Chapter 884

Latin Square Designs

Introduction

This module generates Latin Square and Graeco-Latin Square designs. Designs for three to ten treatments are available.

An introduction to experimental design is presented in Chapter 881 on Two-Level Factorial Designs and will not be repeated here.

Latin Square designs are similar to randomized block designs, except that instead of the removal of one blocking variable, these designs are carefully constructed to allow the removal of two blocking factors. They accomplish this while reducing the number of experimental units needed to conduct the experiment.

Following is an example of a four treatment Latin Square. The experimental layout is as follows:

	Columns			
Rows	**Col1**	**Col2**	**Col3**	**Col4**
Row 1	A	B	C	D
Row 2	B	C	D	A
Row 3	C	D	A	B
Row 4	D	A	B	C

In the above table, the four treatments are represented by the four letters: A, B, C, and D. The letters are arranged so that each letter occurs only once within each row and each column. Notice that a simple random design would require 4 x 4 x 4 = 64 experimental units. This Latin Square needs only 16 experimental units—a reduction of 75%!

The influence of a fourth factor may also be removed from the design by introducing a second set of letters, this time lower case. This design is known as the *Graeco-Latin Square*.

	Columns			
Rows	**Col1**	**Col2**	**Col3**	**Col4**
Row 1	Aa	Bb	Cc	Dd
Row 2	Bd	Ca	Db	Ac
Row 3	Cb	Dc	Ad	Ba
Row 4	Dc	Ad	Ba	Cb

Four factors at four levels each would normally require 256 experimental units, but this design only requires 16—a reduction in experimental units of almost 94%!

The Graeco-Latin Square is formed by combining two orthogonal Latin Squares. Graeco-Latin Squares are available for all numbers of treatments except six.

Latin Square Assumptions

It is important to understand the assumptions that are made when using the Latin Square design. The large reduction in the number of experimental units needed by this design occurs because it assumptions the magnitudes of the interaction terms are small enough that they may be ignored. That is, the Latin Square design is a main effects only design. Another way of saying this is that the treatments, the row factor, and the column factor affect the response independently of one another.

Assuming that there are no interactions is quite restrictive, so before you use this design you should be able to defend this assumption. In practice, the influence of the interactions is averaged into the experimental error of the analysis of variance table. We say that the experimental error is inflated. This results in a reduced F-ratio for testing the treatment factor, and a reduced F-ratio lessens the possibility of achieving statistical significance.

Randomization

Probability statements made during the analysis of the experimental data require strict attention to the randomization process. The randomization process is as follows:

1. Randomly select a design from the set of orthogonal designs available.
2. Randomly assign levels of the row factor to the rows.
3. Randomly assign levels of the column factor to the columns.
4. Randomly assign treatments to the treatment letters (or numbers as the case may be).

Orthogonal Sets

These designs were taken from Rao, Mitra, and Matthai (1966). We have included designs with up to ten treatments. The number of available squares depends on the number of treatments. The following table shows the number of orthogonal squares stored within this procedure.

Number of Treatments	Number of Orthogonal Designs
3	2
4	3
5	4
6	1
7	6
8	7
9	8
10	2

Graeco-Latin Squares are generated by combining two of the available orthogonal squares. Note that there are no six-level Graeco-Latin Squares.

Procedure Options

This section describes the options available in this procedure.

Design Tab

This panel specifies the parameters that will be used to create the design values.

Experimental Setup

Row Values

The values used to represent the rows are specified here. These values may be letters, digits, words, or numbers. The list is delimited by blanks or commas. The number of rows is implied by the number of items in this list. The number of row, column, and treatment values must be equal. From three to ten values are allowed.

Column Values

The values used to represent the columns are specified here. These values may be letters, digits, words, or numbers. The list is delimited by blanks or commas. The number of rows is implied by the number of items in this list. The number of row, column, and treatment values must be equal. From three to ten values are allowed.

Treatment 1 Values

The values used to represent the treatments are specified here. These values may be letters, digits, words, or numbers. The list is delimited by blanks or commas. The number of rows is implied by the number of items in this list. The number of row, column, and treatment values must be equal. From three to ten values are allowed.

Treatment 2 Values

The values used to represent the second set of treatments are specified here. These values may be letters, digits, words, or numbers. The list is delimited by commas. The number of rows is implied by the number of items in this list. The number of row, column, and treatment values must be equal. From three to ten values are allowed.

Note that this value is left blank unless you want to generate a Graeco-Latin Square.

Experimental Setup – Orthogonal Designs

Orthogonal Design Number I

Select one of the available orthogonal designs. The number of available orthogonal designs is given in the table in Orthogonal Sets section above. Good scientific protocol requires that you randomly choose which of these designs is used.

Orthogonal Design Number II

This option is only used when the Treatment 2 Values box is non-blank (when you are generating a Graeco-Latin Square). Select a second of the available orthogonal designs to be combined with the first in forming a Graeco-Latin Square. The value here must be different from the value specified in Orthogonal Design I. Good scientific protocol requires that you randomly choose which of these designs is used.

Data Storage Variables

Store Data on Spreadsheet

Check this box to generate the design data on the spreadsheet. The spreadsheet data will be identical to the design data generated on the output window.

Store First Factor In

The row values are stored in this column of the spreadsheet. The column values are stored in the column immediately to the right. The treatment values are stored in the column immediately to the right of the column column. If specified, the values of the second treatment are stored in the column immediately to the right of the first treatment column.

Warning: The program fills these column with data, so any previous data will be replaced.

Example 1 – Latin Square Design

This section presents an example of how to generate a Latin Square design using this program. **CAUTION: since the purpose of this routine is to generate data, you should begin with an empty output spreadsheet.**

In this example, we will show you how to generate a design with four treatments.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Latin Square Designs** procedure window by expanding **Design of Experiments**, then clicking on **Experimental Design**, and then clicking on **Latin Square Designs**. You may then make the appropriate entries as listed below, or open **Example 1** by going to the **File** menu and choosing **Open Example Template**.

1 Specify the design parameters.

- Set **Row Values** to **R1 R2 R3 R4**.
- Set **Column Values** to **C1 C2 C3 C4**.
- Set **Treatment 1 Values** to **A B C D**.
- Check the **Store Data on Spreadsheet** box.
- Enter **1** in the **Store First Factor In** box.

2 Run the procedure.

- From the Run menu, select **Start Calculation**. Alternatively, just click the Calculate button.

Four-Level Latin Square Design

Experimental Design

ID	Row	Column	Treatment 1
1	R1	C1	A
2	R1	C2	B
3	R1	C3	C
4	R1	C4	D
5	R2	C1	B
6	R2	C2	A
7	R2	C3	D
8	R2	C4	C
9	R3	C1	C
10	R3	C2	D
11	R3	C3	A
12	R3	C4	B
13	R4	C1	D
14	R4	C2	C
15	R4	C3	B
16	R4	C4	A

The values were also produced on the spreadsheet.

These values are also generated on the spreadsheet.