

## Chapter 868

# Multiple Regression using Effect Size

## Introduction

This procedure computes power and sample size for a multiple regression analysis in which the relationship between a dependent variable  $Y$  and a set independent variables  $X_1, X_2, \dots, X_k$  is to be studied. In multiple regression, interest usually focuses on the regression coefficients. However, since the  $X$ 's are usually not available during the planning phase, little is known about these coefficients until after the analysis is run. Hence, this procedure uses the squared multiple correlation coefficient,  $R^2$ , as the measure upon which the power analysis and sample size is based. Gatsonis and Sampson (1989) present power analysis results for two approaches: *unconditional* and *conditional*. Both of these approaches are available in this procedure.

Cohen (1988) defined an effect size  $f^2$  that is calculated from the  $R^2$  or  $\rho^2$  using the relationship

$$f^2 = \frac{R^2}{1 - R^2}$$

This procedure uses the effect size directly rather than  $R^2$  or  $\rho^2$ .

## Unconditional (Random X's) Model

In the unconditional or random  $X$ 's model, the  $X$ 's and  $Y$  have a joint multivariate normal distribution with a specified mean vector and covariance matrix given by

$$\begin{bmatrix} \sigma_Y^2 & \Sigma'_{YX} \\ \Sigma_{YX} & \Sigma_X \end{bmatrix}$$

The study-specific values of  $X$  are unknown at the design phase, so the sample size determination is based on a single, effect-size parameter which represents the expected variations in the  $X$ 's, their interrelationships, and their relationship with  $Y$ . This effect-size parameter is the *squared multiple correlation coefficient* which is defined in terms of the covariance matrix as

$$\rho_{YX}^2 = \frac{\Sigma'_{YX} \Sigma_X^{-1} \Sigma_{YX}}{\sigma_Y^2}$$

If this coefficient is zero, the variables  $X$  provide no information about the linear prediction of  $Y$ . Note that we will use  $\rho^2$  to represent  $\rho_{YX}^2$ .

The sample statistic corresponding to this parameter is  $R^2$ , the *coefficient of determination*. Often, the primary hypothesis involves testing the significance of a subset of  $X$ 's that have been statistically adjusted for a second set of  $X$ 's. The population parameter is then called the *squared multiple partial correlation coefficient*, which is interpreted similarly.

This approach is more common because usually the independent variables are random variables that are observed during the study. If the study were conducted twice, the two set of  $X$ 's would be different.

## Multiple Regression using Effect Size

### Test Statistic in the Unconditional Model

An  $F$ -test with  $k$  and  $N-k-1$  degrees of freedom can be constructed that will test whether all the regression coefficients simultaneously zero as follows

$$F_{k,N-k-1} = \frac{R^2/k}{(1-R^2)/(N-k-1)} = \frac{(N-k-1)}{k} f^2$$

Suppose the independent variables are divided into two sets: C containing  $k_C$  variables and T containing the remaining  $k_T = k - k_C$  variables. That is, we partition  $X = X_T | X_C$ . It can be shown that an  $F$ -test that tests the significance of the T variables adjusted for the C variables is

$$F_{k_T, N-k-1} = \frac{(R_{YX_T|X_C}^2)/k_T}{(1-R_{YX_T|X_C}^2)/(N-k-1)} = \frac{(N-k-1)}{k_T} f^2$$

The quantity  $R_{YX_T|X_C}^2$  is the sample estimate of the population squared multiple partial correlation coefficient  $\rho_{YX_T|X_C}^2$ .

Cohen (1988) shows that  $R_{YX_T|X_C}^2$  can be calculated from the  $R^2$  of fitting all the variables and the  $R^2$  of fitting just the set C variables as follows

$$R_{YX_T|X_C}^2 = \frac{R_{YX}^2 - R_{YX_C}^2}{1 - R_{YX_C}^2}$$

### Calculating the Power in the Unconditional Model

In the unconditional model approach, the statistical hypotheses that is usually of most interest is the set  $H_0: \rho^2 \leq \rho_0^2$  versus  $H_1: \rho^2 > \rho_0^2$  because you want to establish a lower bound for the value, not just established that it is greater than zero.

However, the hypothesis  $H_0: \rho^2 \geq \rho_0^2$  versus  $H_1: \rho^2 < \rho_0^2$  is also valid. In the program, when  $\rho_1^2 > \rho_0^2$  the former hypothesis set is assumed. Otherwise, the later set is assumed.

The calculation of the power of a particular test proceeds as follows:

1. Set  $\rho_0^2 = 0$  and  $\rho_1^2 = \frac{f^2}{1+f^2}$ .
2. Determine the critical value  $r_\alpha$  from the CDF such that  $P(R^2 \leq r_\alpha | N, k, \rho_0^2) = 1 - \alpha$ . Note that we use the value of  $\rho^2$  specified in the null hypothesis.
3. Compute the power using  $\text{Power} = 1 - P(R^2 \leq r_\alpha | N, k, \rho_1^2)$ .

Krishnamoorthy and Xia (2003) give the CDF of  $R^2$  as

$$P(R^2 \leq x | N, k, \rho^2) = \sum_{i=0}^{\infty} P(Y = i) I_x\left(\frac{k-1}{2} + i, \frac{N-k}{2}\right)$$

where

$$I_x(a, b) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \int_0^x t^{a-1} (1-t)^{b-1} dt$$

$$P(Y = i) = \frac{\Gamma\left(\frac{N+1}{2} + i\right)}{\Gamma(i+1)\Gamma\left(\frac{N+1}{2}\right)} (\rho^2)^i (1-\rho^2)^{\frac{N+1}{2}}$$

This formulation does not admit  $\rho^2 = 0$ , so when this occurs, the program inserts  $\rho^2 = 0.000000000001$ .

Finally, when computing the squared multiple *partial* correlation coefficient, Gatsonis and Sampson (1989) indicate you simply need to replace  $N$  with  $N - k_C$  in the above CDF.

## Multiple Regression using Effect Size

### Conditional (Fixed X's) Model

In this approach, the values of the X's are preset by the researchers and are assumed to be known at the planning stage. Since they are known constants, they are not treated as random variables with a probability distribution. Any hypotheses that are tested are conditional on the specific set of X values. The focus in this analysis is how much  $R^2$  increases when a certain set of independent variables is added to the regression model.

We will adopt the following notation: suppose  $C$  (controlled) and  $T$  (tested) are two, non-overlapping subsets of X's. Define  $R_{T|C}^2 = R_{TC}^2 - R_C^2$  to be the  $R^2$  added when  $Y$  is regressed on the variables in set  $T$  after adjusting for the variables in set  $C$ . Here,  $R_C^2$  is the  $R^2$  when  $Y$  is regressed on only those variables in set  $C$  and  $R_{TC}^2$  is the  $R^2$  when  $Y$  is regressed on the variables in both sets.

### Test Statistic in the Conditional Model

You can construct  $F$ -tests that will test whether the regression coefficients corresponding to certain sets of X's are simultaneously zero while controlling for other variables. For example, to test the significance of the X's in set  $T$  while removing the influence of the X's in set  $C$  from experimental error, you would use

$$F_{k_T, N-k_C-k_T-1} = \frac{(R_{T|C}^2)/k_T}{(1 - R_C^2 - R_{T|C}^2)/k_C}$$

where  $k_T$  is the number of variables in  $T$  and  $k_C$  is the number of variables in  $C$ . Most significance tests in regression analysis, correlation analysis, analysis of variance, and analysis of covariance may be constructed using these F-ratios.

### Calculating the Power in the Conditional Model

In this case, power calculations are based on the noncentral-F distribution. The calculation of the power of a particular test proceeds as follows:

1. Determine the critical value  $F_{T, N-k_T-k_C-1, \alpha}$  where  $\alpha$  is the probability of a type-I error.
2. Calculate the noncentrality parameter  $\lambda$  using the formula:
 
$$\lambda = Nf^2$$
3. Compute the power as the probability of being greater than  $F_{u, v, \alpha}$  in a noncentral- $F$  distribution with noncentrality parameter  $\lambda$ .

Note that the formula for  $\lambda$  is different from that used in *PASS 6.0*. The algorithm used in *PASS 6.0* was based on formula (9.3.1) in Cohen (1988) which gives approximate answers. This version of *PASS* using an algorithm that gives exact answers.

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## Effect Size

The above formula includes Cohen's (1988) measure of the effect size in multiple regression,  $f^2$ .

$$f^2 = \left( \frac{R_{T|C}^2}{1 - R_C^2 - R_{T|C}^2} \right)$$

Cohen (1988) defined values near 0.02 as small, near 0.15 as medium, and above 0.35 as large.

## Procedure Options

This section describes the options that are specific to this procedure. These are located on the Design tab. For more information about the options of other tabs, go to the Procedure Window chapter.

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### Design Tab

The Design tab contains most of the parameters and options that you will be concerned with.

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#### Solve For

##### Solve For

This option specifies the parameter to be solved for from the other parameters. The parameters that may be selected are *Power*, *Sample Size*, *Alpha*, and *Effect Size* ( $f^2$ ). Under most situations, you will select either *Power* or *Sample Size*.

Select *Sample Size* when you want to calculate the sample size needed to achieve a given power and alpha level.

Select *Power* when you want to calculate the power of an experiment.

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#### Power and Alpha

##### Power

This option specifies one or more values for power. Power is the probability of rejecting a false null hypothesis, and is equal to one minus Beta. Beta is the probability of a type-II error, which occurs when a false null hypothesis is not rejected.

Values must be between zero and one. Historically, the value of 0.80 (Beta = 0.20) was used for power. Now, 0.90 (Beta = 0.10) is also commonly used.

A single value may be entered here or a range of values such as *0.8 to 0.95 by 0.05* may be entered.

##### Alpha

This option specifies one or more values for the probability of a type-I error (alpha). A type-I error occurs when you reject the null hypothesis when in fact it is true.

Values of alpha must be between zero and one. Historically, the value of 0.05 has been used for alpha. This means that about one test in twenty will falsely reject the null hypothesis. You should pick a value for alpha that represents the risk of a type-I error you are willing to take in your experimental situation.

You may enter a range of values such as *0.01 0.05 0.10* or *0.01 to 0.10 by 0.01*.

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#### Sample Size

##### N (Sample Size)

This option specifies the value(s) for  $N$ , the sample size. Note that this value must be at least one greater than the total number of variables specified in sets  $C$  and  $T$ .

## Multiple Regression using Effect Size

### Effect Size

#### Regression Model Type

Choose between two approaches to the modelling and analysis of multiple regression data. Both approaches result in the same F-test for significance testing. However, they differ in the calculation of power.

- **Unconditional (Random X's)**

This approach is by far the most realistic. It assumes that Y and the X's follow a multivariate normal distribution. Thus, the values of the X's are not known until they are observed during the study. They certainly are not known during the study planning phase.

Power and sample size calculation is based on the distribution of the squared multivariate (partial) correlation coefficient. The extra variation that occurs because the X's are random variables is accounted for in the calculation.

- **Conditional (Fixed X's)**

This approach assumes that the values of the X's are known at the planning phase, usually because they are set by the researchers. This seldom happens in practice, so this approach is usually unjustifiable.

Power and sample size calculation is based on the noncentral-F distribution. It assumes that the X's values are preset by the experimenter.

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### C: Independent Variables (X's) Controlled

These options refer to the independent variables that are controlled for.

#### **$k_C$ (Number of X's Controlled)**

This option specifies the number of variables in set C, variables that are controlled for (or partialled out). This number must be greater than or equal to zero. Note that  $k_C + k_T < N - 1$ .

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### T: Independent Variables (X's) Tested

These options refer to the independent variables that are being tested for statistical significance.

#### **$k_T$ (Number of Independent Variables Tested)**

This option specifies the number of X's in the set T, variables that are tested. This number must be greater than or equal to one. Note that  $k_C + k_T < N - 1$ .

#### **$f^2$**

Enter one or more values of the effect size parameter  $f^2$ . The value must be greater than zero. Cohen (1988) defined values near 0.02 as *small*, near 0.15 as *medium*, and above 0.35 as *large*.

You can enter a single value such as *0.15* or a series of values such as *0.02 0.15 0.35* or *0.05 to 0.35 by 0.05*.

## Multiple Regression using Effect Size

## Example 1 – Finding Sample Size in the Conditional Model

Suppose researchers are planning a multiple regression study to look at the impact of a fifth independent variable on the overall  $F$  test. They want to determine the sample size requirements needed to detect a small, medium, or large effect. They want to consider power values of either 0.8 or 0.9 and a significance level is 0.05. They know the  $X$ 's in advance so they want to use the conditional model for power calculations.

### Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Multiple Regression using Effect Size** procedure window. You may then make the appropriate entries as listed below, or open **Example 1** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
<b>Design Tab</b>	
Solve For .....	<b>Sample Size</b>
Power.....	<b>0.8 0.9</b>
Alpha.....	<b>0.05</b>
Regression Model Type.....	<b>Conditional (Fixed X's)</b>
kc .....	<b>4</b>
k <sub>T</sub> .....	<b>1</b>
f <sup>2</sup> .....	<b>0.02 0.15 0.35</b>

### Annotated Output

Click the Calculate button to perform the calculations and generate the following output.

### Numeric Results

#### Numeric Results

Model: Conditional (Fixed X's)

		Independent Variables Controlled	Independent Variables Tested	Effect Size			
Power	N	kc	k <sub>T</sub>	f <sup>2</sup>	Alpha	Beta	
0.8006	395	4	1	0.020	0.050	0.1994	
0.8039	55	4	1	0.150	0.050	0.1961	
0.8012	25	4	1	0.350	0.050	0.1988	
0.9004	528	4	1	0.020	0.050	0.0996	
0.9034	73	4	1	0.150	0.050	0.0966	
0.9058	33	4	1	0.350	0.050	0.0942	

#### References

- Cohen, Jacob. 1988. *Statistical Power Analysis for the Behavioral Sciences*, Lawrence Erlbaum Associates, Hillsdale, New Jersey.
- Gatsonis, C. and Sampson, A.R. 1989. 'Multiple Correlation: Exact Power and Sample Size Calculations.' *Psychological Bulletin*, Vol. 106, No. 3, Pages 516-524.

#### Report Definitions

- Power is the probability of rejecting a false null hypothesis.
- N is the number of observations on which the multiple regression is computed.
- kc is the number of independent variables controlled.
- k<sub>T</sub> is the number of independent variables tested.
- Ind. Variables Controlled are those variables whose influence is removed from experimental error.
- Ind. Variables Tested are those variables whose regression coefficients are tested against zero.
- f<sup>2</sup> is the effect size. It is calculated using  $[R^2(T|C) / \{1 - R^2(C) - R^2(T|C)\}]$ .
- R<sup>2</sup>(C) is the R<sup>2</sup> value of only the control variables.
- R<sup>2</sup>(T|C) is the amount added to the overall R<sup>2</sup> value by the treatment variables after the control variables.

## Multiple Regression using Effect Size

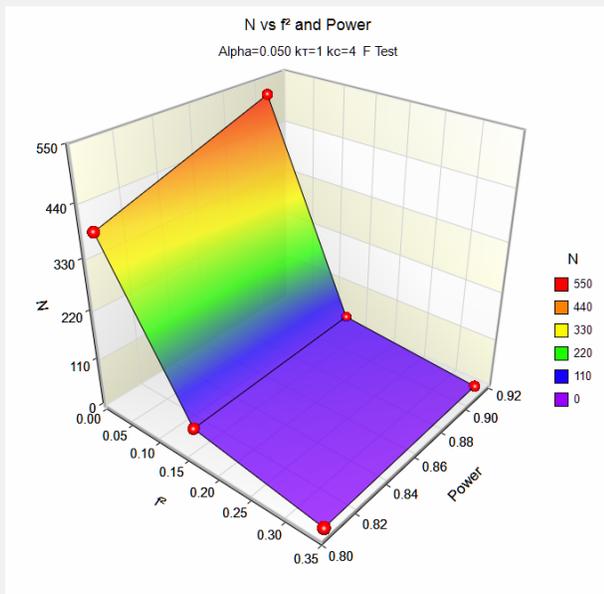
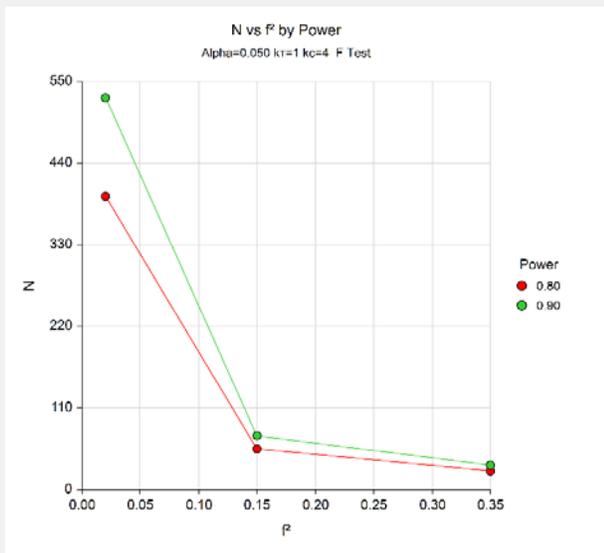
Cohen's  $f^2$  interpretation: 0.02 = small, 0.15 = medium, 0.35 = large.  
 Alpha is the probability of rejecting a true null hypothesis. It should be small.  
 Beta is the probability of accepting a false null hypothesis. It should be small.

### Summary Statements

A sample size of 395 achieves 80% power to detect an  $f^2$  of 0.020 attributable to 1 independent variable(s) using an F-Test with a significance level (alpha) of 0.050. The variables tested are adjusted for an additional 4 independent variable(s). The calculations assume a conditional (fixed X's) model.

This report shows the necessary sample sizes. The definitions of each of the columns is given in the Report Definitions section.

## Plots Section



These plots show the relationship between sample size, effect size, and power.

## Multiple Regression using Effect Size

## Example 2 – Validation

We will use an example from the *Multiple Regression* procedure to validate this procedure. Example 5 of that procedure calculates a power of 0.9683 when  $\alpha = 0.05$ ,  $N = 15$ ,  $k_T = 2$ , and  $R^2 = 0.6$ . To use this procedure, we must translate the  $R^2$  value to an  $f^2$  value. Using the relationship

$$f^2 = \left( \frac{R_{T|C}^2}{1 - R_C^2 - R_{T|C}^2} \right)$$

we find

$$f^2 = \left( \frac{0.6}{1 - 0.0 - 0.6} \right) = 1.5$$

## Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Multiple Regression using Effect Size** procedure window. You may then make the appropriate entries as listed below, or open **Example 2** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
<b>Design Tab</b>	
Solve For .....	<b>Power</b>
Alpha .....	<b>0.05</b>
N .....	<b>15</b>
Regression Model Type.....	<b>Conditional (Fixed X's)</b>
kc .....	<b>0</b>
$k_T$ .....	<b>2</b>
$f^2$ .....	<b>1.5</b>

## Output

Click the Calculate button to perform the calculations and generate the following output.

### Numeric Results

Numeric Results		Independent Variables Controlled	Independent Variables Tested	Effect Size $f^2$	Alpha	Beta
Power	N	kc	$k_T$			
0.9683	15	0	2	1.500	0.050	0.0317

The power of 0.9683 matches the result in the other procedure.