots & 0 \\
0 & \rho & 1 & \rho & \cdots & 0 \\
0 & 0 & \rho & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & 0 & \cdots & 1
\end{bmatrix}_{M \times M}
$$

where $\rho$ is the baseline correlation.

**Banded(2)**
A Banded(2) (banded order 2) correlation model assumes that all variances on the diagonal are equal, covariances for observations one time period or two periods apart are equal to $\rho$, and covariances for measurements greater than one time period apart are equal to zero. That is

$$
[p_{jj'}] = \begin{bmatrix}
1 & \rho & \rho & 0 & \cdots & 0 \\
\rho & 1 & \rho & \rho & \cdots & 0 \\
\rho & \rho & 1 & \rho & \cdots & 0 \\
0 & \rho & \rho & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & 0 & \cdots & 1
\end{bmatrix}_{M \times M}
$$

where $\rho$ is the baseline correlation.
AR1 (Traditional)
This version of AR1 (autoregressive order 1) correlation model assumes that all variances on the diagonal are equal and that correlations \( t \) time periods apart are equal to \( \rho^t \). That is

\[
\begin{bmatrix}
1 & \rho & \rho^2 & \rho^3 & \cdots & \rho^{M-1} \\
\rho & 1 & \rho & \rho^2 & \cdots & \rho^{M-2} \\
\rho^2 & \rho & 1 & \rho & \cdots & \rho^{M-3} \\
\rho^3 & \rho^2 & \rho & 1 & \cdots & \rho^{M-4} \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\rho^{M-1} & \rho^{M-2} & \rho^{M-3} & \rho^{M-4} & \cdots & 1
\end{bmatrix}_{M \times M}
\]

where \( \rho \) is the baseline correlation.

AR1 (Proportional)
This version of AR1 (autoregressive order 1) correlation model is described in the book by Ahn et al. (2015). It assumes that all variances on the diagonal are equal and that correlations \( |t_j - t_k| \) time periods apart are equal to \( \rho^{|t_j - t_k|} \). That is

\[
\begin{bmatrix}
\rho_{jk} \end{bmatrix} = \begin{bmatrix}
\rho^{|t_j - t_k|}
\end{bmatrix}_{M \times M}
\]

where \( \rho \) is the baseline correlation. Note that in this pattern, the value of \( \rho \) is shown in the final column since in this case \( t_j = 0 \) and \( t_k = 1 \), so \( |t_j - t_k| = 1 \).

Dampened Exponential
A dampened exponential is an extension of the AR(1) correlation model in which the exponents are raised to the power \( Dexp (\theta = Dexp \) in the diagram below). This causes the resulting correlations to be reduced (dampened). Here is an example

\[
\begin{bmatrix}
1 & \rho & \rho^{2\theta} & \rho^{3\theta} & \cdots & \rho^{(M-1)\theta} \\
\rho & 1 & \rho & \rho^{2\theta} & \cdots & \rho^{(M-2)\theta} \\
\rho^{2\theta} & \rho & 1 & \rho & \cdots & \rho^{(M-3)\theta} \\
\rho^{3\theta} & \rho^{2\theta} & \rho & 1 & \cdots & \rho^{(M-4)\theta} \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\rho^{(M-1)\theta} & \rho^{(M-2)\theta} & \rho^{(M-3)\theta} & \rho^{(M-4)\theta} & \cdots & 1
\end{bmatrix}_{M \times M}
\]

where \( \rho \) is the baseline correlation.

Dampened Exponential (Proportional)
This version of the dampened exponential correlation model is described in the book by Ahn et al. (2015). It assumes that all variances on the diagonal are equal and that correlations \( |t_j - t_k| \) time periods apart are equal to \( \rho^{|t_j - t_k|} \). That is

\[
\begin{bmatrix}
\rho_{jk} \end{bmatrix} = \begin{bmatrix}
\rho^{|t_j - t_k|}\theta
\end{bmatrix}_{M \times M}
\]

where \( \rho \) is the baseline correlation. Note that in this pattern, the value of \( \rho^{|t_j - t_k|}\theta \) turns up in the final column since in this case \( t_j = 0 \) and \( t_k = 1 \), so \( |t_j - t_k| = 1 \).
Linear Exponential Decay

A linear exponential decay correlation model is one in which the exponent of the correlation decays according to a linear equation from 1 at the *Base Time Proportion* to a final value, $E_{max}$. The resulting pattern looks similar to the dampened exponential. Note that the exponents are applied to the absolute difference between the Measurement Time Proportions. This method allows you to easily construct comparable correlation matrices of different dimensions. Otherwise, differences in the resulting power would be more strongly due to differences in the correlation matrices.

Here is an example. Suppose $M$ is 6, $\rho = 0.5$, $E_{max} = 3$, the *Base Time Proportion* is 0.20, and the Measurement Time Proportions are (0, 0.2, 0.4, 0.6, 0.8, 1). The following correlation matrix would be obtained

$$
\begin{bmatrix}
1 & 0.5 & 0.3536 & 0.25 & 0.1768 & 0.125 \\
0.5 & 1 & 0.5 & 0.3536 & 0.25 & 0.1768 \\
0.3536 & 0.5 & 1 & 0.5 & 0.3536 & 0.25 \\
0.25 & 0.3536 & 0.5 & 1 & 0.5 & 0.3536 \\
0.1768 & 0.25 & 0.3536 & 0.5 & 1 & 0.5 \\
0.125 & 0.1768 & 0.25 & 0.3536 & 0.5 & 1 \\
\end{bmatrix}_{M \times M}
$$

Note that in the top row, the correlation is 0.5 for the second (0.2 - 0) time point and 0.125 ($0.5^{3}$) at the last (1 - 0) time points. The correlations are obtained by raising 0.5 to the appropriate exponent. The linear equation from 1 to 3 results in the exponents 1, 1.5, 2, 2.5, 3 correspondent to the time proportions 0, 0.2, 0.4, 0.6, 0.8, and 1.

As a further example, note that the correlation for the 0.4 time point is, $0.5^{1.5} = 0.35355339 \approx 0.3536$.

This method allows you to compare various values of $M$ while keeping the correlation matrix similar. To see what we mean, consider what the correlation matrix looks like when $M$ is reduced to 4 and the measurement time proportions are set to (0, 0.2, 0.6, 1). It becomes

$$
\begin{bmatrix}
1 & 0.5 & 0.25 & 0.125 \\
0.5 & 1 & 0.5 & 0.25 \\
0.25 & 0.5 & 1 & 0.5 \\
0.125 & 0.25 & 0.5 & 1 \\
\end{bmatrix}_{M \times M}
$$

Note that the correlation at a measurement time difference of 0.6 is equal to 0.25 in both matrices.

Missing Data Patterns

The problem of missing data occurs for several reasons. In longitudinal studies in which a subject is measured multiple times, missing data becomes more complicated to model because it is possible that a subject is measured only some of the time. In fact, it is probably more common for data to be incomplete than complete. The approach of omitting subjects with incomplete data during the planning phase is very inaccurate. Certainly, subjects with partial measurements are included in the analysis. This procedure provides several missing data patterns to choose from so that your sample size calculations are more realistic.

In the presentation to following, we denote the percent of subjects with a missing response at time point $t_j$ as $\kappa_j$. The proportion non-missing at a particular time point is $\phi_j = 1 - \kappa_j$. We will refer to $\phi_j$ as the *marginal observant probability* at time $t_j$ and $\phi_{jj'}$ as a *joint observant probability* at times $t_j$ and $t_j'$.

Pairwise Missing Pattern

The program provides three options for how the pairwise (joint) observant probabilities $\phi_{jj'}$ are calculated. These are

**Independent (Ind):** $\phi_{jj'} = \phi_j \phi_{j'}$, $\phi_{jj} = \phi_j$

**Monotonic (Mon):** $\phi_{jj'} = \phi_k$ where $k = \max(j, j')$

**Mixture:** $\phi_{jj'} = W(\text{Ind}) + (1 - W)(\text{Mon})$ for weighting factor $W$.  

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Missing Input Type

The are several ways in which the missing value pattern can be specified. Each missing value pattern is a list of missing proportions at each of the \( M \) time points. Each value in the list must be non-negative and less than 1. Possible input choices are

- **Constant = 0**
  All missing proportions are set to 0. That is, there are no missing values.

- **Constant**
  All missing proportions are set to constant value.

- **Piecewise Constant on Spreadsheet**
  A set of missing proportions are defined for several time intervals using the spreadsheet. One column contains the missing proportions for the interval, going down the rows. Another column defines the corresponding upper limit of time proportion of the interval. The lower limit is implied by the limit given immediately above. The program assumes that the first time interval starts at 0 percent.

- **Linear (Steady Change)**
  The missing proportions fall along a straight-line between 0 and 1 elapsed time. Only the first and last proportions are entered.

- **Piecewise Linear on Spreadsheet**
  The missing proportions fall along a set of connected straight-lines that are defined by two columns on the spreadsheet.

- **List**
  Enter a list of \( M \) missing proportions, one for each time point.

- **Multiple Lists on Spreadsheet**
  Select multiple columns containing vertical lists of missing proportions. Each column contains a set of missing proportions in rows, one for each time point.

- **Pairwise Observed Proportions on Spreadsheet**
  Enter an \( M \times M \) matrix of observant probabilities by selecting \( M \) columns. These observant probabilities are the proportion of the responses for both the row and column time points that are observed.

Sample Size Calculations

The details of the calculation of sample size and power is given in Wang, Zhang, and Ahn (2018). The formula for the sample size is

\[
N = \frac{C'VC \left( z_{1-\alpha} + z_{1-\gamma} \right)^2}{(C'\beta)^2}
\]

where

- \( \gamma \) 1 – power
- \( \alpha \) significance level
- \( z_{1-\alpha/2} \) is the 100(1 – \( \alpha/2 \))th percentile of a standard normal distribution.
The above formula is easily rearranged to obtain a formula for power.

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**Procedure Options**

This section describes the options that are specific to this procedure. These are located on the Design tab. For more information about the options of other tabs, go to the Procedure Window chapter.

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**Design Tab**

The Design tab contains most of the parameters and options that you will be concerned with.

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**Solve For**

This option specifies the parameter to be solved for. When you choose to solve for Sample Size, the program searches for the lowest sample size that meets the alpha and power criterion you have specified. The Solve For parameter is displayed on the vertical axis of the plot.

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**Power and Alpha**

**Power**

This option specifies one or more values for power. Power is the probability of rejecting a false null hypothesis, and is equal to one minus Beta. Beta is the probability of a type-II error, which occurs when a false null hypothesis is not rejected. In this procedure, a type-II error occurs when you fail to reject the null hypothesis of equal slopes when in fact the slopes are different.

Values must be between zero and one. Historically, the value of 0.80 (beta = 0.20) was used for power. Now, 0.90 (beta = 0.10) is also commonly used.

A single value may be entered here or a range of values such as 0.8 to 0.95 by 0.05 may be entered.

**Alpha**

This option specifies one or more values for the probability of a type-I error. A type-I error occurs when a true null hypothesis is rejected. In this procedure, a type-I error occurs when you reject the null hypothesis of equal slopes when in fact the slopes are equal.

Values must be between zero and one. Historically, the value of 0.05 has been used for alpha. This means that about one test in twenty will falsely reject the null hypothesis. You should pick a value for alpha that represents the risk of a type-I error you are willing to take in your experimental situation.

You may enter a range of values such as 0.01 0.05 0.10 or 0.01 to 0.10 by 0.01.
Sample Size and Group Allocation

G (Number of Groups)
This is the number of groups (arms) whose means are being compared. The number of items used in the Group Allocation boxes is controlled by this number.

Range
G ≥ 2.

Group Allocation Input Type (when Solve For = Power)
Specify how you want to enter the information about how the subjects are allocated to each of the G groups.
Possible options are:

- **Equal (n1 = n2 = ... = nG = n)**
  The sample size of every group is n. All group sample sizes are equal. Enter one or more values for n.

- **Enter group multipliers and n**
  Enter a list of group multipliers (r1, r2, ..., rG) and one or more values of n. The individual group sample sizes are found by multiplying the multipliers by n. For example, n1 = r1 x n.

- **Enter n1, n2, ..., nG**
  Enter a list of group sample sizes, one for each group.

- **Enter columns of ni's**
  Select one or more columns of the spreadsheet that each contain a set of group sample sizes going down the column. Each column is analyzed separately.

n (Sample Size Per Group)
Enter n, the number of subjects in each group. The total number of subjects, N, is equal to n x (number of groups).
You can specify a single value or a list.

Single Value
Enter a value for the sample size of all groups. If you enter '10' here and there are five groups, then each group will be assigned 10 subjects and the total sample size will be 50.

List of Values
A separate power analysis is calculated for each value of n in the list. All analyses assume that the common, group sample size is n.

Range of n
n > 1

Group Multipliers (r1, r2, ..., rG)
Enter a set of G multipliers, one for each group.
The individual group sample sizes is computed as ni = ceiling[r1 x n], where ceiling[y] is the first integer greater than or equal to y. For example, the multipliers {1, 1, 2, 2.95} and base n of 10 would result in the sample sizes {10, 10, 20, 30}.

Incomplete List
If the number of items in the list is less than G, the missing multipliers are set equal to the last entry in the list.
Range
The items in the list must be positive. The resulting sample sizes must be at least 1.

\( n \) (Base Sample Size Per Group)
Enter \( n \), the base number of subjects in each group. The number of subjects in the group is found by multiplying this number by the corresponding group multiplier, \( \{r_1, r_2, \ldots, r_G\} \), and rounding up to the next integer.

You can specify a single value or a list.

Single Value
Enter a value for the base group sample size.

List of Values
A separate power analysis is calculated for each value of \( n \) in the list.

Range
Ceiling\( [n \times r_i] \geq 1 \).

\( n_1, n_2, \ldots, n_G \) (List)
Enter a list of \( G \) sample sizes, one for each group.

Incomplete List
If the number of items in the list is less than \( G \), the missing sample sizes are set equal to the last entry in the list.

Range
The items in the list must be positive. At least one item in the list must be greater than 1.

Columns of \( n_i \)’s
Enter one or more spreadsheet columns containing vertical lists of group sample sizes.

Press the Spreadsheet icon (directly to the right) to select the columns and then enter the values.

Press the Input Spreadsheet icon (to the right and slightly up) to view/edit the spreadsheet. Also note that you can obtain the spreadsheet by selecting "Tools", then "Input Spreadsheet", from the menus.

On the spreadsheet, the group sample sizes are entered going down.

Examples (assuming \( G = 3 \))

\[
\begin{array}{ccc}
C1 & C2 & C3 \\
111 & 1 & 28 \\
115 & 20 & 68 \\
100 & 30 & 46 \\
\end{array}
\]

Definition of a Single Column
Each column gives one list. Each column results in a new scenario. The columns are not connected, but all should have exactly \( G \) rows.

Each entry in the list is the sample size of that group.

Incomplete List
If the number of items in the list is less than \( G \), the missing entries are set equal to the last entry in the list.
Valid Entries
All values should be positive integers. At least one value must be greater than one.

Note
The column names (C1, C2, ...) can be changed by clicking on them in the spreadsheet.

Group Allocation Input Type (when Solve For = Sample Size)
Specify how you want to enter the information about how the subjects are allocated to each of the G groups.

Options

- **Equal (n1 = n2 = ... = nG = n)**
  
  All group sample sizes are equal to n. The value of n will be found by conducting a search.

- **Enter group allocation pattern**
  
  Enter an allocation pattern (r1, r2, ..., rG). The pattern consists of a set of G numbers. These numbers will be rescaled into proportions by dividing each item by the sum of all items. The individual group sample sizes are found by multiplying these proportions by N (the total sample size) and rounding up.

- **Enter columns of allocation patterns**
  
  Select one or more columns of the spreadsheet that each contain a group allocation pattern going down the column. Each column is analyzed separately.

Group Allocation Pattern (r1, r2, …, rG)

Enter an allocation pattern (r1, r2, ..., rG). The pattern consists of a set of G numbers. These numbers will be rescaled into proportions of total sample size by dividing each item by the sum of all items. The individual group sample sizes are found by multiplying these proportions by N (the total sample size) and rounding up.

For example, the pattern {1, 3, 4} will be rescaled to {0.125, 0.375, 0.5}. The group sample sizes will be constrained to these proportions (within rounding) during the search for the sample configuration that just meets the power requirement.

Incomplete List
If the number of items in the list is less than G, the missing numbers are set equal to the last entry in the list.

Range
The items in the list must be positive. The resulting sample sizes must be at least 1.

Columns of Group Allocation Patterns

Enter one or more spreadsheet columns containing vertical lists of group allocation patterns.

Press the Spreadsheet icon (directly to the right) to select the columns and then enter the values.

Press the Input Spreadsheet icon (to the right and slightly up) to view/edit the spreadsheet. Also note that you can obtain the spreadsheet by selecting "Tools", then "Input Spreadsheet", from the menus.

On the spreadsheet, the group allocation patterns are entered going down.

Examples (assuming G = 3)

<table>
<thead>
<tr>
<th>C1</th>
<th>C2</th>
<th>C3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>
Definition of a Single Column
Each column gives one allocation pattern. Each column results in a new scenario. The columns are not connected, but all should have exactly G rows.

Incomplete List
If the number of items in a list is less than G, the missing numbers are set equal to the last entry in the list before they are rescaled.

Valid Entries
All values should be positive numbers. You can enter decimal values.

Note
The column names (C1, C2, ...) can be changed by clicking on them in the spreadsheet

Measurement Times Schedule

Measurement Time Input Type
Specify how you want to enter the information about the measurement proportion times.

- Equally Spaced Measurement Times
  Enter one or more values for M, the number of measurements per subject. Equally-spaced measurement time proportions will automatically be generated.

- List of Measurement Time Proportions
  Enter a list of measurement time proportions in the box below, one for each time point. The number of measurement times, M, is set equal to the number of items in this list.

- Columns of Measurement Time Proportions
  Select one or more columns of the spreadsheet that contain a set of measurement time proportions. Each column is analyzed separately. The number of measurement times, M, is equal to the number of rows in the active column.

M (Measurements)
Specify one or more values for M, the total number of measurements scheduled for each subject. It is assumed that these measurements are made at identical time points or under the same conditions.

At least one value must be entered. If multiple values are entered, a separate analysis is run for each value.

The value must be an integer greater than 1. A reasonable range is from 2 to 50.

Some examples are
3
3 4 5
3 to 10 by 1

Measurement Time Proportions
Enter a set of measurement time proportions, one for each time point. The number of measurements, M, is the number of items in the list. Each entry in the list is the proportion that that time is of the maximum time.

For example, suppose a 24-month study is planned with anticipated measurements at the beginning and then after each six-month period. Hence, the five measurement times are 0, 6, 12, 18, and 24 months. The entry must be proportions of 24, so the actual entry would be ‘0, 0.25, 0.50, 0.75, 1.0’.
M, the number of repeated measurements, is set to the number of items in the list.

Once M is set here, it must match its implicit value in the Correlations and Missing Proportions.

**Auto Rescaling**

You can enter any increasing series that is convenient and it will be rescaled so that each entry is a proportion of the maximum. In the example above, you could have entered ‘0, 6, 12, 18, 24’ or even ‘1, 2, 3, 4, 5’ and the program would have rescaled the values into proportions beginning at 0 and ending at 1.

**Valid Entries**

Since the numbers are automatically rescaled, the only requirement is that the series be strictly increasing.

**Examples**

0 0.25 0.50 0.75 1.0
0 10 50 90 100
0 0.20 0.40 0.90 0.95 1.0
1 2 3 4 5
0 10 20 40

**Column(s) of Time Proportions**

Enter one or more spreadsheet columns containing lists of measurement time proportions.

Press the Spreadsheet icon (directly to the right) to select the columns and then enter the values. Press the Input Spreadsheet icon (to the right and slightly up) to view/edit the spreadsheet. Also note that you can obtain the spreadsheet by selecting Tools, then Input Spreadsheet, from the menus. On the spreadsheet, the measurement times are entered going down.

**Examples**

C1  C2   C3
0.0  1    0.1
0.2  2    0.4
0.4  3    0.5
0.6  4
0.8
1.0

**Definition of a Single Column**

Each column gives one list of time proportions. Each column results in a new scenario. Different columns are not connected and may be of different lengths.

Each entry in the list is the proportion that that time is of the maximum time. For example, suppose a 24-month study is planned with anticipated measurements at the beginning and then after each six-month period. Hence, the five measurement times are 0, 6, 12, 18, and 24 months. The entry must be proportions of 24, so the actual entry would be ‘0, 0.25, 0.50, 0.75, 1.0’ (entered vertically in a particular column).

**Auto Rescaling**

You can enter any increasing series that is convenient and it will be rescaled so that each entry is a proportion of the maximum. In the example above, you could have entered ‘0 6 12 18 24’, or even ‘1 2 3 4 5’, going down the column and the program would have rescaled the values into proportions beginning at 0 and ending at 1.

**Valid Entries**

Since the numbers are automatically rescaled, the only requirement is that the series be strictly increasing.
Notes

M, the number of repeated measurements, is set to the number of items in the active column.

Once M is set here, it must match its implicit value in the Correlations and Missing Proportions.

The column names (C1, C2, ...) can be changed by clicking on them in the spreadsheet.

Effect Size - P1, P2, ..., PG (Mean Time-Averaged Responses) and Contrast

Pi's Input Type
Specify how you want to enter P1, P2, ..., PG, the time-averaged response probabilities for each of the G groups.

Options

- **P1, P2, ..., PG**
  Specify the values of the time-averaged response probabilities for each group. The differences among these values represent the effect size that you want to detect.

- **Columns Containing Sets of Pi's**
  Select one or more columns of the spreadsheet that each contain a set of Pi's going down the column. Each column is analyzed separately.

P1, P2, ..., PG
Enter the values of the time-averaged response probabilities, one for each of the G groups. These entries provide values assumed by the alternative hypothesis. The null hypothesis assumes that all of the values are equal. The effect size detected by the study is a function of the differences among these values.

A time-averaged response probability for a particular individual is the average of all of the repeatedly-measured responses of that individual. This value is the anticipated mean response probability of all subjects in a particular group.

The actual analysis uses the logit of each probability where logit(P) = log(P/(1-P)).

Range
Each Pi can be any number between 0 and 1. Also, at least one of the values must be different from the rest.

Example
0.2 0.1 0.1 0.1

Incomplete List
If the number of items in a list is less than G, the missing numbers are set equal to the last entry.

Columns Containing Sets of Pi's
Enter one or more spreadsheet columns containing vertical lists of P1, P2, ..., PG.
Each Pi can be any number between 0 and 1. Also, at least one of the values must be different from the rest.
Press the Spreadsheet icon (directly to the right) to select the columns and then enter the values.
Press the Input Spreadsheet icon (to the right and slightly up) to view/edit the spreadsheet. Also note that you can obtain the spreadsheet by selecting "Tools", then "Input Spreadsheet", from the menus.
On the spreadsheet, the Pi's are entered going down.
Examples (assuming G = 3)

<table>
<thead>
<tr>
<th></th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>0.3</td>
<td>0.4</td>
<td></td>
</tr>
</tbody>
</table>

**Definition of a Single Column**

Each column gives one set of probabilities. Each column results in a new scenario. The columns are not connected, but all should have exactly G rows.

**Incomplete List**

If the number of items in a column is less than G, the missing numbers are set equal to the last entry in the column.

**Valid Entries**

Numbers between 0 and 1.

**Note**

The column names (C1, C2, ...) can be changed by clicking on them in the spreadsheet.

**Contrast Input Type**

Select how you want to enter the contrast coefficients. Possible options are as follows.

**Contrast**

A contrast is a weighted average of the logits of the group response probabilities in which the weights sum to zero. For example, suppose you are studying four groups and that the main hypothesis of interest is whether there is a linear trend across the groups. You might use "-3, -1, 1, 3" here. This would form the weighted average of the logits:

\[-3 \text{logit}(P_1) - \text{logit}(P_2) + \text{logit}(P_3) + 3 \text{logit}(P_4)\]

These weights (the coefficients) are used to calculate a specific weighted average of the logits which is to be tested against zero using a standard z (or t) test. Hence, G coefficients (weights) must be defined. These coefficients must sum to zero. Also, the scale of the coefficients does not matter. That is the powers of "0.5 0.25 0.25", "-2 1 1", and "-200 100 100" are the same.

**Logit**

The logit of P is defined as \(\text{Log}(P/(1-P))\).

**Options**

- **List of Contrast Coefficients**
  Enter a list of coefficients, separated by commas or blanks.

- **Multiple Lists of Contrast Coefficients**
  Enter several contrasts, one per column, on the spreadsheet. Each contrast is entered down a column, one coefficient per row.

- **First Group vs Rest**
  A contrast is generated appropriate for testing that the logit of the first group is different from the average of the remaining groups. For example, if there were four groups, the generated coefficients would be "-3, 1, 1, 1".
• **Last Group vs Rest**
  A contrast is generated appropriate for testing that the logit of the last group is different from the average logit of the remaining groups. For example, if there were four groups, the generated coefficients would be "1, 1, 1, -3".

• **Linear Trend**
  A set of coefficients is generated appropriate for testing the alternative hypothesis that there is a linear (straight-line) trend across the group logits. These coefficients assume that the groups are equally spaced across some unspecified, quantitative variable associated with the groups.

• **Maximum Power (if ni's equal)**
  A set of coefficients is generated which will result in the maximum possible power. This contrast is based on a knowledge of the actual population logits, so in practice it cannot be attained. However, it lets you determine how close your power is compared with the maximum possible.

**(List of) Contrast Coefficients**
Specify how you want to enter the contrast coefficients.

Enter a list of numbers, separated by blanks or commas, one number for each group. These numbers must sum to zero. The attained power is highly related to the specific contrast that is entered.

**Definition**
A contrast is a weighted average of the logits of the G group response probabilities in which the weights (coefficients) sum to zero. Each successive coefficient is applied to the corresponding group response probability. For example, suppose G = 3 and the first group is a control group. A contrast that might be of interest is "2 -1 -1". This contrast is interpreted as "2 Logit(P1) - Logit(P2) - Logit(P3)". The coefficients must sum to zero.

**Incomplete List**
If the list is not long enough, the list is padded with zeros.

**Examples**
-1 -1 2
-2 -1 0 1 2
-3 -1 1 3
-3 1 1 1

**Multiple Lists of (Contrast) Coefficients**
Select a set of columns on the spreadsheet which each contain a contrast defined vertically.
Press the Spreadsheet icon (directly to the right) to select the columns and then enter the values.
Press the Input Spreadsheet icon (to the right and slightly up) to view/edit the spreadsheet. Also note that you can obtain the spreadsheet by selecting "Tools", then "Input Spreadsheet", from the menus.

**Definition**
A contrast is a weighted average of the logits of the G group response probabilities in which the weights (coefficients) sum to zero. Each successive coefficient is applied to the corresponding group mean. For example, suppose G = 3 and the first group is a control group. A contrast that might be of interest is "2 -1 -1". This contrast is interpreted as "2 Logit(P1) - Logit(P2) - Logit(P3)". The coefficients must sum to zero. The attained power is highly related to the specific contrast that is entered.
Incomplete List
If the list is not long enough, it is padded with zeros.

Examples
C1 - C3
C1, C2, C3
Con1, Con2, Con3

Spreadsheet Example
If G = 4 and there are three columns specified, the spreadsheet might look as follows.

C1  C2  C3
1   -3   1
1   -1  -1
1    1  -1
-3    3   1

Effect Size - \( \rho \) (Correlation Between Measurements)

Pattern of \( \rho \)'s Across Time (or Space)
Specify the type of pattern for the \( \rho \)'s, the correlations between responses of an individual at two points in time or between subjects within a cluster. This pattern often depends on the measurement time proportions. Usually, the correlation shrinks as two points are further and further apart in time. The options are

- **Compound Symmetry (All \( \rho \)'s Equal)**
  A single value of \( \rho \) is used for all correlations. This pattern is commonly used for cluster-randomized trials in which case \( m \) is the cluster size and \( \rho \) is the intracluster correlation.

  \[
  \begin{pmatrix}
  1 & \rho & \rho & \rho \\
  \rho & 1 & \rho & \rho \\
  \rho & \rho & 1 & \rho \\
  \rho & \rho & \rho & 1 \\
  \end{pmatrix}
  \]

  Example first row of correlation matrix

  1.0000, 0.5000, 0.5000, 0.5000, 0.5000, 0.5000

- **Banded (1 Time Period)**
  This is similar to all \( \rho \)'s equal, except that all elements are set to zero after the first off-diagonal element.

  \[
  \begin{pmatrix}
  1 & \rho & 0 & 0 \\
  \rho & 1 & \rho & 0 \\
  0 & \rho & 1 & \rho \\
  0 & 0 & \rho & 1 \\
  \end{pmatrix}
  \]

  Example first row of correlation matrix

  1.0000, 0.5000, 0.0000, 0.0000, 0.0000, 0.0000
Banded (2 Time Periods)
This is similar to all \( \rho \)'s equal, except that all elements are set to zero after the second off-diagonal element.

\[
\begin{bmatrix}
1 & \rho & \rho & 0 \\
\rho & 1 & \rho & \rho \\
\rho & \rho & 1 & \rho \\
0 & \rho & \rho & 1 \\
\end{bmatrix}
\]

Example first row of correlation matrix
1.0000, 0.5000, 0.5000, 0.0000, 0.0000, 0.0000

AR1 (Traditional) 1st Order Autocorrelation
A single value of \( \rho \) is used to generate a first order autocorrelation pattern in the correlation matrix. This pattern produces the autocorrelation at each successive step by multiplying the value at the last step by \( \rho \). The basic formula is \( \rho_{jk} = \rho^{|j-k|} \). Thus the pattern is \( \rho, \rho^2, \rho^3 \), etc.

\[
\begin{bmatrix}
1 & \rho & \rho^2 & \rho^3 \\
\rho & 1 & \rho & \rho^2 \\
\rho^2 & \rho & 1 & \rho \\
\rho^3 & \rho^2 & \rho & 1 \\
\end{bmatrix}
\]

Example first row of correlation matrix
1.0000, 0.5000, 0.2500, 0.1250, 0.0625, 0.0313

AR1 (Proportional)
A single value of \( \rho \) is used to generate a first order autocorrelation pattern in the correlation matrix. This pattern produces the autocorrelation at each successive step by multiplying the value at the last step by \( \rho \). The basic formula is \( \rho_{jk} = \rho^{t_j-t_k} \), where \( t_j \) and \( t_k \) are time proportions at which measurements are planned. An example is

\[
\begin{bmatrix}
\rho^{0\cdot3} & \rho^{1\cdot3} & \rho^{2\cdot3} & \rho^{3\cdot3} \\
\rho^{1\cdot3} & \rho^{0\cdot3} & \rho^{1\cdot3} & \rho^{2\cdot3} \\
\rho^{2\cdot3} & \rho^{1\cdot3} & \rho^{0\cdot3} & \rho^{1\cdot3} \\
\rho^{3\cdot3} & \rho^{2\cdot3} & \rho^{1\cdot3} & \rho^{0\cdot3} \\
\end{bmatrix}
\]

Example first row of correlation matrix when \( M = 6 \) and \( \rho = 0.1 \). Note that \( \rho \) shows up at the end of the list in this case.
1.0000, 0.6310, 0.3981, 0.2512, 0.1585, 0.1000

Dampened Exponential (Traditional)
This pattern is similar to the AR(1), except that the exponents are raised to the power of a second parameter which we label as Dexp. The formula is \( \rho_{jk} = \rho^{1\cdot3} \cdot D_{exp} \).

Example first row of correlation matrix with Dexp = 1.1
1.0000, 0.5000, 0.2012, 0.0729, 0.0245, 0.0078
• **Dampened Exponential (Proportional)**
  A single value of $\rho$ is used to generate a first order autocorrelation pattern in the correlation matrix. This pattern produces the autocorrelation at each successive step by multiplying the value at the last step by $\rho$. The basic formula is $\rho_{jk} = \rho^{\left|\frac{t_j - t_k}{D_{\text{exp}}}\right|}$, where $t_j$ and $t_k$ are time proportions at which measurements are planned. An example with $D_{\text{exp}} = 1$ is

$$
\begin{array}{cccc}
\rho^{00} & \rho^{10} & \rho^{20} & \rho^{30} \\
\rho^{10} & \rho^{00} & \rho^{10} & \rho^{20} \\
\rho^{20} & \rho^{10} & \rho^{00} & \rho^{10} \\
\rho^{30} & \rho^{20} & \rho^{10} & \rho^{00}
\end{array}
$$

Example first row of correlation matrix when $M = 6$, $\rho = 0.1$, and $D_{\text{exp}} = 1$. Note that $\rho$ shows up at the end of the list in this case.

$1.0000, 0.6310, 0.3981, 0.2512, 0.1585, 0.1000$

• **Linear Exponential Decay**
  This pattern is similar to AR(1). The formula is $\rho_{jk} = \rho^{\left|\frac{t_j}{P} - \frac{t_k}{P}\right|}E_{\text{max}}$. $P$ is called the Base Time Proportion. The exponents of $\rho$ range from 1 at $P$ to $E_{\text{max}}$ at $M$.

Example first row of correlation matrix with $E_{\text{max}} = 4$

$1.0000, 0.5000, 0.2973, 0.1768, 0.1051, 0.0625$

This pattern is particularly useful when you want to compare different values of $M$ since the exponent only depends on the difference. That is, a difference of 0.10 in the time proportions will always result in the same exponent, no matter what the value of $M$. This is NOT the case for AR(1).

• **Matrix on Spreadsheet**
  A symmetric matrix is entered in the spreadsheet. The number of columns selected must match $M$, the number of time periods.

### $\rho$ Pattern = Compound Symmetry, Banded (1 Time Period), Banded (2 Time Periods), AR(1)

**$\rho$ (Base Correlation)**

This is the base correlation, $\rho$, between measurements on the same subject taken at two time points. In several cases, this value is modified according to a special formula.

At least one value must be entered. If multiple values are entered, a separate analysis is performed for each value.

**Range**

$0 < \rho < 1$ (negative values are not used). A value near 0 indicates low correlation. A value near 1 indicates high correlation.

**Recommended**

The value of this parameter depends on time or location pattern at which measurements are taken. In their book on sample size, Machin and Campbell comment that values between 0.60 and 0.75 are typical. Some authors recommend using 0.2 when nothing is known about the actual value.

**Examples**

- 0.5
- 0.5 0.6 0.7
- 0 to 0.9 by 0.1
**ρ Pattern = Damped Exponential**

**ρ (Base Correlation)**
This is the base correlation, ρ, between measurements on the same subject taken at two time points. In several cases, this value is modified according to a special formula.

At least one value must be entered. If multiple values are entered, a separate analysis is performed for each value.

**Range**
0 < ρ < 1 (negative values are not used). A value near 0 indicates low correlation. A value near 1 indicates high correlation.

**Recommended**
The value of this parameter depends on time or location pattern at which measurements are taken. In their book on sample size, Machin and Campbell comment that values between 0.60 and 0.75 are typical. Some authors recommend using 0.2 when nothing is known about the actual value.

**Examples**
0.5
0.5 0.6 0.7
0 to 0.9 by 0.1

**Dexp (Dampening Exponent)**
This exponent is applied to the AR(1) formula so that the resulting correlations are reduced faster. The formula become ρjk = ρ^|j-k|^Dexp.

**Range**
The valid range is Dexp > 0. The typical range is 1 to 3.

**ρ Pattern = Linear Exponential Decay**

**ρ (Base Correlation)**
This is the base correlation, ρ, between measurements on the same subject taken at two time points. In several cases, this value is modified according to a special formula.

At least one value must be entered. If multiple values are entered, a separate analysis is performed for each value.

**Range**
0 < ρ < 1 (negative values are not used). A value near 0 indicates low correlation. A value near 1 indicates high correlation.

**Recommended**
The value of this parameter depends on time or location pattern at which measurements are taken. In their book on sample size, Machin and Campbell comment that values between 0.60 and 0.75 are typical. Some authors recommend using 0.2 when nothing is known about the actual value.

**Examples**
0.5
0.5 0.6 0.7
0 to 0.9 by 0.1
Base Time Proportion
The base time proportion is used by the Linear Exponential Decay formula to specify the time proportion at which the correlation exponent is one. Thus, at this proportion the base correlation is reproduced.

Range
$0 < \text{Time} < 0.5$.

Typical Value
$1/M$

Note
When you are comparing values of $M$, choose a value for $M$ near the smallest value of $1/M$.

Emax (Max Decay Exponent)
This specifies the exponent of the base correlation when the time proportion is 1. Thus, the exponents go linearly from 1 at the base time to Emax at the maximum time.

Range
Emax $> 0$.

Typical
Typical values of Emax are between 1 and 10.

$p$ Pattern = Matrix on Spreadsheet

Columns Containing the $p_{jk}$'s
Select a set of $M$ columns from the spreadsheet that contain a correlation matrix. Each column represents a column of the correlation matrix so it must contain $M$ rows.

Since this is a correlation matrix, the diagonal elements must be set to one and the absolute values of the off-diagonal elements must be less than one. Also, the matrix must be symmetric.

Missing Proportions Across Measurement Times

Missing Input Type
Specify the way in which you want to specify the pattern of missing values.

Each missing value pattern is a list of missing proportions at each of the $M$ time points. Each value in the list must be non-negative and less than 1.

Possible input choices are

- **Constant = 0**
  All missing proportions are set to 0. That is, there are no missing values.

- **Constant**
  All missing proportions are set to constant value.
• Piecewise Constant on Spreadsheet
  A set of missing proportions are defined for several time intervals using the spreadsheet. One column contains the missing proportions for the interval, going down the rows. Another column defines the corresponding upper limit of time proportion of the interval. The lower limit is implied by the limit given immediately above. The program assumes that the first time interval starts at 0 percent.

• Linear (Steady Change)
  The missing proportions fall along a straight-line between 0 and 1 elapsed time. Only the first and last proportions are entered.

• Piecewise Linear on Spreadsheet
  The missing proportions fall along a set of connected straight-lines that are defined by two columns on the spreadsheet.

• List
  Enter a list of M missing proportions, one for each time point.

• Multiple Lists on Spreadsheet
  Select multiple columns containing vertical lists of missing proportions. Each column contains a set of missing proportions in rows, one for each time point.

• Pairwise Observed Proportions on Spreadsheet
  Enter an M x M matrix of observant probabilities by selecting M columns. These observant probabilities are the proportion of the responses for both the row and column time points that are observed.

**Missing Input Type = Constant Additional Options**

**Constant Missing Proportion**
Enter one or more values of the missing value proportion that is to be used for all time points. If multiple values are entered, a separate analysis is performed for each value.

**Range**
0 ≤ Missing Proportion < 1.

**Examples**
0.2
0.1 0.2 0.4
0 to 0.4 by 0.1

**Missing Input Type = Piecewise Constant on Spreadsheet Additional Options**

**Pairwise Missing Pattern**
See the Pairwise Missing Pattern section of above.

**Column of Missing Proportions (Pairwise)**
Enter the column name on the spreadsheet containing a column of missing proportions, one for each segment. The program assigns this missing proportion to all time points in this time segment. The number of segments is arbitrary and determined by the number of rows.

Note that the usual pattern is to have the missing proportions increase over time.
Press the Spreadsheet icon (directly to the right) to select the column and then enter the missing proportion values. Press the Input Spreadsheet icon (to the right and slightly up) to view/edit the spreadsheet. Note that you can obtain also the spreadsheet window by selecting "Tools", then "Input Spreadsheet", from the menus.

(Pairwise)
This refers to the two types of pairwise missing patterns: Ind(ependent) and Mon(otonic).

Example
Suppose that 'C1' is selected for the Missing column and 'C2' is selected for the Time column. Suppose that these columns contain the following values:

<table>
<thead>
<tr>
<th>C1</th>
<th>C2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>0.3</td>
<td>0.5</td>
</tr>
<tr>
<td>0.35</td>
<td>0.75</td>
</tr>
<tr>
<td>0.4</td>
<td>0.9</td>
</tr>
<tr>
<td>0.6</td>
<td>1.0</td>
</tr>
</tbody>
</table>

This would be interpreted by the program as

<table>
<thead>
<tr>
<th>Time Interval</th>
<th>Missing</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 ≤ t ≤ 0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>0.2 &lt; t ≤ 0.5</td>
<td>0.3</td>
</tr>
<tr>
<td>0.5 &lt; t ≤ 0.75</td>
<td>0.35</td>
</tr>
<tr>
<td>0.75 &lt; t ≤ 0.9</td>
<td>0.4</td>
</tr>
<tr>
<td>0.9 &lt; t ≤ 1.0</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Column of Time Proportions (Pairwise)
Enter the column name on the spreadsheet containing a column of Upper Limit Time Proportions, one for each segment. The program assigns the missing proportion to all time points in the time segment defined by this upper limit. The number of segments is arbitrary and determined by the number of rows. Note that the final upper limit must be 1.

Press the Spreadsheet icon (directly to the right) to select the column and then enter the missing proportion values. Press the Input Spreadsheet icon (to the right and slightly up) to view/edit the spreadsheet. Note that you can obtain also the spreadsheet window by selecting "Tools", then "Input Spreadsheet", from the menus.

(Pairwise)
This refers to the two types of pairwise missing patterns: Ind(ependent) and Mon(otonic).

Example
Suppose that 'C1' is selected for the Missing column and 'C2' is selected for the Time column. Suppose that these columns contain the following values:

<table>
<thead>
<tr>
<th>C1</th>
<th>C2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>0.3</td>
<td>0.5</td>
</tr>
<tr>
<td>0.35</td>
<td>0.75</td>
</tr>
<tr>
<td>0.4</td>
<td>0.9</td>
</tr>
<tr>
<td>0.6</td>
<td>1.0</td>
</tr>
</tbody>
</table>
This would be interpreted by the program as

<table>
<thead>
<tr>
<th>Time Interval</th>
<th>Missing</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 ≤ t ≤ 0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>0.2 &lt; t ≤ 0.5</td>
<td>0.3</td>
</tr>
<tr>
<td>0.5 &lt; t ≤ 0.75</td>
<td>0.35</td>
</tr>
<tr>
<td>0.75 &lt; t ≤ 0.9</td>
<td>0.4</td>
</tr>
<tr>
<td>0.9 &lt; t ≤ 1.0</td>
<td>0.6</td>
</tr>
</tbody>
</table>

**Missing Input Type = Linear (Steady Change) Additional Options**

**Pairwise Missing Pattern**
See the Pairwise Missing Pattern section of above.

**First (or Last) Missing Proportion (Pairwise)**
Specify a set of M missing proportions by entering only the first and last values of the list. A straight-line formula will be used to calculate the missing proportions for each time point in the interval.

**Range**
0 ≤ First ≤ Last
First ≤ Last < 1

**Example**
Suppose a study is being planned with 5, evenly spaced, measurements. Further suppose that the missing data proportions are anticipated to be 0.1 initially and increase to 0.6 at the end of the study, with the missing proportions in between arranged linearly. If M = 5, First = 0.1, and Last = 0.6, then the resulting Missing Proportions will be 0.1, 0.225, 0.35, 0.475, and 0.60. Note that these proportions are evenly spaced between the two boundaries.

Note that using this option allows the value of M to be varied and reasonable comparisons in power and sample size to be made.

**Missing Input Type = Piecewise Linear on Spreadsheet Additional Options**

**Pairwise Missing Pairwise**
See the Pairwise Missing Pairwise section of above.

**Column of Missing (or Time) Proportions (Pairwise)**
These options let you easily define separate missing proportions for various time intervals of a study.

Select the column name on the spreadsheet of a column containing missing proportions. Also select the column name of the corresponding measurement time proportions. The program calculates a straight-line defined by each successive pair of points (see example below). This set of connected line segments becomes a prediction equation for the missing proportion based on the corresponding time proportion.

The number of line segments is arbitrary and determined from the number of rows in the selected columns. The number of rows does not have to match M, the number of time points.

**(Pairwise)**
This designates which of the two types of pairwise missing patterns, Ind(ependent) or Mon(otonic), is being entered in this option.
Notes
The usual pattern is to for the missing proportions to increase over time. The time value of the first row of the time column must be 0. The time value of the last row of the time column must be 1.

Shortcuts
Press the Spreadsheet icon (directly to the right) to select the column and then enter the missing proportion values. Press the Input Spreadsheet icon (to the right and slightly up) to view/edit the spreadsheet. Note that you can obtain also the spreadsheet window by selecting "Tools", then "Input Spreadsheet", from the menus.

Example
Suppose that you have enter data in two columns on the spreadsheet labeled Miss and Time as follows.

<table>
<thead>
<tr>
<th>Miss</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0</td>
</tr>
<tr>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>0.3</td>
<td>0.5</td>
</tr>
<tr>
<td>0.35</td>
<td>0.75</td>
</tr>
<tr>
<td>0.4</td>
<td>0.9</td>
</tr>
<tr>
<td>0.6</td>
<td>1</td>
</tr>
</tbody>
</table>

Now suppose that Miss is selected for the Missing column and Time is selected for the Time column. This would be interpreted by the program as the following segmented-prediction equation.

\[
\begin{array}{ccc}
\text{Time Interval} & \text{Slope} & \text{Intercept} \\
0 \leq T \leq 0.2 & 0.25=(0.1-0.05)/(0.2-0) & 0.1-0.2*0.25 = 0.05 \\
0.2 < T \leq 0.5 & 0.67=(0.3-0.1)/(0.5-0.2) & 0.3-0.5*0.67 = -0.0333 \\
0.5 < T \leq 0.75 & 0.2=(0.35-0.3)/(0.75-0.5) & 0.35-0.75*.20 = 0.2 \\
0.75 < T \leq 0.9 & 0.33=(0.4-0.35)/(0.9-0.75) & 0.4-0.9*.33 = 0.1 \\
0.9 < T \leq 1 & 2=(0.6-0.4)/(1-0.9) & 0.6-1*2 = -1.4 \\
\end{array}
\]

A Missing Proportion would then be calculated using the formula

\[Missing = Intercept + Slope \times Time\]

For example, using the above values, the missing proportions at time points 0, 0.1, 0.3, 0.8, and 1.0 would be found as follows:

<table>
<thead>
<tr>
<th>Time</th>
<th>Equation</th>
<th>Resulting Missing Proportion</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.05 + 0.25(0)</td>
<td>0.05</td>
</tr>
<tr>
<td>0.1</td>
<td>0.05 + 0.25(0.1)</td>
<td>0.075</td>
</tr>
<tr>
<td>0.3</td>
<td>-0.0333 + 0.67(0.3)</td>
<td>0.1667</td>
</tr>
<tr>
<td>0.8</td>
<td>0.1 + 0.33(0.8)</td>
<td>0.3667</td>
</tr>
<tr>
<td>1.0</td>
<td>-1.4 + 2(1)</td>
<td>0.6</td>
</tr>
</tbody>
</table>

**Missing Input Type = List Additional Options**

**Pairwise Missing Pairwise**
See the Pairwise Missing Pairwise section of above.

**List of Missing Proportions (Pairwise)**
These options let you easily define separate missing proportions for various time intervals of a study.

Select the column name on the spreadsheet of a column containing missing proportions. Also select the column name of the corresponding measurement time proportions. The program calculates a straight-line defined by each successive pair of points (see example below). This set of connected line segments becomes a prediction equation for the missing proportion based on the corresponding time proportion.
Missing Input Type = Multiple Lists on Spreadsheet Additional Options

Pairwise Missing Pairwise
See the Pairwise Missing Pairwise section of above.

Columns of Missing Proportions (Pairwise)
Enter lists of missing value proportions by first entering them on the spreadsheet and then selecting those columns here. The missing value proportions are entered down the rows of a column, one for each time point. Each value is the anticipated proportion of missing responses at the corresponding time point. Although you can enter any pattern you want, the usual pattern is that the proportion missing increases over time. When you specify more than $M$ values in a list (where $M$ is specified in the Measurement Times section), the extra values are ignored. When you specify too few values, the last value you specify is copied forward.

Press the Spreadsheet icon (directly to the right) to select the column and then enter the missing proportion values. Press the Input Spreadsheet icon (to the right and slightly up) to view/edit the spreadsheet. Note that you can obtain also the spreadsheet window by selecting "Tools", then "Input Spreadsheet", from the menus.

(Pairwise)
This designates which of the two types of pairwise missing patterns, Ind(ependent) or Mon(otonic), is being entered in this option.

Range
$0 \leq \text{missing proportion} < 1.$

Missing Input Type = Pairwise Observed Proportions on Spreadsheet Additional Options

Columns of Pairwise Observed
Enter an M-by-M matrix of pairwise observed proportions on the spreadsheet and then select it by selecting those M columns here. Each entry gives the proportion (or probability) that the corresponding row and column time point values are both observed (non-missing). The diagonal elements of this matrix contain the probability that an observation is observed at that point (marginal probability).

Range
$0 < \text{each value} \leq 1.$

Note
In other missing entries, you enter the proportion missing. This is the only case that requires the proportion observed.
Example 1 – Determining Sample Size

Researchers are planning a study comparing three heart-rate medications: a standard drug and two experimental drugs. The experimental drugs are expected to have about the same impact on heart rate. Each subject will receive four applications of only one drug, two days apart. The response that will be analyzed is whether the heart rate is above 60 bpm. With the standard drug, the percentage is 40%. The researchers want a sample size large enough to detect a difference of 20 percentage points in the time-averaged response. They will use the response percentages of 40, 20, and 20 to represent this difference. The contrast coefficients that they will use are -2, 1, 1.

Similar studies have shown an autocorrelation between adjacent measurements on the same individual of 0.7, so the researchers want to try values of 0.6, 0.7, and 0.8. The researchers assume that first-order autocorrelation adequately represents the autocorrelation pattern. The test will be conducted at the 0.05 significance level and powered at 90%. The subjects will be randomly split equally among the three groups.

The researchers anticipate that the missing pattern across time will begin at 0% missing and increase steadily to 20% at the fourth measurement. They assume that the pairwise missing is independent.

What are the sample size requirements for this study?

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the GEE Tests for the TAD of Multiple Groups in a Repeated Measures Design (Binary Outcome) procedure window. You may then make the appropriate entries as listed below, or open Example 1 by going to the File menu and choosing Open Example Template.

<table>
<thead>
<tr>
<th>Option</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design Tab</td>
<td></td>
</tr>
<tr>
<td>Solve For</td>
<td>Sample Size</td>
</tr>
<tr>
<td>Power</td>
<td>0.90</td>
</tr>
<tr>
<td>Alpha</td>
<td>0.05</td>
</tr>
<tr>
<td>G (Number of Groups)</td>
<td>3</td>
</tr>
<tr>
<td>Group Allocation Input Type</td>
<td>Equal (n1 = n2 = ⋯ = nG = n)</td>
</tr>
<tr>
<td>Measurement Time Input Type</td>
<td>Equally Spaced Measurement Times</td>
</tr>
<tr>
<td>M (Number of Measurement Times)</td>
<td>4</td>
</tr>
<tr>
<td>P1's Input Type</td>
<td>P1, P2, ⋯, PG</td>
</tr>
<tr>
<td>P1, P2, ⋯, PG</td>
<td>0.4 0.2 0.2</td>
</tr>
<tr>
<td>Contrast Input Type</td>
<td>List of Contrast Coefficients</td>
</tr>
<tr>
<td>Contrast Coefficients</td>
<td>-2 1 1</td>
</tr>
<tr>
<td>Pattern of ρ's Across Time</td>
<td>AR1 (Traditional)</td>
</tr>
<tr>
<td>ρ (Base Correlation)</td>
<td>0.6 0.7 0.8</td>
</tr>
<tr>
<td>Missing Input Type</td>
<td>Linear (Steady Change)</td>
</tr>
<tr>
<td>Pairwise Missing Pattern</td>
<td>Independent (Ind)</td>
</tr>
<tr>
<td>First Missing Proportion (Ind)</td>
<td>0</td>
</tr>
<tr>
<td>Last Missing Proportion (Ind)</td>
<td>0.2</td>
</tr>
</tbody>
</table>
Annotated Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results for a Time-Averaged Response Test of Binary Data using GEE

<table>
<thead>
<tr>
<th>Contrast: List of Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement Times: Equally spaced</td>
</tr>
<tr>
<td>Correlation: AR(1): ( \rho(j,k) = \rho^{</td>
</tr>
<tr>
<td>Missing Pattern: Range of missing proportions</td>
</tr>
<tr>
<td>Observant Proportions: Assume independence</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9037</td>
<td>150</td>
<td>ni1</td>
<td>3</td>
<td>4</td>
<td>P1</td>
<td>Con1</td>
<td>0.6000</td>
<td>( \rho(T1) )</td>
<td>0.050</td>
<td>M1(T1)</td>
<td>T1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.9042</td>
<td>171</td>
<td>ni2</td>
<td>3</td>
<td>4</td>
<td>P1</td>
<td>Con1</td>
<td>0.7000</td>
<td>( \rho(T1) )</td>
<td>0.050</td>
<td>M1(T1)</td>
<td>T1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.9009</td>
<td>192</td>
<td>ni3</td>
<td>3</td>
<td>4</td>
<td>P1</td>
<td>Con1</td>
<td>0.8000</td>
<td>( \rho(T1) )</td>
<td>0.050</td>
<td>M1(T1)</td>
<td>T1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Footnotes

T1: 0.00, 0.33, 0.67, 1.00
M1(T1): 0.00, 0.07, 0.13, 0.20
ni1: 50, 50, 50
ni2: 57, 57, 57
ni3: 64, 64, 64
P1: 0.40, 0.20, 0.20
Con1: -2.00, 1.00, 1.00
\( \rho(T1) \): 1.0000, 0.6000, 0.3600, 0.2160
\( \rho(T1) \): 1.0000, 0.7000, 0.4900, 0.3430
\( \rho(T1) \): 1.0000, 0.8000, 0.6400, 0.5120

References


Report Definitions

Power is the probability of rejecting a false null hypothesis.
N is the total number of subjects in the study.
ni's gives the name of the footnote containing the individual group sample sizes.
G is the number of treatment groups.
M is the number of time points at which each subject is scheduled to be measured.
P1's gives the name of the footnote containing the time-averaged response probabilities for each group.
Contrast Coef gives the name of the footnote containing the contrast coefficients that are combined with the group response probabilities.
\( \rho \) is the base correlation between two responses on the same subject. It may be transformed based on the correlation pattern.
Alpha is the significance level of the test. The probability of rejecting the null hypothesis when the null hypothesis is true.
Missing gives the name of the footnote containing the missing value proportions across time.
Time gives the name of the footnote containing the time proportions. The time values represent the proportion of the total study time that has elapsed just before the measurement.

Summary Statements

A total sample of 150 subjects, divided among 3 groups and each measured 4 times, achieve a power of 0.9037 when using a normal test from a GEE analysis to determine whether a contrast of the group time-averaged response probabilities differs significantly from zero at a significance level of 0.050. The group sample sizes are 50, 50, 50. The group response probabilities under the alternative hypothesis are 0.40, 0.20, 0.20. The contrast coefficients are -2.00, 1.00, 1.00. The measurements of each subject will be made at the following times expressed as proportions of the total study time: 0.00, 0.33, 0.67, 1.00. Missing values are assumed to occur completely at random (MCAR). These missing value proportions will be combined to form the pairwise observant probabilities using the Independent method. The anticipated proportions missing at each measurement time are 0.00, 0.07, 0.13, 0.20. The first row of the autocorrelation matrix of the responses within a subject is assumed to be 1.0000, 0.6000, 0.3600, 0.2160. Other rows follow the same pattern.

This report gives the sample size for each value of the other parameters.
This shows the relationship among the design parameters.

### Autocorrelation Matrix for Report Row 1

<table>
<thead>
<tr>
<th>Time</th>
<th>T(0.00)</th>
<th>T(0.33)</th>
<th>T(0.67)</th>
<th>T(1.00)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T(0.00)</td>
<td>1.0000</td>
<td>0.6000</td>
<td>0.3600</td>
<td>0.2160</td>
</tr>
<tr>
<td>T(0.33)</td>
<td>0.6000</td>
<td>1.0000</td>
<td>0.6000</td>
<td>0.3600</td>
</tr>
<tr>
<td>T(0.67)</td>
<td>0.3600</td>
<td>0.6000</td>
<td>1.0000</td>
<td>0.6000</td>
</tr>
<tr>
<td>T(1.00)</td>
<td>0.2160</td>
<td>0.3600</td>
<td>0.6000</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

These reports show the autocorrelation matrix for the indicated row of the report.
Example 2 – Finding the Power

Continuing with Example 1, the researchers want to determine the power corresponding to group sample sizes ranging from 40 to 70 for the middle values of the other parameters.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the GEE Tests for the TAD of Multiple Groups in a Repeated Measures Design (Binary Outcome) procedure window. You may then make the appropriate entries as listed below, or open Example 2 by going to the File menu and choosing Open Example Template.

<table>
<thead>
<tr>
<th>Option</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design Tab</td>
<td></td>
</tr>
<tr>
<td>Solve For</td>
<td>Power</td>
</tr>
<tr>
<td>Alpha</td>
<td>0.05</td>
</tr>
<tr>
<td>G (Number of Groups)</td>
<td>3</td>
</tr>
<tr>
<td>Group Allocation Input Type</td>
<td>Equal (n1 = n2 = \cdots = nG = n)</td>
</tr>
<tr>
<td>n (Sample Size Per Group)</td>
<td>40 50 60 70</td>
</tr>
<tr>
<td>Measurement Time Input Type</td>
<td>Equally Spaced Measurement Times</td>
</tr>
<tr>
<td>M (Number of Measurement Times)</td>
<td>4</td>
</tr>
<tr>
<td>Pi's Input Type</td>
<td>P1, P2, ..., PG</td>
</tr>
<tr>
<td>P1, P2, ..., PG</td>
<td>0.4 0.2 0.2</td>
</tr>
<tr>
<td>Contrast Input Type</td>
<td>List of Contrast Coefficients</td>
</tr>
<tr>
<td>Contrast Coefficients</td>
<td>-2 1 1</td>
</tr>
<tr>
<td>Pattern of ρ's Across Time</td>
<td>AR1 (Traditional)</td>
</tr>
<tr>
<td>ρ (Base Correlation)</td>
<td>0.7</td>
</tr>
<tr>
<td>Missing Input Type</td>
<td>Linear (Steady Change)</td>
</tr>
<tr>
<td>Pairwise Missing Pattern</td>
<td>Independent (Ind)</td>
</tr>
<tr>
<td>First Missing Proportion (Ind)</td>
<td>0</td>
</tr>
<tr>
<td>Last Missing Proportion (Ind)</td>
<td>0.2</td>
</tr>
</tbody>
</table>
### Numeric Results for a Time-Averaged Response Test of Binary Data using GEE

<table>
<thead>
<tr>
<th>Power</th>
<th>N</th>
<th>ni's</th>
<th>G</th>
<th>M</th>
<th>Pi's</th>
<th>Coef</th>
<th>Base Corr</th>
<th>First Row of Corr</th>
<th>Alpha</th>
<th>Missing</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7810</td>
<td>120</td>
<td>n1</td>
<td>3</td>
<td>4</td>
<td>Pi1</td>
<td>Con1</td>
<td>0.7000</td>
<td>p1(T1)</td>
<td>0.050</td>
<td>M1(T1)</td>
<td>T1</td>
</tr>
<tr>
<td>0.8640</td>
<td>150</td>
<td>n2</td>
<td>3</td>
<td>4</td>
<td>Pi1</td>
<td>Con1</td>
<td>0.7000</td>
<td>p1(T1)</td>
<td>0.050</td>
<td>M1(T1)</td>
<td>T1</td>
</tr>
<tr>
<td>0.9178</td>
<td>180</td>
<td>n3</td>
<td>3</td>
<td>4</td>
<td>Pi1</td>
<td>Con1</td>
<td>0.7000</td>
<td>p1(T1)</td>
<td>0.050</td>
<td>M1(T1)</td>
<td>T1</td>
</tr>
<tr>
<td>0.9514</td>
<td>210</td>
<td>n4</td>
<td>3</td>
<td>4</td>
<td>Pi1</td>
<td>Con1</td>
<td>0.7000</td>
<td>p1(T1)</td>
<td>0.050</td>
<td>M1(T1)</td>
<td>T1</td>
</tr>
</tbody>
</table>

### Footnotes

- **T1**: 0.00, 0.33, 0.67, 1.00
- **M1(T1)**: 0.00, 0.07, 0.13, 0.20
- **ni1**: 40, 40, 40
- **ni2**: 50, 50, 50
- **ni3**: 60, 60, 60
- **ni4**: 70, 70, 70
- **Pi1**: 0.40, 0.20, 0.20
- **Con1**: -2.00, 1.00, 1.00
- **p1(T1)**: 1.0000, 0.7000, 0.4900, 0.3430

Note that the horizontal axis is scaled according to the *ni* footnote index. Hence, 1 is for *ni1*, 2 is for *ni2*, and so on. The same is true for the *Pi’s* and the *Linear Miss* in the subtitle.
Example 3 – Comparing Various Effect Sizes

Continuing with Examples 1 and 2, the researchers want to compare the sample size of various sets of group response probabilities. To do this, they will compare the four sets of probabilities shown in the following table. The first row is the set of time-averaged response for the standard medication. The second and third rows give the anticipated response to the experimental medications.

The values in this table must be loaded into the spreadsheet.

<table>
<thead>
<tr>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.40</td>
<td>0.40</td>
<td>0.40</td>
<td>0.40</td>
</tr>
<tr>
<td>0.10</td>
<td>0.20</td>
<td>0.30</td>
<td>0.35</td>
</tr>
<tr>
<td>0.10</td>
<td>0.20</td>
<td>0.30</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Table of Four Sets of Time-Averaged Response Probabilities

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the GEE Tests for the TAD of Multiple Groups in a Repeated Measures Design (Binary Outcome) procedure window. You may then make the appropriate entries as listed below, or open Example 3 by going to the File menu and choosing Open Example Template.

<table>
<thead>
<tr>
<th>Option</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design Tab</td>
<td></td>
</tr>
<tr>
<td>Solve For ..................................</td>
<td>Sample Size</td>
</tr>
<tr>
<td>Power .....................................</td>
<td>0.90</td>
</tr>
<tr>
<td>Alpha .....................................</td>
<td>0.05</td>
</tr>
<tr>
<td>G (Number of Groups) .................</td>
<td>3</td>
</tr>
<tr>
<td>Group Allocation Input Type ..........</td>
<td>Equal (n1 = n2 = ··· = nG = n)</td>
</tr>
<tr>
<td>Measurement Time Input Type ..........</td>
<td>Equally Spaced Measurement Times</td>
</tr>
<tr>
<td>M (Number of Measurement Times) ......</td>
<td>4</td>
</tr>
<tr>
<td>Pi's Input Type ........................</td>
<td>Columns containing sets of Pi's</td>
</tr>
<tr>
<td>Columns Containing Sets of Pi's .....</td>
<td>C1-C4</td>
</tr>
<tr>
<td>Pattern of ρ's Across Time ..........</td>
<td>AR1 (Traditional)</td>
</tr>
<tr>
<td>ρ (Base Correlation) ..................</td>
<td>0.7</td>
</tr>
<tr>
<td>Missing Input Type ...................</td>
<td>Linear (Steady Change)</td>
</tr>
<tr>
<td>Pairwise Missing Pattern ............</td>
<td>Independent (Ind)</td>
</tr>
<tr>
<td>First Missing Proportion (Ind) ......</td>
<td>0</td>
</tr>
<tr>
<td>Last Missing Proportion (Ind) .......</td>
<td>0.2</td>
</tr>
</tbody>
</table>
Output

Click the Calculate button to perform the calculations and generate the following output.

### Numeric Results for a Time-Averaged Response Test of Binary Data using GEE

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9069 69 ni1 3</td>
<td>4</td>
<td>C1</td>
<td>Con1</td>
<td>0.7000</td>
<td>p1(T1)</td>
<td>0.050</td>
<td>M1(T1)</td>
<td>T1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.9042 171 ni2 3</td>
<td>4</td>
<td>C2</td>
<td>Con1</td>
<td>0.7000</td>
<td>p1(T1)</td>
<td>0.050</td>
<td>M1(T1)</td>
<td>T1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.9005 747 ni3 3</td>
<td>4</td>
<td>C3</td>
<td>Con1</td>
<td>0.7000</td>
<td>p1(T1)</td>
<td>0.050</td>
<td>M1(T1)</td>
<td>T1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.9000 3102 ni4 3</td>
<td>4</td>
<td>C4</td>
<td>Con1</td>
<td>0.7000</td>
<td>p1(T1)</td>
<td>0.050</td>
<td>M1(T1)</td>
<td>T1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Footnotes**

T1: 0.00, 0.33, 0.67, 1.00
M1(T1): 0.00, 0.07, 0.13, 0.20
ni1: 23, 23, 23
ni2: 57, 57, 57
ni3: 249, 249, 249
ni4: 1034, 1034, 1034
C1: 0.40, 0.10, 0.10
C2: 0.40, 0.20, 0.20
C3: 0.40, 0.30, 0.30
C4: 0.40, 0.35, 0.35
Con1: -2.00, 1.00, 1.00
p1(T1): 1.0000, 0.7000, 0.4900, 0.3430

Note that the horizontal axis gives the index of the probability footnotes. Thus, ‘1.0’ is column C1, ‘2.0’ is column C2, and so on.
Example 4 – Validation of Sample Size Calculation using Wang, Zhang, and Ahn (2018)

Wang, Zhang, and Ahn (2018) present an example in which $G = 4$, $m = 6$, $P_1 = \exp(0)/(1 + \exp(0)) = 0.5$, $P_2 = P_3 = P_4 = \exp(0.5)/(1 + \exp(0.5)) = 0.62245933$, compound symmetry, $\rho = 0.3$ and 0.5, missing = \{0.0, 0.05, 0.10, 0.15, 0.20, 0.25\} and assumes independence, $\alpha = 0.05$, and power = 0.8. The contrast uses the coefficient -3, 1, 1, 1. They obtained sample sizes of 300 and 413.

Setup

This section presents the values of each of the parameters needed to generate the upper-left-quarter of this example. First, from the PASS Home window, load the GEE Tests for the TAD of Multiple Groups in a Repeated Measures Design (Binary Outcome) procedure window. You may then make the appropriate entries as listed below, or open Example 4 by going to the File menu and choosing Open Example Template.

<table>
<thead>
<tr>
<th>Option</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Design Tab</strong></td>
<td></td>
</tr>
<tr>
<td>Solve For</td>
<td>Sample Size</td>
</tr>
<tr>
<td>Power</td>
<td>0.8</td>
</tr>
<tr>
<td>Alpha</td>
<td>0.05</td>
</tr>
<tr>
<td>G (Number of Groups)</td>
<td>4</td>
</tr>
<tr>
<td>Group Allocation Input Type</td>
<td>Equal (n1 = n2 = \ldots = nG = n)</td>
</tr>
<tr>
<td>Measurement Time Input Type</td>
<td>Equally Spaced Measurement Times</td>
</tr>
<tr>
<td>M (Number of Measurement Times)</td>
<td>6</td>
</tr>
<tr>
<td>Pi's Input Type</td>
<td>P1, P2, ..., PG</td>
</tr>
<tr>
<td>P1, P2, ..., PG</td>
<td>0.5 0.62245933</td>
</tr>
<tr>
<td>Contrast Input Type</td>
<td>List of Contrast Coefficients</td>
</tr>
<tr>
<td>Contrast Coefficients</td>
<td>-3 1 1 1</td>
</tr>
<tr>
<td>Pattern of $\rho$'s Across Time</td>
<td>Compound Symmetry (All $\rho$'s Equal)</td>
</tr>
<tr>
<td>$\rho$ (Base Correlation)</td>
<td>0.3 0.5</td>
</tr>
<tr>
<td>Missing Input Type</td>
<td>Linear (Steady Change)</td>
</tr>
<tr>
<td>Pairwise Missing Pattern</td>
<td>Independent (Ind)</td>
</tr>
<tr>
<td>First Missing Proportion (Ind)</td>
<td>0</td>
</tr>
<tr>
<td>Last Missing Proportion (Ind)</td>
<td>0.25</td>
</tr>
</tbody>
</table>
Output

Click the Calculate button to perform the calculations and generate the following output.

### Numeric Results for a Time-Averaged Response Test of Binary Data using GEE

<table>
<thead>
<tr>
<th>Power</th>
<th>N</th>
<th>n1's</th>
<th>GjM</th>
<th>Pi's</th>
<th>Contrast</th>
<th>Coef</th>
<th>Base Corr</th>
<th>First Row of Corr</th>
<th>Alpha</th>
<th>Missing</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8008</td>
<td>300</td>
<td>ni1</td>
<td>4</td>
<td>6</td>
<td>Pi1</td>
<td>Con1</td>
<td>0.3000</td>
<td>p1(T1)</td>
<td>0.050</td>
<td>M1(T1)</td>
<td>T1</td>
</tr>
<tr>
<td>0.8032</td>
<td>416</td>
<td>ni2</td>
<td>4</td>
<td>6</td>
<td>Pi1</td>
<td>Con1</td>
<td>0.5000</td>
<td>p2(T1)</td>
<td>0.050</td>
<td>M1(T1)</td>
<td>T1</td>
</tr>
</tbody>
</table>

**Footnotes**

T1: 0.00, 0.20, 0.40, 0.60, 0.80, 1.00
M1(T1): 0.00, 0.05, 0.10, 0.15, 0.20, 0.25
ni1: 75, 75, 75, 75
ni2: 104, 104, 104, 104
Pi1: 0.50, 0.62, 0.62, 0.62
Con1: -3.00, 1.00, 1.00, 1.00
p1(T1): 1.0000, 0.3000, 0.3000, 0.3000, 0.3000, 0.3000
p2(T1): 1.0000, 0.5000, 0.5000, 0.5000, 0.5000, 0.5000

The first sample size of 300 matches the validation answer exactly, but the second value of 416 is slightly higher than 413 since PASS’s sample size search enforces the equal group allocation requirement.
Example 5 – Comparing Contrast Coefficients

This example will show how important the contrast coefficients are in achieving a certain power.

This example uses $G = 4$, $M = 6$, and $\alpha = 0.05$. The missing input type will be set to Linear from 0 to 30% and the pairwise missing assumption is independent. Group Pi’s are 0.5, 0.5, 0.6, 0.7. The per group sample sizes range from 25 to 225. The correlation pattern will be Linear Exponential Decay with a base correlation of 0.8, Base Time Proportion of 0.1, and Emax set to 4. The Table of Contrasts shows the contrasts that will be compared. Note that C4 was found by finding the Maximum Power contrast in an initial run.

Table of Contrasts

<table>
<thead>
<tr>
<th></th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-3</td>
<td>-3</td>
<td>1</td>
<td>-0.6</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>1</td>
<td>-0.6</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.2</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>3</td>
<td>-3</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the GEE Tests for the TAD of Multiple Groups in a Repeated Measures Design (Binary Outcome) procedure window. You may then make the appropriate entries as listed below, or open Example 6 by going to the File menu and choosing Open Example Template.

<table>
<thead>
<tr>
<th>Option</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design Tab</td>
<td></td>
</tr>
<tr>
<td>Solve For</td>
<td>Power</td>
</tr>
<tr>
<td>Alpha</td>
<td>0.05</td>
</tr>
<tr>
<td>G (Number of Groups)</td>
<td>4</td>
</tr>
<tr>
<td>Group Allocation Input Type</td>
<td>Equal (n1 = n2 = ⋯ = nG = n)</td>
</tr>
<tr>
<td>n (Sample Size Per Group)</td>
<td>25 75 125 175 225</td>
</tr>
<tr>
<td>Measurement Time Input Type</td>
<td>Equally Spaced Measurement Times</td>
</tr>
<tr>
<td>M (Number of Measurement Times)</td>
<td>6</td>
</tr>
<tr>
<td>Pi's Input Type</td>
<td>P1, P2, ..., PG</td>
</tr>
<tr>
<td>P1, P2, ..., PG</td>
<td>0.5, 0.5, 0.6, 0.7</td>
</tr>
<tr>
<td>Contrast Input Type</td>
<td>Multiple Lists of Contrast Coefficients</td>
</tr>
<tr>
<td>Multiple Lists of Coefficients</td>
<td>C1-C4</td>
</tr>
</tbody>
</table>

The above table must be entered into the spreadsheet. Press the Input Spreadsheet icon to the right to view the spreadsheet.

<table>
<thead>
<tr>
<th>Option</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pattern of ρ's Across Time</td>
<td>Linear Exponential Decay</td>
</tr>
<tr>
<td>ρ (Base Correlation)</td>
<td>0.8</td>
</tr>
<tr>
<td>Base Time Proportion</td>
<td>0.10</td>
</tr>
<tr>
<td>Emax (Max Decay Exponent)</td>
<td>4</td>
</tr>
<tr>
<td>Missing Input Type</td>
<td>Linear (Steady Change)</td>
</tr>
<tr>
<td>Pairwise Missing Pattern</td>
<td>Independent (Ind)</td>
</tr>
<tr>
<td>First Missing Proportion (Ind)</td>
<td>0</td>
</tr>
<tr>
<td>Last Missing Proportion (Ind)</td>
<td>0.30</td>
</tr>
</tbody>
</table>
Output

Click the Calculate button to perform the calculations and generate the following output.

### Numeric Results for a Time-Averaged Response Test of Binary Data using GEE

- **Contrast:** Multiple Lists of Coefficients: \{C1-C4\}
- **Measurement Times:** Equally spaced
- **Correlation:** Linear exponential decay, with $E_{\text{max}} = 4$ and Base Time Prop = 0.10
- **Missing Pattern:** Range of missing proportions
- **Observant Proportions:** Assume independence

<table>
<thead>
<tr>
<th>Power N</th>
<th>N</th>
<th>ni's</th>
<th>gjm</th>
<th>p1's</th>
<th>contrast coef</th>
<th>base corr p</th>
<th>matrix alpha</th>
<th>missing</th>
<th>time</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1845</td>
<td>100</td>
<td>ni1</td>
<td>4</td>
<td>6</td>
<td>p1</td>
<td>C1</td>
<td>0.8000</td>
<td>p1(T1)</td>
<td>0.050</td>
</tr>
<tr>
<td>0.4652</td>
<td>100</td>
<td>ni1</td>
<td>4</td>
<td>6</td>
<td>p1</td>
<td>C2</td>
<td>0.8000</td>
<td>p1(T1)</td>
<td>0.050</td>
</tr>
<tr>
<td>0.4002</td>
<td>100</td>
<td>ni1</td>
<td>4</td>
<td>6</td>
<td>p1</td>
<td>C3</td>
<td>0.8000</td>
<td>p1(T1)</td>
<td>0.050</td>
</tr>
<tr>
<td>0.5037</td>
<td>100</td>
<td>ni1</td>
<td>4</td>
<td>6</td>
<td>p1</td>
<td>C4</td>
<td>0.8000</td>
<td>p1(T1)</td>
<td>0.050</td>
</tr>
<tr>
<td>0.4517</td>
<td>100</td>
<td>ni2</td>
<td>4</td>
<td>6</td>
<td>p1</td>
<td>C1</td>
<td>0.8000</td>
<td>p1(T1)</td>
<td>0.050</td>
</tr>
<tr>
<td>0.9003</td>
<td>300</td>
<td>ni2</td>
<td>4</td>
<td>6</td>
<td>p1</td>
<td>C2</td>
<td>0.8000</td>
<td>p1(T1)</td>
<td>0.050</td>
</tr>
<tr>
<td>0.8406</td>
<td>300</td>
<td>ni2</td>
<td>4</td>
<td>6</td>
<td>p1</td>
<td>C3</td>
<td>0.8000</td>
<td>p1(T1)</td>
<td>0.050</td>
</tr>
<tr>
<td>0.9266</td>
<td>300</td>
<td>ni2</td>
<td>4</td>
<td>6</td>
<td>p1</td>
<td>C4</td>
<td>0.8000</td>
<td>p1(T1)</td>
<td>0.050</td>
</tr>
<tr>
<td>0.6604</td>
<td>500</td>
<td>ni3</td>
<td>4</td>
<td>6</td>
<td>p1</td>
<td>C1</td>
<td>0.8000</td>
<td>p1(T1)</td>
<td>0.050</td>
</tr>
<tr>
<td>0.9870</td>
<td>500</td>
<td>ni3</td>
<td>4</td>
<td>6</td>
<td>p1</td>
<td>C2</td>
<td>0.8000</td>
<td>p1(T1)</td>
<td>0.050</td>
</tr>
<tr>
<td>0.9684</td>
<td>500</td>
<td>ni3</td>
<td>4</td>
<td>6</td>
<td>p1</td>
<td>C3</td>
<td>0.8000</td>
<td>p1(T1)</td>
<td>0.050</td>
</tr>
<tr>
<td>0.9927</td>
<td>500</td>
<td>ni3</td>
<td>4</td>
<td>6</td>
<td>p1</td>
<td>C4</td>
<td>0.8000</td>
<td>p1(T1)</td>
<td>0.050</td>
</tr>
<tr>
<td>0.8019</td>
<td>700</td>
<td>ni4</td>
<td>4</td>
<td>6</td>
<td>p1</td>
<td>C1</td>
<td>0.8000</td>
<td>p1(T1)</td>
<td>0.050</td>
</tr>
<tr>
<td>0.9986</td>
<td>700</td>
<td>ni4</td>
<td>4</td>
<td>6</td>
<td>p1</td>
<td>C2</td>
<td>0.8000</td>
<td>p1(T1)</td>
<td>0.050</td>
</tr>
<tr>
<td>0.9947</td>
<td>700</td>
<td>ni4</td>
<td>4</td>
<td>6</td>
<td>p1</td>
<td>C3</td>
<td>0.8000</td>
<td>p1(T1)</td>
<td>0.050</td>
</tr>
<tr>
<td>0.9994</td>
<td>700</td>
<td>ni4</td>
<td>4</td>
<td>6</td>
<td>p1</td>
<td>C4</td>
<td>0.8000</td>
<td>p1(T1)</td>
<td>0.050</td>
</tr>
<tr>
<td>0.8896</td>
<td>900</td>
<td>ni5</td>
<td>4</td>
<td>6</td>
<td>p1</td>
<td>C1</td>
<td>0.8000</td>
<td>p1(T1)</td>
<td>0.050</td>
</tr>
<tr>
<td>0.9999</td>
<td>900</td>
<td>ni5</td>
<td>4</td>
<td>6</td>
<td>p1</td>
<td>C2</td>
<td>0.8000</td>
<td>p1(T1)</td>
<td>0.050</td>
</tr>
<tr>
<td>0.9992</td>
<td>900</td>
<td>ni5</td>
<td>4</td>
<td>6</td>
<td>p1</td>
<td>C3</td>
<td>0.8000</td>
<td>p1(T1)</td>
<td>0.050</td>
</tr>
<tr>
<td>1.0000</td>
<td>900</td>
<td>ni5</td>
<td>4</td>
<td>6</td>
<td>p1</td>
<td>C4</td>
<td>0.8000</td>
<td>p1(T1)</td>
<td>0.050</td>
</tr>
</tbody>
</table>

**Footnotes**

- **T1:** 0.00, 0.20, 0.40, 0.60, 0.80, 1.00
- **M1(T1):** 0.00, 0.06, 0.12, 0.18, 0.24, 0.30
- **ni1:** 25, 25, 25, 25
- **ni2:** 75, 75, 75, 75
- **ni3:** 125, 125, 125, 125
- **ni4:** 175, 175, 175, 175
- **ni5:** 225, 225, 225, 225
- **p1(T1):** 1.0000, 0.7427, 0.6400, 0.5515, 0.4753, 0.4096
Note that the horizontal axis is scaled according to the $ni$ footnote index. Hence, 1 is for $ni1$, 2 is for $ni2$, and so on. The same is true for the $Pi$'s and the Linear Miss in the subtitle.

This example shows the huge impact that the choice of contrast coefficients can have on the power.

**Further Examples of GEE Options**

The PASS GEE procedures offer many options that allow you to investigate various designs in detail. These are available at the end of Chapter 399, “GEE Tests for the Slope of Two Groups in a Repeated Measures Design (Continuous Outcome).” We suggest that you take time to look through those examples.