

Chapter 105

Non-Inferiority Tests for One Proportion

Introduction

This module provides power analysis and sample size calculation for non-inferiority tests in one-sample designs in which the outcome is binary. Users may choose from among five popular test statistics commonly used for running the hypothesis test.

Approximate sample size formulas for non-inferiority tests of a single proportion are presented in Chow et al. (2008) page 85. However, only large sample (normal approximation) results are given there. This procedure has the capability for computing power using both the normal approximation and binomial enumeration for all tests. Some sample size programs use only the normal approximation to the binomial distribution for power and sample size estimates. The normal approximation is accurate for large sample sizes and for proportions between 0.2 and 0.8, roughly. When the sample sizes are small or the proportions are extreme (i.e. less than 0.2 or greater than 0.8) the binomial calculations are much more accurate.

Example

A non-inferiority test example will set the stage for the discussion of the terminology that follows. Suppose that the current treatment for a disease is effective 70% of the time. Unfortunately, this treatment is expensive and occasionally exhibits serious side-effects. A promising new treatment has been developed to the point where it can be tested. One of the first questions that must be answered is whether the new treatment is as good as the current treatment. In other words, do at least 70% of treated subjects respond to the new treatment?

Because of the many benefits of the new treatment, clinicians are willing to adopt the new treatment even if it is slightly less effective than the current treatment. They must determine, however, how much less effective the new treatment can be to still be adopted. Should it be adopted if 69% respond? 68%? 65%? 60%? There is a percentage below 70% at which the difference between the two treatments is no longer considered ignorable. After thoughtful discussion with several clinicians, it was decided that if a response of at least 63% was achieved, the new treatment would be adopted. The difference between these two percentages is called the *margin of equivalence*. The margin of equivalence in this example is 7%.

The developers must design an experiment to test the hypothesis that the response rate of the new treatment is at least 0.63. The statistical hypothesis to be tested is

$$H_0: P \leq 0.63 \text{ versus } H_1: P > 0.63$$

Notice that when the null hypothesis is rejected, the conclusion is that the response rate is at least 0.63. Note that even though the response rate of the current treatment is 0.70, the hypothesis test is about a response rate of 0.63. Also notice that a rejection of the null hypothesis results in the conclusion of interest.

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Binomial Model

A binomial variable should exhibit the following four properties:

1. The variable is binary --- it can take on one of two possible values.
2. The variable is observed a known number of times. Each observation or replication is called a Bernoulli trial. The number of replications is n . The number of times that the outcome of interest is observed is r . Thus r takes on the possible values 0, 1, 2, ..., n .
3. The probability, P , that the outcome of interest occurs is constant for each trial.
4. The trials are independent. The outcome of one trial does not influence the outcome of the any other trial.

A binomial probability is calculated using the formula

$$b(r; n, P) = \binom{n}{r} P^r (1 - P)^{n-r}$$

where

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

The Hypergeometric Model

When samples are taken without replacement from a population of known size, N , the *hypergeometric* distribution should be used in place of the binomial distribution. The properties of a variable that is distributed according to the hypergeometric distribution are

1. The variable is binary--it can take on one of two possible values.
2. The variable is observed a known number of times. Each observation or replication is called a Bernoulli trial. The number of replications is n . The number of times that the outcome of interest is observed is r . Thus r takes on the possible values 0, 1, 2, ..., n .
3. The total number of items is N . The proportion of items with the characteristic of interest is P .

The hypergeometric probability of obtaining exactly r of n items with the characteristic of interest is calculated using

$$h(r; N, n, P) = \frac{\binom{NP}{r} \binom{N - NP}{n - r}}{\binom{N}{n}}$$

Note that the quantity NP is rounded to the nearest integer.

Hypothesis Testing

Parameterizations of the Proportions

In the discussion that follows, let P represent the proportion responding as a success. That is, P is the actual probability of a *success* in a binomial experiment. Let PB represent the *baseline* proportion. In a non-inferiority experiment, the baseline proportion is the response rate of the current treatment. The power of a test is computed at a specific value of the proportion. Let $P0$ represent the proportion at which the power is computed.

$P0$ represents the smallest value of P that still results in the conclusion that the new treatment is non-inferior to the current treatment. The statistical hypotheses that are tested are

$$H0: P \leq P0 \text{ versus } H1: P > P0$$

or equivalently

$$H0: P - PB \leq d0 \text{ versus } H1: P - PB > d0$$

where $d0$ is the equal to $P0 - PB$.

There are three common methods of specifying the margin of non-inferiority. The most direct is to simply assign values for PB and $P0$. However, it is often more meaningful to identify PB and then specify $P0$ implicitly by giving their difference, ratio, or odds ratio. Mathematically, the definitions of these parameterizations are

<u>Parameter</u>	<u>Computation</u>	<u>Hypotheses</u>
Difference	$d0 = P0 - PB$	$H0: P \leq PB + d0$ vs $H1: P > PB + d0$
Ratio	$r0 = P0 / PB$	$H0: P \leq r0(PB)$ vs $H1: P > r0(PB)$
Odds Ratio	$o0 = Odds0 / OddsB$	$H0: P \leq A$ vs $H1: P > A$

where $A = \frac{(o0)(PB)}{1 - PB + (o0)(PB)}$.

Difference

The difference is perhaps the most direct method of comparison between two proportions. It is easy to interpret and communicate. It gives the absolute impact of the treatment. However, there are subtle difficulties that can arise with its interpretation.

One difficulty arises when the event of interest is rare. If a difference of 0.001 occurs when the baseline probability is 0.40, it would be dismissed as being trivial. That is, there is usually little interest in a treatment that only decreases the probability from 0.400 to 0.399. However, if the baseline probability of a disease is 0.002, a 0.001 decrease would represent a reduction of 50%. Thus, interpretation of the difference depends on the baseline probability of the event. As a rule of thumb, the difference is best suited for those cases in which $0.20 < P < 0.80$.

Non-Inferiority using a Difference

The following example might help you understand the concept of a *non-inferiority* test. Suppose 60% of patients respond to the current treatment method ($PB = 0.60$). If the response rate of the new treatment is no less than five percentage points worse ($d0 = -0.05$) than the existing treatment, it will be considered non-inferior. Substituting these figures into the statistical hypotheses gives $H0: d \leq -0.05$ versus $H1: d > -0.05$. The relationship $P0 = PB + d0$ gives $H0: P \leq 0.55$ versus $H1: P > 0.55$.

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In this example, when the null hypothesis is rejected, the concluded alternative is that the response rate is at least 55%.

Ratio

The ratio $r_0 = P_0 / P_B$ gives the relative change in the probability of the response. Testing non-inferiority and superiority use the same formulation $H_0: r \leq r_0$ versus $H_1: r > r_0$.

Non-Inferiority using a Ratio

The following example might help you understand the concept of *non-inferiority* as defined by the ratio. Suppose that 60% of patients ($P_B = 0.60$) respond to the current treatment method. If a new treatment decreases the response rate by no more than 10% ($r_0 = 0.90$), it will be considered to be non-inferior to the standard treatment. Substituting these figures into the statistical hypotheses gives $H_0: r \leq 0.90$ versus $H_1: r > 0.90$. The relationship $P_0 = (r_0)(P_B)$ gives $H_0: P \leq 0.54$ versus $H_1: P > 0.54$.

In this example, when the null hypothesis is rejected, the concluded alternative is that the response rate is at least 54%. That is, the conclusion of non-inferiority is that the new treatment's response rate is no worse than 10% less than that of the standard treatment.

Odds Ratio

The odds ratio, $o_0 = (P_0 / (1 - P_0)) / (P_B / (1 - P_B))$, gives the relative change in the odds of the response. Testing non-inferiority and superiority use the same formulation $H_0: o \leq o_0$ versus $H_1: o > o_0$. The only difference is that for non-inferiority tests $o_0 < 1$, while for superiority tests $o_0 > 1$.

Test Statistics

Many different test statistics have been proposed for non-inferiority tests of a single proportion. Most of these were proposed before computers or hand calculators were widely available. Although these legacy methods are still presented in textbooks, their power and accuracy should be compared against modern exact methods before they are adopted for serious research. To make this comparison easy, the power and significance of several tests of a single proportion are available in this procedure.

Exact Test

The test statistic is r , the number of successes in n trials. This test should be the standard against which other test statistics are judged. The significance level and power are computed by enumerating the possible values of r , computing the probability of each value, and then computing the corresponding value of the test statistic. Hence the values that are reported in the output for these tests are exact, not approximate.

Z-Tests

Several z statistics have been proposed that use the central limit theorem. This theorem states that for large sample sizes, the distribution of the z statistic is approximately normal. All of these tests take the following form:

$$z = \frac{p - P_B - d_0}{s} = \frac{p - P_0}{s}$$

Although these z tests were developed because the distribution of z is approximately normal in large samples, the actual significance level and power can be computed exactly using the binomial distribution.

We include four z tests which are based on two methods for computing s and whether a continuity correction is applied.

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Z-Test using S(P0)

This test statistic uses the value of $P0$ to compute s .

$$z_1 = \frac{p - PB - d0}{\sqrt{(PB + d0)(1 - (PB + d0))/n}} = \frac{p - P0}{\sqrt{P0(1 - P0)/n}}$$

Z-Test using S(P0) with Continuity Correction

This test statistic is similar to the one above except that a continuity correction is applied to make the normal distribution more closely approximate the binomial distribution.

$$z_2 = \frac{(p - PB - d0) + c}{\sqrt{(PB + d0)(1 - (PB + d0))/n}} = \frac{(p - P0) + c}{\sqrt{P0(1 - P0)/n}}$$

where

$$c = \begin{cases} \frac{-1}{2n} & \text{if } p > P0 \\ \frac{1}{2n} & \text{if } p < P0 \\ 0 & \text{if } |p - P0| < \frac{1}{2n} \end{cases}$$

Z-Test using S(Phat)

This test statistic uses the value of p to compute s .

$$z_3 = \frac{p - PB - d0}{\sqrt{p(1 - p)/n}} = \frac{p - P0}{\sqrt{p(1 - p)/n}}$$

Z-Test using S(Phat) with Continuity Correction

This test statistic is similar to the one above except that a continuity correction is applied to make the normal distribution more closely approximate the binomial distribution.

$$z_4 = \frac{(p - PB - d0) + c}{\sqrt{p(1 - p)/n}} = \frac{(p - P0) + c}{\sqrt{p(1 - p)/n}}$$

where

$$c = \begin{cases} \frac{-1}{2n} & \text{if } p > P0 \\ \frac{1}{2n} & \text{if } p < P0 \\ 0 & \text{if } |p - P0| < \frac{1}{2n} \end{cases}$$

Power Calculation

Normal Approximation Method

Power may be calculated for one-sample proportions non-inferiority tests using the normal approximation to the binomial distribution. This section provides the power calculation formulas for the various test statistics available in this procedure. In each case, power is presented for the lower and upper one-sided hypothesis tests. In the equations that follow, $\Phi()$ represents the standard normal cumulative distribution function, and z_α represents the value that leaves α in the upper tail of the standard normal distribution. All power values are evaluated at $P = P_1$.

Exact Test

Power for the higher proportions worse hypothesis test of $H_0: P \geq P_0$ vs. $H_1: P < P_0$ is

$$Power_{ET, \text{ Lower One-Sided}} = \Phi \left(\frac{\sqrt{n}(P_0 - P_1) - z_\alpha \sqrt{P_0(1 - P_0)FPC}}{\sqrt{P_1(1 - P_1)FPC}} \right)$$

where $FPC = (N - n)/(N - 1)$ if the population size, N , is finite, and $FPC = 1$ if the population size is infinite.

Power for the higher proportions better hypothesis test of $H_0: P \leq P_0$ vs. $H_1: P > P_0$ is

$$Power_{ET, \text{ Upper One-Sided}} = 1 - \Phi \left(\frac{\sqrt{n}(P_0 - P_1) + z_\alpha \sqrt{P_0(1 - P_0)FPC}}{\sqrt{P_1(1 - P_1)FPC}} \right)$$

Z Test using S(P0)

Power for the higher proportions worse hypothesis test of $H_0: P \geq P_0$ vs. $H_1: P < P_0$ is

$$Power_{ZS(P_0), \text{ Lower One-Sided}} = \Phi \left(\frac{\sqrt{n}(P_0 - P_1) - z_\alpha \sqrt{P_0(1 - P_0)}}{\sqrt{P_1(1 - P_1)}} \right)$$

Power for the higher proportions better hypothesis test of $H_0: P \leq P_0$ vs. $H_1: P > P_0$ is

$$Power_{ZS(P_0), \text{ Upper One-Sided}} = 1 - \Phi \left(\frac{\sqrt{n}(P_0 - P_1) + z_\alpha \sqrt{P_0(1 - P_0)}}{\sqrt{P_1(1 - P_1)}} \right)$$

Z Test using S(P0) with Continuity Correction

Power for the higher proportions worse hypothesis test of $H_0: P \geq P_0$ vs. $H_1: P < P_0$ is

$$Power_{ZS(P_0)CC, \text{ Lower One-Sided}} = \Phi \left(\frac{\sqrt{n}(P_0 - P_1) - z_\alpha \sqrt{P_0(1 - P_0)} - c}{\sqrt{P_1(1 - P_1)}} \right)$$

Power for the higher proportions better hypothesis test of $H_0: P \leq P_0$ vs. $H_1: P > P_0$ is

$$Power_{ZS(P_0)CC, \text{ Upper One-Sided}} = 1 - \Phi \left(\frac{\sqrt{n}(P_0 - P_1) + z_\alpha \sqrt{P_0(1 - P_0)} + c}{\sqrt{P_1(1 - P_1)}} \right)$$

where $c = 1/2\sqrt{n}$ if $|P_1 - P_0| < 1/2n$ otherwise $c = 0$.

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Z Test using S(Phat)

Power for the higher proportions worse hypothesis test of $H_0: P \geq P_0$ vs. $H_1: P < P_0$ is

$$Power_{ZS(P_1), \text{ Lower One-Sided}} = \Phi \left(\frac{\sqrt{n}(P_0 - P_1) - z_\alpha \sqrt{P_1(1 - P_1)}}{\sqrt{P_1(1 - P_1)}} \right)$$

Power for the higher proportions better hypothesis test of $H_0: P \leq P_0$ vs. $H_1: P > P_0$ is

$$Power_{ZS(P_1), \text{ Upper One-Sided}} = 1 - \Phi \left(\frac{\sqrt{n}(P_0 - P_1) + z_\alpha \sqrt{P_1(1 - P_1)}}{\sqrt{P_1(1 - P_1)}} \right)$$

Z Test using S(Phat) with Continuity Correction

Power for the higher proportions worse hypothesis test of $H_0: P \geq P_0$ vs. $H_1: P < P_0$ is

$$Power_{ZS(P_1)CC, \text{ Lower One-Sided}} = \Phi \left(\frac{\sqrt{n}(P_0 - P_1) - z_\alpha \sqrt{P_1(1 - P_1)} - c}{\sqrt{P_1(1 - P_1)}} \right)$$

Power for the higher proportions better hypothesis test of $H_0: P \leq P_0$ vs. $H_1: P > P_0$ is

$$Power_{ZS(P_1)CC, \text{ Upper One-Sided}} = 1 - \Phi \left(\frac{\sqrt{n}(P_0 - P_1) + z_\alpha \sqrt{P_1(1 - P_1)} + c}{\sqrt{P_1(1 - P_1)}} \right)$$

where $c = 1/2\sqrt{n}$ if $|P_1 - P_0| < 1/2n$ otherwise $c = 0$.

Steps to Calculate Power using Binomial Enumeration of All Possible Outcomes

Historically, power and sample size calculations for a one-sample proportion test have been based on normal approximations to the binomial. However, with the speed of modern computers using the normal approximation is unnecessary, especially for small samples. Rather, the significance level and power can be computed using complete enumeration of all possible values of x , the number of successes in a sample of size n .

This is done as follows.

1. The critical value of the test is computed using standard techniques.
2. For each possible value of x , the value of the test statistic (z -test or exact test) is computed along with its associated probability of occurrence.
3. The significance level and power are computed by summing the probabilities of occurrence for all values of the test statistic that are greater than (or less than) the critical value. Each probability of occurrence is calculated using P_0 for the significance level and P_1 for the power.

Other variables such as the sample size are then found using an efficient search algorithm. Although this method is not as elegant as a closed-form solution, it is completely accurate.

Non-Inferiority Tests for One Proportion

Procedure Options

This section describes the options that are specific to this procedure. These are located on the Design tab. For more information about the options of other tabs, go to the Procedure Window chapter.

Design Tab

The Design tab contains the parameters associated with this test such as the proportions, sample sizes, alpha, and power.

Solve For

Solve For

This option specifies the parameter to be solved for using the other parameters. The parameters that may be selected are *Power*, *Sample Size*, and *Effect Size*.

Power Calculation

Power Calculation Method

Select the method to be used to calculate power. When the sample sizes are reasonably large (i.e. greater than 50) and the proportions are between 0.2 and 0.8 the two methods will give similar results. For smaller sample sizes and more extreme proportions (less than 0.2 or greater than 0.8), the normal approximation is not as accurate so the binomial calculations may be more appropriate.

The choices are

- **Binomial Enumeration**
Power for each test is computed using binomial enumeration of all possible outcomes when $n \leq \text{Max } n$ for Binomial Enumeration (otherwise, the normal approximation is used). Binomial enumeration of all outcomes is possible because of the discrete nature of the data.
- **Normal Approximation**
Approximate power for each test is computed using the normal approximation to the binomial distribution.

Actual alpha values are only computed when “Binomial Enumeration” is selected.

Max n for Binomial Enumeration

Only shown when Power Calculation Method = “Binomial Enumeration”

When n is less than or equal to this value, power is calculated using the binomial distribution and enumeration of all possible outcomes. This is possible because of the discrete nature of the data. “Actual Alpha” values are only computed when binomial power calculations are made.

When n is greater than this value, the normal approximation to the binomial is used when calculating power.

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Test

Higher Proportions Are

This option specifies whether proportions represent successes or failures.

- **Better**

When proportions represent successes, the higher proportions are better. In this case, a non-inferior treatment is one whose proportion is at least as high as the baseline. The alternative hypothesis of non-inferiority is $H1: P > P_0$, where P_0 is slightly less than P_B .

- **Worse**

When proportions represent failures, the lower proportions are better. In this case, a non-inferior treatment is one whose proportion is at least as low as the baseline or lower. The alternative hypothesis of non-inferiority is $H1: P < P_0$, where P_0 is slightly greater than P_B .

Test Type

Specify the type of test that will be used in reporting. Note that *C.C.* is short for *Continuity Correction*. This refers to the adding or subtracting $1/(2n)$ to (or from) the numerator of the z-value to bring the normal approximation closer to the binomial distribution.

N (Population Size)

Only shown when Test Type = "Exact Test"

Enter the total number of items in the population from which the sample of n items is selected. Enter *Infinite* to signify an infinite population so that no correction factor is applied. An *infinite* population is generally one in which the number in the population is large and unknown.

Note that N must be greater than n .

When samples are drawn from a very large (infinite) population, calculations are based on the binomial distribution.

When samples are drawn from a population of known size, specified here as N , calculations are based on the hypergeometric distribution.

Power and Alpha

Power

This option specifies one or more values for power. Power is the probability of rejecting a false null hypothesis, and is equal to one minus Beta. Beta is the probability of a type-II error, which occurs when a false null hypothesis is not rejected.

Values must be between zero and one. Historically, the value of 0.80 (Beta = 0.20) was used for power. Now, 0.90 (Beta = 0.10) is also commonly used.

A single value may be entered here or a range of values such as *0.8 to 0.95 by 0.05* may be entered.

Alpha

This option specifies one or more values for the probability of a type-I error. A type-I error occurs when a true null hypothesis is rejected.

Values must be between zero and one. Historically, the value of 0.05 has been used for alpha. This means that about one test in twenty will falsely reject the null hypothesis. You should pick a value for alpha that represents the risk of a type-I error you are willing to take in your experimental situation.

Note that because of the discrete nature of the binomial distribution, the alpha level rarely will be achieved exactly.

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A single value may be entered here or a range of values such as *0.05 to 0.2 by 0.05* may be entered.

Sample Size

n (Sample Size)

Enter a value (or range of values) for the sample size n . This is the number of individuals sampled in the study. Values must be integers greater than one.

You may a single value or a range of values such as *10, 50, 100* or *10 to 100 by 10*.

Effect Size

Input Type

Indicate what type of values to enter to specify the non-inferiority margin and effect size. Regardless of the entry type chosen, the test statistics used in the power and sample size calculations are the same. This option is simply given for convenience in specifying the non-inferiority margin and effect size.

PB (Baseline Proportion)

Enter a value (or range of values) for the baseline proportion. In a non-inferiority study, this is the response rate of the standard (existing) treatment. Note that this is not the value of P_0 . Instead, this value is used in the calculation of P_0 .

Proportions must be between zero and one.

You may enter a range of values such as *0.1 0.2 0.3* or *0.1 to 0.9 by 0.1*.

P0 (Non-Inferiority Proportion)

This option sets the smallest value which is still trivially different from PB by directly setting the value of P_0 . If 'Higher Proportions Are' is set to 'Better', specify a value of P_0 that is less than PB.

For example, if PB (baseline proportion) is 0.50, you might consider 0.49, 0.48, 0.47, and 0.46 to be close enough so that the fact that they are less than 0.50 can be overlooked. However, you might decide that if the value is 0.45 or less, the treatment is inferior. Thus, this value would be set to 0.45.

Since this value is a proportion, it must be a positive value less than one. It cannot be equal to PB.

P1 (Actual Proportion)

This is the value of the proportion (P_1) at which the power is calculated. The power calculations assume that this is the actual value of the proportion. For non-inferiority tests, this value is often set equal to PB.

Proportions must be between zero and one.

You may enter a range of values such as *0.1 0.2 0.3* or *0.1 to 0.9 by 0.1*.

d0 (Non-Inferiority Difference)

This option sets the smallest value which is still trivially different from PB by setting the difference between P_0 and PB. If 'Higher Proportions Are' is set to 'Better', specify a difference that is less than zero.

For example, if PB (baseline proportion) is 0.50, you might consider -.01, -.02, or -.04 to be small enough so that the fact that P_0 is less than 0.50 can be overlooked. However, you might decide that if the difference is -.05 or less, the treatment is inferior. Thus, this value would be set to -.05.

Since this value is a difference between two proportions, it must be between -1 and 1.

Non-Inferiority Tests for One Proportion

d1 (Actual Difference)

This option specifies the value of P1 (the proportion at which the power is calculated) by specifying the difference between the two proportions, P1 and PB. This difference is used with PB to calculate the value of P1 using the formula, $P1 = PB + \text{Difference}$. This value is often set equal to zero.

Differences must be between -1 and 1.

You may enter a range of values such as *.03 .05 .10* or *.01 to .05 by .01*.

r0 (Non-Inferiority Ratio)

This option sets the smallest value which is still trivially different from PB by setting the ratio of P0 to PB. If 'Higher Proportions Are' is set to 'Better', specify a ratio that is less than one.

For example, if PB (baseline proportion) is 0.50, you might consider ratios of 0.99, 0.98, or even 0.96 to be small enough so that the fact that P0 is less than PB can be overlooked (the difference is trivial). However, you might decide that if the ratio is 0.95 or less, the treatment is inferior. Thus, this value would be set to 0.95.

Since this value is a ratio between two proportions, it must be positive. Since it is a margin, it cannot be one. It cannot be so large that the calculated value of P0 is greater than one.

r1 (Actual Ratio)

This option specifies the value of P1 (the actual proportion) by specifying the ratio between the two proportions, P1 and PB. This ratio is used with PB to calculate the value of P1 using the formula, $P1 = (PB)(\text{Ratio})$. This value is often set equal to one.

Ratios must greater than zero. Note that the ratios must be small enough so that P1 is less than one.

You may enter a range of values such as *.5 .6 .7 .8* or *1.25 to 2.0 by .25*.

o0 (Non-Inferiority Odds Ratio)

This option sets the smallest value that is still trivially different from PB by setting the odds ratio of P0 and PB. If 'Higher Proportions Are' is set to 'Better', specify a ratio that is less than one.

For example, if PB (baseline proportion) is 0.50, you might consider odds ratios of 0.99, 0.90, or even 0.81 to be small enough so that the fact that P0 is less than PB can be overlooked (the difference is trivial). However, you might decide that if the odds ratio is 0.80 or less, the treatment is inferior. Thus, this value would be set to 0.80.

Since this value is a ratio between two odds, it must be positive. Since it is a margin, it cannot be one.

o1 (Actual Odds Ratio)

This option specifies the value of P1 (the actual proportion) by specifying the odds ratio between the two proportions, P1 and PB. This ratio is used with PB to calculate the value of P1. For non-inferiority tests, this value is often set equal to one.

Odds ratios must greater than zero.

You may enter a range of values such as *.5 .6 .7 .8* or *1.25 to 2.0 by .25*.

Non-Inferiority Tests for One Proportion

Example 1 – Finding the Power

Suppose 50% of patients with a certain type of cancer survive two years using the current treatment. The current treatment is expensive and has several severe side effects. A new treatment has fewer side effects and is less expensive. A non-inferiority trial is to be conducted to show that the two-year survival rate of the new treatment is as good as the current treatment. After serious consideration, the margin of non-inferiority is set at 5%. What power will be achieved by sample sizes of 50, 100, 200, 300, 500, or 800 and a significance level of 0.05? For comparative purposes, also calculate the power for a margin of non-inferiority of 10%. Assume that the true survival rate of the new treatment is the same as that of the current (baseline) treatment.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Non-Inferiority Tests for One Proportion** procedure window by expanding **Proportions**, then **One Proportion**, then **Non-Inferiority**, and then clicking on **Non-Inferiority Tests for One Proportion**. You may then make the appropriate entries as listed below, or open **Example 1** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
Design Tab	
Solve For	Power
Power Calculation Method.....	Normal Approximation
Higher Proportions Are	Better
Test Type.....	Z-test using S(P0)
Alpha.....	0.05
n (Sample Size)	50 100 200 300 500 800
Input Type.....	Differences
PB (Baseline Proportion)	0.50
d0 (Non-Inferiority Difference)	-0.10 -0.05
d1 (Actual Difference).....	0

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results for Testing Non-Inferiority of One Proportion using the Z-Test with S(P0)

Higher Proportions are Better

Alternative Hypothesis: One-Sided (H0: $P \leq P_0$ vs. H1: $P > P_0$)

Power*	n	Baseline Proportion PB	Non-Inf. Difference d0	Actual Difference d1	Alpha	Reject H0 If Z >
0.42175	50	0.5000	-0.1000	0.0000	0.0500	1.6449
0.17631	50	0.5000	-0.0500	0.0000	0.0500	1.6449
0.65113	100	0.5000	-0.1000	0.0000	0.0500	1.6449
0.26219	100	0.5000	-0.0500	0.0000	0.0500	1.6449
0.88816	200	0.5000	-0.1000	0.0000	0.0500	1.6449
0.41200	200	0.5000	-0.0500	0.0000	0.0500	1.6449
0.96802	300	0.5000	-0.1000	0.0000	0.0500	1.6449
0.53802	300	0.5000	-0.0500	0.0000	0.0500	1.6449
0.99789	500	0.5000	-0.1000	0.0000	0.0500	1.6449
0.72557	500	0.5000	-0.0500	0.0000	0.0500	1.6449
0.99997	800	0.5000	-0.1000	0.0000	0.0500	1.6449
0.88333	800	0.5000	-0.0500	0.0000	0.0500	1.6449

Non-Inferiority Tests for One Proportion

* Power was computed using the normal approximation method.

References

- Blackwelder, W.C. 1998. 'Equivalence Trials.' In Encyclopedia of Biostatistics, John Wiley and Sons. New York. Volume 2, 1367-1372.
- Chow, S. C., Shao, J., and Wang, H. 2008. Sample Size Calculations in Clinical Research, Second Edition. Chapman & Hall/CRC. Boca Raton, Florida.
- Fleiss, J. L., Levin, B., and Paik, M.C. 2003. Statistical Methods for Rates and Proportions. Third Edition. John Wiley & Sons. New York.

Report Definitions

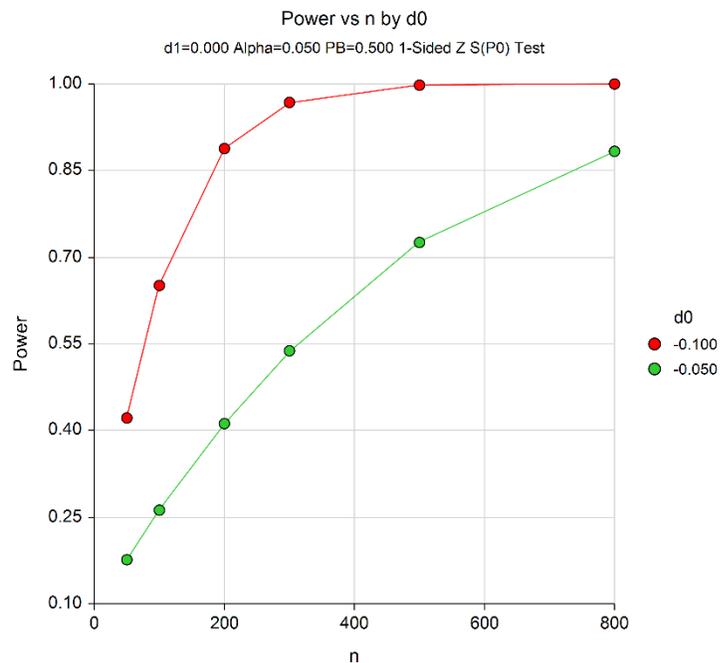
Power is the probability of rejecting the null hypothesis when it is false. It should be close to one.
 n is the size of the sample drawn from the population. To conserve resources, it should be as small as possible.
 PB is the baseline or standard value of the proportion.
 $d_0 = P_0 - PB$ is the distance below PB that is still considered non-inferior.
 $d_1 = P_1 - PB$ is the value of the difference at which the power is calculated.
 α (significance level) is the probability of rejecting the null hypothesis when it is true. It should be small.
 Reject H_0 if... gives the critical value(s) for the test.

Summary Statements

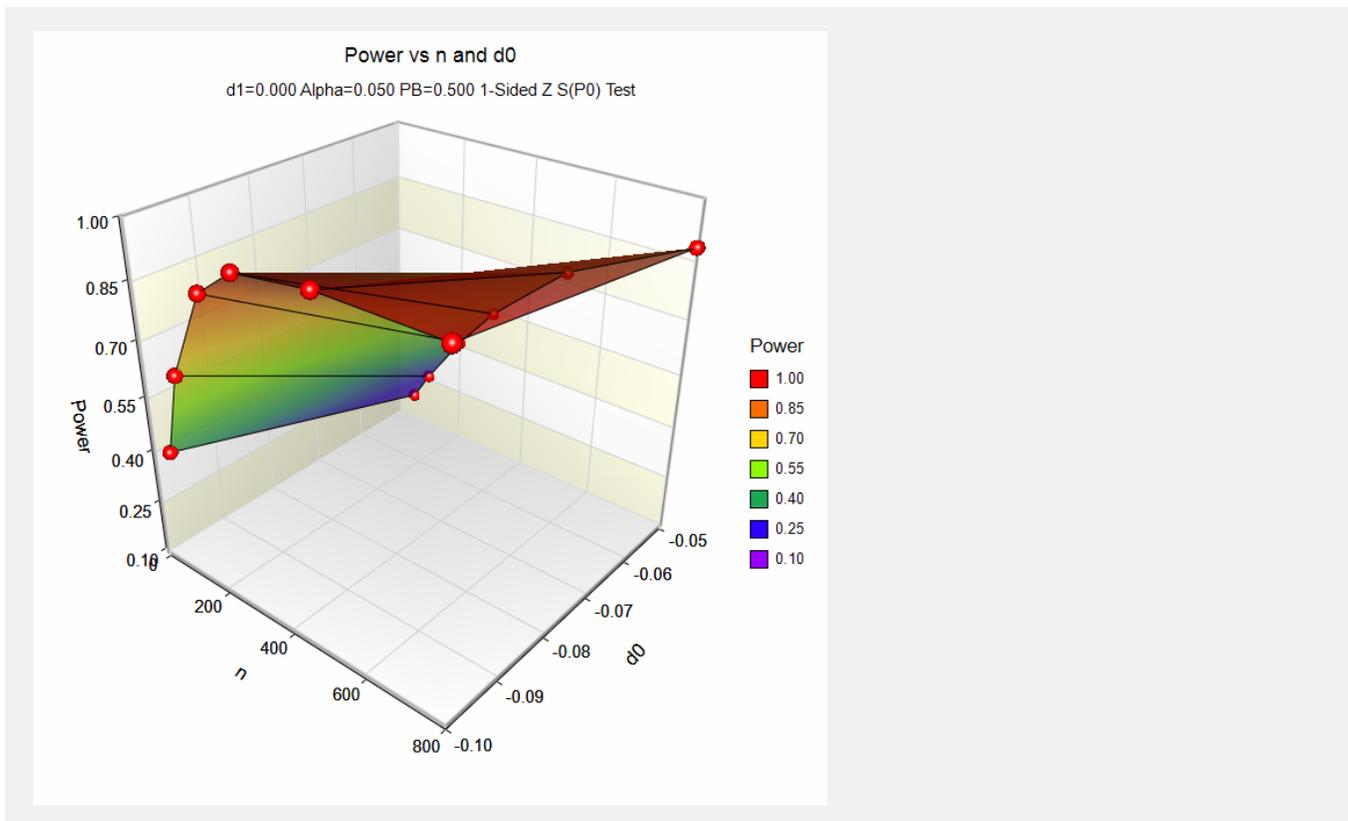
A sample size of 50 achieves 42.175% power to detect a non-inferiority difference ($P_0 - PB$) of -0.1000 using a one-sided Z-test that uses $S(P_0)$ to estimate the standard deviation with a significance level (α) of 0.0500. These results assume a baseline proportion (PB) of 0.5000 and that the actual difference ($P_1 - PB$) is 0.0000.

This report shows the values of each of the parameters, one scenario per row. The values from this table are plotted in the chart below.

Chart Section



Non-Inferiority Tests for One Proportion



These plots show the relationship between power, sample size, and the trivial difference. Note that 90% power is achieved with an n of about 200 when the trivial difference is $-.10$ and about 800 when the trivial difference is -0.05 .

Non-Inferiority Tests for One Proportion

Example 2 – Finding the Sample Size

Continuing with Example 1, suppose you want to find the exact sample size necessary to achieve 90% power when the non-inferiority difference is -0.05.

In this example we'll show you how to setup the calculation by inputting proportions, differences, ratios, and odds ratios. In all cases, you'll see that the sample sizes are exactly the same. The only difference is in the way the non-inferiority margin and effect size are specified.

Setup (Proportions)

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Non-Inferiority Tests for One Proportion** procedure window by expanding **Proportions**, then **One Proportion**, then **Non-Inferiority**, and then clicking on **Non-Inferiority Tests for One Proportion**. You may then make the appropriate entries as listed below, or open **Example 2a** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
Design Tab	
Solve For	Sample Size
Power Calculation Method.....	Normal Approximation
Higher Proportions Are	Better
Test Type.....	Z-test using S(P0)
Power.....	0.90
Alpha.....	0.05
Input Type.....	Proportions
P0 (Non-Inferiority Proportion).....	0.45
P1 (Actual Proportion)	0.5

Output (Proportions)

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results for Testing Non-Inferiority of One Proportion using the Z-Test with S(P0)

Higher Proportions are Better

Alternative Hypothesis: One-Sided (H0: $P \leq P0$ vs. H1: $P > P0$)

Power*	n	Non-Inf. Proportion P0	Actual Proportion P1	Alpha	Reject H0 If Z >
0.90013	852	0.4500	0.5000	0.0500	1.6449

* Power was computed using the normal approximation method.

This report shows that a sample size of 852 will be necessary to achieve the design requirements.

Non-Inferiority Tests for One Proportion

Setup (Differences)

To input $P_0 = 0.45$ and $P_1 = 0.5$ using differences, recall the relationships between P_0 , P_1 , PB , d_0 and d_1 :

$$d_0 = P_0 - PB = 0.45 - 0.5 = -0.05$$

$$d_1 = P_1 - PB = 0.5 - 0.5 = 0$$

The setup for differences is exactly the same as that for proportions except for the following inputs on the Design tab. You may then make the appropriate entries as listed below, or open **Example 2b** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
Design Tab	
Input Type	Differences
PB (Baseline Proportion)	0.50
d_0 (Non-Inferiority Difference)	-0.05
d_1 (Actual Difference)	0

Output (Differences)

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results for Testing Non-Inferiority of One Proportion using the Z-Test with S(P0) Higher Proportions are Better Alternative Hypothesis: One-Sided (H0: $P \leq P_0$ vs. H1: $P > P_0$)						
Power*	n	Baseline Proportion PB	Non-Inf. Difference d0	Actual Difference d1	Alpha	Reject H0 If Z >
0.90013	852	0.5000	-0.0500	0.0000	0.0500	1.6449
* Power was computed using the normal approximation method.						

This report shows the same sample size when the difference $d_0 = -0.05$ is entered as the non-inferiority difference and $d_1 = 0.0$ is entered as the actual difference.

Setup (Ratios)

To input $P_0 = 0.45$ and $P_1 = 0.5$ using ratios, recall the relationships between P_0 , P_1 , PB , r_0 and r_1 :

$$r_0 = P_0/PB = 0.45/0.5 = 0.9$$

$$r_1 = P_1/PB = 0.5/0.5 = 1$$

The setup for ratios is exactly the same as that for proportions except for the following inputs on the Design tab. You may then make the appropriate entries as listed below, or open **Example 2c** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
Design Tab	
Input Type	Ratios
PB (Baseline Proportion)	0.50
r_0 (Non-Inferiority Ratio)	0.9
r_1 (Actual Ratio)	1

Non-Inferiority Tests for One Proportion

Output (Ratios)

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results for Testing Non-Inferiority of One Proportion using the Z-Test with S(P0)

Higher Proportions are Better

Alternative Hypothesis: One-Sided (H0: $P \leq P_0$ vs. H1: $P > P_0$)

Power*	n	Baseline Proportion PB	Non-Inf. Ratio r0	Actual Ratio r1	Alpha	Reject H0 If Z >
0.90013	852	0.5000	0.9000	1.0000	0.0500	1.6449

* Power was computed using the normal approximation method.

This report shows the same sample size when the ratio $r_0 = 0.9$ is entered as the non-inferiority ratio and $r_1 = 1$ is entered as the actual ratio.

Setup (Odds Ratios)

To input $P_0 = 0.45$ and $P_1 = 0.5$ using odds ratios, recall the relationships between P_0 , P_1 , P_B , o_0 and o_1 :

$$o_0 = \text{odds}_0 / \text{odds}_B = [0.45 / (1 - 0.45)] / [0.5 / (1 - 0.5)] = 0.8181818$$

$$o_1 = \text{odds}_1 / \text{odds}_B = [0.5 / (1 - 0.5)] / [0.5 / (1 - 0.5)] = 1$$

The setup for odds ratios is exactly the same as that for proportions except for the following inputs on the Design tab. You may then make the appropriate entries as listed below, or open **Example 2d** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
Design Tab	
Input Type.....	Odds Ratios
PB (Baseline Proportion)	0.50
o_0 (Non-Inferiority Odds Ratio).....	0.8181818
o_1 (Actual Odds Ratio)	1

Output (Odds Ratios)

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results for Testing Non-Inferiority of One Proportion using the Z-Test with S(P0)

Higher Proportions are Better

Alternative Hypothesis: One-Sided (H0: $P \leq P_0$ vs. H1: $P > P_0$)

Power*	n	Baseline Proportion PB	Non-Inf. Odds Ratio o0	Actual Odds Ratio o1	Alpha	Reject H0 If Z >
0.90013	852	0.5000	0.8182	1.0000	0.0500	1.6449

* Power was computed using the normal approximation method.

This report shows the same sample size when the ratio $o_0 = 0.8181818$ is entered as the non-inferiority odds ratio and $o_1 = 1$ is entered as the actual odds ratio.

Non-Inferiority Tests for One Proportion

Example 3 – Comparing Test Statistics

One important decision that must be made before conducting the experiment is to decide which of the available test statistics to use. This procedure makes it easy to make this comparison. The differences in the characteristics of test statistics are most noticeable in small samples. Hence, the investigation done here is for sample sizes of 20 to 200 in steps of 20. The trivial difference will be set to -0.10. We'll calculate exact power using binomial enumeration when the baseline proportion (PB) is 0.5, the actual difference (d1) is 0.0, and alpha = 0.05.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Non-Inferiority Tests for One Proportion** procedure window by expanding **Proportions**, then **One Proportion**, then **Non-Inferiority**, and then clicking on **Non-Inferiority Tests for One Proportion**. You may then make the appropriate entries as listed below, or open **Example 3** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
Design Tab	
Solve For	Power
Power Calculation Method.....	Binomial Enumeration
Max n for Binomial Enumeration	10000
Higher Proportions Are	Better
Test Type.....	Z-test using S(P0)
Alpha.....	0.05
n (Sample Size)	20 to 200 by 20
Input Type.....	Differences
PB (Baseline Proportion)	0.50
d0 (Non-Inferiority Difference)	-0.10
d1 (Actual Difference).....	0
Reports Tab	
Show Comparative Reports.....	Checked
Comparative Plots Tab	
Show Comparative Plots	Checked

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Power Comparison of Five Different Non-Inferiority Tests for One Proportion

Higher Proportions are Better

Alternative Hypothesis: One-Sided (H0: $P \leq P_0$ vs. H1: $P > P_0$)

n	Baseline Prop. PB	Non-Inf. Diff. d0	Actual Diff. d1	Target Alpha	Exact Test Power	Z-Test S(P0) Power	Z-Test S(P0)C Power	Z-Test S(P) Power	Z-Test S(P)C Power
20	0.5000	-0.1000	0.0000	0.0500	0.13159	0.25172	0.13159	0.25172	0.13159
40	0.5000	-0.1000	0.0000	0.0500	0.31791	0.31791	0.31791	0.31791	0.31791
60	0.5000	-0.1000	0.0000	0.0500	0.44871	0.44871	0.44871	0.44871	0.44871
80	0.5000	-0.1000	0.0000	0.0500	0.54446	0.54446	0.54446	0.54446	0.54446
100	0.5000	-0.1000	0.0000	0.0500	0.61782	0.61782	0.61782	0.61782	0.61782
120	0.5000	-0.1000	0.0000	0.0500	0.67587	0.73850	0.67587	0.73850	0.67587
140	0.5000	-0.1000	0.0000	0.0500	0.72287	0.77651	0.72287	0.77651	0.72287

Non-Inferiority Tests for One Proportion

160	0.5000	-0.1000	0.0000	0.0500	0.80772	0.80772	0.80772	0.80772	0.80772
180	0.5000	-0.1000	0.0000	0.0500	0.83371	0.86825	0.83371	0.83371	0.83371
200	0.5000	-0.1000	0.0000	0.0500	0.88538	0.88538	0.88538	0.88538	0.85559

Note: Power was computed using binomial enumeration of all possible outcomes.

Actual Alpha Comparison of Five Different Non-Inferiority Tests for One Proportion

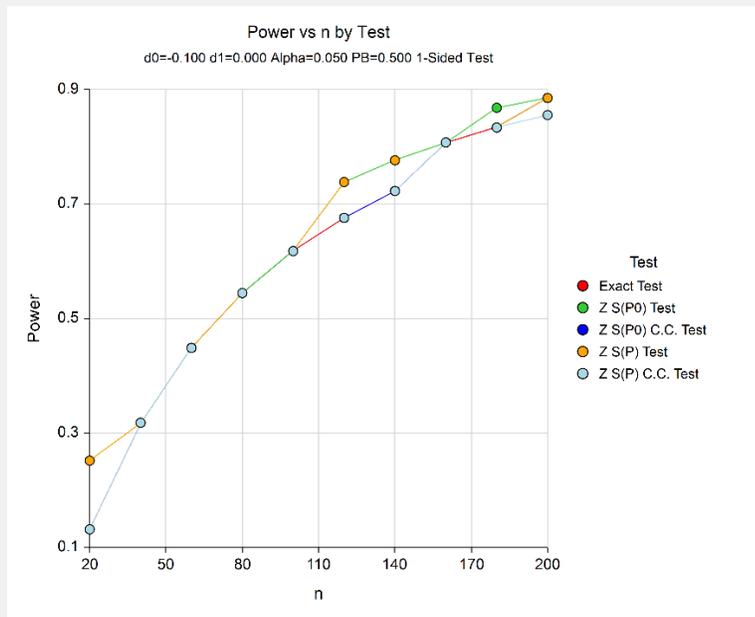
Higher Proportions are Better

Alternative Hypothesis: One-Sided (H0: P ≤ P0 vs. H1: P > P0)

n	Baseline Prop. PB	Non-Inf. Diff. d0	Actual Diff. d1	Target Alpha	Exact Test Alpha	Z-Test S(P0) Alpha	Z-Test S(P0)C Alpha	Z-Test S(P) Alpha	Z-Test S(P)C Alpha
20	0.5000	-0.1000	0.0000	0.0500	0.0210	0.0565	0.0210	0.0565	0.0210
40	0.5000	-0.1000	0.0000	0.0500	0.0392	0.0392	0.0392	0.0392	0.0392
60	0.5000	-0.1000	0.0000	0.0500	0.0445	0.0445	0.0445	0.0445	0.0445
80	0.5000	-0.1000	0.0000	0.0500	0.0445	0.0445	0.0445	0.0445	0.0445
100	0.5000	-0.1000	0.0000	0.0500	0.0423	0.0423	0.0423	0.0423	0.0423
120	0.5000	-0.1000	0.0000	0.0500	0.0392	0.0575	0.0392	0.0575	0.0392
140	0.5000	-0.1000	0.0000	0.0500	0.0358	0.0514	0.0358	0.0514	0.0358
160	0.5000	-0.1000	0.0000	0.0500	0.0459	0.0459	0.0459	0.0459	0.0459
180	0.5000	-0.1000	0.0000	0.0500	0.0408	0.0558	0.0408	0.0558	0.0408
200	0.5000	-0.1000	0.0000	0.0500	0.0492	0.0492	0.0492	0.0492	0.0363

Note: Actual alpha was computed using binomial enumeration of all possible outcomes.

Chart Section



The first numeric report shows the power for each test statistic. The second shows the actual alpha achieved by the design.

An examination of the first report shows that the power is often different for at least one of the tests. Also notice that the exact test always has the minimum power in each row. This would lead us to discard this test statistic. However, consider the second report which shows the actual alpha level (the target was 0.05) for each test. By inspecting corresponding entries in both tables, we see that whenever a test statistic achieves a better power than the exact test, it also yields an actual alpha level larger than the target alpha.

For example, look at the powers for n = 20. The z-test using s(phat) has an unusually large power = 0.25172. This is a much larger power than the exact test's value of 0.13159. However, note that the actual alpha for this test is 0.0565 which is larger than the target alpha of 0.05.

We conclude that indeed, the exact test is consistently the best test since it always achieves a significance level that is less than the target value.

Non-Inferiority Tests for One Proportion

Example 4 – Comparing Power Calculation Methods

Continuing with Example 3, let's see how the results compare if we were to use approximate power calculations instead of power calculations based on binomial enumeration.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Non-Inferiority Tests for One Proportion** procedure window by expanding **Proportions**, then **One Proportion**, then **Non-Inferiority**, and then clicking on **Non-Inferiority Tests for One Proportion**. You may then make the appropriate entries as listed below, or open **Example 4** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
Design Tab	
Solve For	Power
Power Calculation Method.....	Binomial Enumeration
Max n for Binomial Enumeration	10000
Higher Proportions Are	Better
Test Type.....	Z-test using S(P0)
Alpha.....	0.05
n (Sample Size)	20 to 200 by 20
Input Type.....	Differences
PB (Baseline Proportion)	0.50
d0 (Non-Inferiority Difference)	-0.10
d1 (Actual Difference).....	0
Reports Tab	
Show Power Detail Report	Checked

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results and Plots

Power Detail Report for Testing Non-Inferiority of One Proportion using the Z-Test with S(P0)

Higher Proportions are Better

Alternative Hypothesis: One-Sided (H0: $P \leq P_0$ vs. H1: $P > P_0$)

n	PB	d0	d1	Normal Approximation		Binomial Enumeration	
				Power	Alpha	Power	Alpha
20	0.5000	-0.1000	0.0000	0.23663	0.0500	0.25172	0.0565
40	0.5000	-0.1000	0.0000	0.36440	0.0500	0.31791	0.0392
60	0.5000	-0.1000	0.0000	0.47511	0.0500	0.44871	0.0445
80	0.5000	-0.1000	0.0000	0.57034	0.0500	0.54446	0.0445
100	0.5000	-0.1000	0.0000	0.65113	0.0500	0.61782	0.0423
120	0.5000	-0.1000	0.0000	0.71880	0.0500	0.73850	0.0575
140	0.5000	-0.1000	0.0000	0.77482	0.0500	0.77651	0.0514
160	0.5000	-0.1000	0.0000	0.82074	0.0500	0.80772	0.0459
180	0.5000	-0.1000	0.0000	0.85806	0.0500	0.86825	0.0558
200	0.5000	-0.1000	0.0000	0.88816	0.0500	0.88538	0.0492

Notice that the power values are pretty consistent for both methods. For some sample sizes, however, the actual alpha level computed using binomial enumeration is greater than the target of 0.05.

Non-Inferiority Tests for One Proportion

Example 5 – Finding Power after an Experiment

The proportion of successes of the current treatment is known to be 0.74 based on years of treatment use. Researchers have developed a new method of treatment which costs about half the current treatment price. Before the new treatment can be approved it must be shown that the success of the proposed treatment is not inferior to that of the current treatment. It is determined that use of the new treatment is justifiable if it is shown that it is effective more than 70% of the time. Sixty individuals are randomly selected to receive the new method of treatment. Forty-three (71.67%) of the 60 individuals responded positively to the treatment. The p-value for the test based on exact binomial probabilities is 0.4514. Because the researchers were unable to show the new treatment is non-inferior, they desire to know the power of the test.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Non-Inferiority Tests for One Proportion** procedure window by expanding **Proportions**, then **One Proportion**, then **Non-Inferiority**, and then clicking on **Non-Inferiority Tests for One Proportion**. You may then make the appropriate entries as listed below, or open **Example 5** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
Design Tab	
Solve For	Power
Power Calculation Method.....	Binomial Enumeration
Max n for Binomial Enumeration	10000
Higher Proportions Are	Better
Test Type.....	Exact Test
N (Population Size).....	Infinite
Alpha.....	0.05
n (Sample Size)	60
Input Type.....	Differences
PB (Baseline Proportion)	0.74
d0 (Non-Inferiority Difference)	-0.04
d1 (Actual Difference)	0

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results**Numeric Results for Testing Non-Inferiority of One Proportion using the Exact Test**

Higher Proportions are Better

Alternative Hypothesis: One-Sided (H0: $P \leq P_0$ vs. H1: $P > P_0$)

		Baseline Proportion PB	Non-Inf. Difference d0	Actual Difference d1	Target Alpha	Actual Alpha*	Reject H0 If $R \geq$
Power*	n	0.7400	-0.0400	0.0000	0.0500	0.0295	49
0.11120	60						

* Power and actual alpha were computed using binomial enumeration of all possible outcomes.

This report shows that the power for the test was only 0.11120.

Non-Inferiority Tests for One Proportion

Example 6 – Finding Sample Size based on the Odds Ratio

The odds for success of the current treatment is known be 4.31. A new treatment is developed to compete with the current treatment with respect to cost and reduction in side effects. It must be shown to be non-inferior to the current treatment. The researchers want to determine the sample size necessary to achieve 80% power in this test of non-inferiority. The researchers determine that the new treatment will be considered non-inferior if the odds for success are no less than 90% the odds for success of the current treatment. The baseline proportion is calculated as $PB = \text{odds}/(1+\text{odds}) = 4.31/(1+4.31) = 0.8117$.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Non-Inferiority Tests for One Proportion** procedure window by expanding **Proportions**, then **One Proportion**, then **Non-Inferiority**, and then clicking on **Non-Inferiority Tests for One Proportion**. You may then make the appropriate entries as listed below, or open **Example 6** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
Design Tab	
Solve For	Sample Size
Power Calculation Method.....	Binomial Enumeration
Max n for Binomial Enumeration	10000
Higher Proportions Are	Better
Test Type.....	Exact Test
N (Population Size).....	Infinite
Power.....	0.80
Alpha.....	0.05
Input Type.....	Odds Ratios
PB (Baseline Proportion)	0.8117
o0 (Non-Inferiority Odds Ratio).....	0.9
o1 (Actual Odds Ratio)	1.0

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results**Numeric Results for Testing Non-Inferiority of One Proportion using the Exact Test**

Higher Proportions are Better

Alternative Hypothesis: One-Sided (H0: $P \leq P_0$ vs. H1: $P > P_0$)

		Baseline Proportion PB	Non-Inf. Odds Ratio o0	Actual Odds Ratio o1	Target Alpha	Actual Alpha*	Reject H0 If R \geq
Power*	n	0.8117	0.9000	1.0000	0.0500	0.0499	2860
0.80041	3547						

* Power and actual alpha were computed using binomial enumeration of all possible outcomes.

A sample size of 3547 is required to show non-inferiority under these conditions.

Non-Inferiority Tests for One Proportion

Example 7 – Validation using Chow, Shao, and Wang (2008)

Chow, Shao, and Wang (2008) page 87 gives the result of a sample size calculation for the z-test with S(Phat). They calculate a sample size of 18 when $\alpha = 0.05$, power = 0.80, PB = 0.30, non-inferiority difference = -0.10, and actual difference = 0.20 (such that actual proportion = 0.50).

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Non-Inferiority Tests for One Proportion** procedure window by expanding **Proportions**, then **One Proportion**, then **Non-Inferiority**, and then clicking on **Non-Inferiority Tests for One Proportion**. You may then make the appropriate entries as listed below, or open **Example 7** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
Design Tab	
Solve For	Sample Size
Power Calculation Method.....	Normal Approximation
Higher Proportions Are	Better
Test Type.....	Z-test using S(Phat)
Power.....	0.80
Alpha.....	0.05
Input Type.....	Differences
PB (Baseline Proportion)	0.30
d0 (Non-Inferiority Difference)	-0.10
d1 (Actual Difference)	0.20

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results**Numeric Results for Testing Non-Inferiority of One Proportion using the Z-Test with S(Phat)**

Higher Proportions are Better

Alternative Hypothesis: One-Sided (H0: $P \leq P_0$ vs. H1: $P > P_0$)

		Baseline Proportion PB	Non-Inf. Difference d0	Actual Difference d1	Alpha	Reject H0 If Z >
Power*	n					
0.81613	18	0.3000	-0.1000	0.2000	0.0500	1.6449

* Power was computed using the normal approximation method.

PASS also calculated a sample size of 18.