PASS Sample Size Software NCSS.com

Chapter 171

Non-Inferiority Tests for the Odds Ratio of Two Proportions in a 2x2 Cross-Over Design

Introduction

Senn (2002) defines a *cross-over* design as one in which each subject receives all treatments and the objective is to study differences among the treatments. The name *cross-over* comes from the most common case in which there are only two treatments. In this case, each subject *crosses over* from one treatment to the other. It is assumed that there is a *washout* period between treatments during which the response returns back to its baseline value. If this does not occur, there is said to be a *carry-over* effect.

A 2×2 cross-over design contains to two *sequences* (treatment orderings) and two time periods (occasions). One sequence receives treatment A followed by treatment B. The other sequence receives B and then A. The design includes a washout period between responses to make certain that the effects of the first drug do not carry over to the second. Thus, the groups in this design are defined by the sequence in which the drugs are administered, not by the treatments they receive. Indeed, higher-order cross-over designs have been used in which the same treatment is used at both occasions.

Cross-over designs are employed because, if the no-carryover assumption is met, treatment differences are measured within a subject rather than between subjects—making a more precise measurement. Examples of the situations that might use a cross-over design are the comparison of anti-inflammatory drugs in arthritis and the comparison of hypotensive agents in essential hypertension. In both cases, symptoms are expected to return to their usual baseline level shortly after the treatment is stopped.

The sample size calculations in the procedure are based on the formulas presented in Lui (2016).

Advantages of Cross-Over Designs

A comparison of treatments on the same subject is expected to be more precise. The increased precision often translates into a smaller sample size. Also, patient enrollment into the study may be easier because each patient will receive both treatments. Finally, it is often more difficult to obtain a subject than to obtain a measurement.

Disadvantages of Cross-Over Designs

The statistical analysis of a cross-over experiment is more complex than a parallel-group experiment and requires additional assumptions. It may be difficult to separate the treatment effect from the period effect, the carry-over effect of the previous treatment, and the interaction between period and treatment.

The design cannot be used when the treatment (or the measurement of the response) alters the subject permanently. Hence, it should not be used to compare treatments that are intended to provide a cure.

Because subjects must be measured at least twice, it is often more difficult to keep patients enrolled in the study. It is arguably simpler to measure a subject once than to obtain their measurement twice. This is particularly true when the measurement process is painful, uncomfortable, embarrassing, or time consuming.

Technical Details

The 2×2 crossover design may be described as follows. Randomly assign the subjects to one of two sequence groups so that there are n_1 subjects in sequence one and n_2 subjects in sequence two. In order to achieve design balance, the sample sizes n_1 and n_2 are assumed to be equal so that $n_1 = n_2 = n = N/2$.

Sequence one is given the control (A) followed by the treatment (B). Sequence two is given the treatment (B) followed by the control (A).

The design can be analyzed using a simple *z*-test if we ignore period and sequence effects or using a more complex random effects logistic regression model that adjusts for period and sequence effects. The sample size calculations herein ignore period and sequence effects. Julious (2010) suggests on page 175 that the bias due to ignoring period effects if a period-adjusted analysis is planned is not great and that sample size calculations that ignore period effects are adequate.

Cross-Over Design

The discussions that follow summarize the results in Lui (2016). Consider a 2×2 cross-over design and let $x_{ij}^{(g)}$ represent the binary response (0 or 1) from the j^{th} subject, $j = 1, ..., n_g$, in the i^{th} period (i = 1, 2), in sequence g(g = 1, 2). Let $n_{rc}^{(g)}$ represent the number of subjects among n_g subjects in sequence g with the response vector ($x_{1j} = r, x_{2j} = c$). We can then summarize the results in terms of counts from a cross-over design with the following table for sequences 1 and 2 as

SEQUENCE 1 (Control → Treatment)

		(7	Period 2 Freatmer	='
		Yes	No	Total
Period 1	Yes	$n_{11}^{(1)}$	$n_{10}^{(1)}$	$n_{1\cdot}^{(1)}$
(Control)	No	$n_{01}^{(1)}$	$n_{00}^{(1)}$	$n_{0}^{(1)}$
	Total	$n^{(1)}_{\cdot 1}$	$n_{\cdot 0}^{(1)}$	n_1

SEQUENCE 2 (Treatment → Control)

			Period 2 (Control	
		Yes	No	Total
Period 1	Yes	$n_{11}^{(2)}$	$n_{10}^{(2)}$	$n_{1}^{(2)}$
(Treatment)	No	$n_{01}^{(2)}$	$n_{00}^{(2)}$	$n_{0}^{(2)}$
	Total	$n^{(2)}_{\cdot 1}$	$n_{\cdot 0}^{(2)}$	n_2

Non-Inferiority Tests for the Odds Ratio of Two Proportions in a 2x2 Cross-Over Design

In terms of proportions, the 2×2 cross-over design tables can be summarized as

SEQUENCE 1 (Control → Treatment)

SEQUENCE 2 (Treatment → Control)

		(7	Period 2 Freatmer	-
		Yes	No	Total
Period 1	Yes	$P_{11}^{(1)}$	$P_{10}^{(1)}$	$P_{1}^{(1)}$
(Control)	No	$P_{01}^{(1)}$	$P_{00}^{(1)}$	$P_{0}^{(1)}$
	Total	$P_{\cdot 1}^{(1)}$	$P_{.0}^{(1)}$	1

			Period 2 (Control	=
		Yes	No	Total
Period 1	Yes	$P_{11}^{(2)}$	$P_{10}^{(2)}$	$P_{1}^{(2)}$
(Treatment)	No	$P_{01}^{(2)}$	$P_{00}^{(2)}$	$P_{0}^{(2)}$
	Total	$P_{\cdot 1}^{(2)}$	$P_{\cdot 0}^{(2)}$	1

with the individual proportions estimated as

$$\widehat{P}_{rc}^{(g)} = \frac{n_{rc}^{(g)}}{n_g}.$$

Lui (2016) indicates on pages 32-42 that the odds ratio for the treatment versus the control (O_T/O_C) is defined for a 2×2 cross-over design as

$$OR = \sqrt{\frac{P_{01}^{(1)}P_{10}^{(2)}}{P_{10}^{(1)}P_{01}^{(2)}}}$$

with estimate

$$\widehat{OR} = \sqrt{\frac{\widehat{P}_{01}^{(1)}\widehat{P}_{10}^{(2)}}{\widehat{P}_{10}^{(1)}\widehat{P}_{01}^{(2)}}}.$$

The estimated log odds ratio, $\log(\widehat{OR})$, has asymptotic variance σ^2/n with

$$\sigma^2 = \frac{1}{4} \left(\frac{1}{P_{01}^{(1)}} + \frac{1}{P_{10}^{(1)}} + \frac{1}{P_{01}^{(2)}} + \frac{1}{P_{10}^{(2)}} \right)$$

which can be estimated as

$$\hat{\sigma}^2 = \frac{1}{4} \left(\frac{1}{\hat{p}_{01}^{(1)}} + \frac{1}{\hat{p}_{10}^{(1)}} + \frac{1}{\hat{p}_{01}^{(2)}} + \frac{1}{\hat{p}_{10}^{(2)}} \right).$$

The standard deviation, then, is

$$SD = \sigma = \sqrt{\sigma^2}$$

with estimate

$$\widehat{SD} = \widehat{\sigma} = \sqrt{\widehat{\sigma}^2}.$$

Non-Inferiority Test Statistics

Higher Proportions Better

When higher proportions are better, the null and alternative hypotheses for a one-sided non-inferiority test are

$$H_0: OR \leq OR_0$$
 vs $H_A: OR > OR_0$

where OR_0 is the lower non-inferiority bound (i.e. the smallest odds ratio (Ot/Oc) for which the treatment will still be considered non-inferior to the standard or control). When higher proportions are better, OR_0 should be less than one.

The power and sample size calculations are based on the test statistic

$$Z = \frac{\log(\widehat{OR}) - \log(OR_0)}{\frac{\widehat{SD}}{\sqrt{n}}}$$

which is asymptotically distributed as standard normal under the null hypothesis. The null hypothesis is rejected in favor of the alternative at level α if

$$\frac{\log(\widehat{OR}) - \log(OR_0)}{\frac{\widehat{SD}}{\sqrt{n}}} > Z_{1-\alpha}$$

where $Z_{1-\alpha}$ is the upper $1-\alpha$ percentile of the standard normal distribution.

Higher Proportions Worse

When higher proportions are worse, the null and alternative hypotheses for a one-sided non-inferiority test are

$$H_0: OR \ge OR_0$$
 vs $H_A: OR < OR_0$

where OR_0 is the upper non-inferiority bound (i.e. the largest odds ratio (Ot/Oc) for which the treatment will still be considered non-inferior to the standard or control). When higher proportions are worse, OR_0 should be greater than one.

The power and sample size calculations are based on the test statistic

$$Z = \frac{\log(\widehat{OR}) - \log(OR_0)}{\frac{\widehat{SD}}{\sqrt{n}}}$$

which is asymptotically distributed as standard normal under the null hypothesis. The null hypothesis is rejected in favor of the alternative at level α if

$$\frac{\log(\widehat{OR}) - \log(OR_0)}{\frac{\widehat{SD}}{\sqrt{n}}} < Z_{\alpha}$$

where Z_{α} is the lower α percentile of the standard normal distribution.

Non-Inferiority Power Calculations

Higher Proportions Better

Derived from the sample size formula given in Lui (2016) on pages 42 and 43, the power for the one-sided non-inferiority test of H_0 : $OR \le OR_0$ versus H_A : $OR > OR_0$ is

$$\Phi\left(\frac{\log(OR_1) - \log(OR_0)}{\frac{SD}{\sqrt{n}}} - Z_{1-\alpha}\right)$$

where $\Phi()$ is the standard normal distribution function, OR_1 is the actual value of the odds ratio under the alternative hypothesis, and $Z_{1-\alpha}$ is the upper $1-\alpha$ percentile of the standard normal distribution. The sample size calculation formula is

$$n = Ceiling \left\{ \left(\frac{\left(Z_{1-\alpha} + Z_{1-\beta} \right) SD}{\log(OR_1) - \log(OR_0)} \right)^2 \right\}.$$

Higher Proportions Worse

Derived from the sample size formula given in Lui (2016) on pages 42 and 43, the power for the one-sided non-inferiority test of H_0 : $OR \ge OR_0$ versus H_A : $OR < OR_0$ is

$$\Phi\left(\frac{\log(OR_0) - \log(OR_1)}{\frac{SD}{\sqrt{n}}} + Z_{\alpha}\right)$$

where $\Phi()$ is the standard normal distribution function, OR_1 is the actual value of the odds ratio under the alternative hypothesis, and Z_{α} is the lower α percentile of the standard normal distribution. The sample size calculation formula is

$$n = Ceiling \left\{ \left(\frac{\left(Z_{1-\alpha} + Z_{1-\beta} \right) SD}{\log(OR_0) - \log(OR_1)} \right)^2 \right\}.$$

Procedure Options

This section describes the options that are specific to this procedure. These are located on the Design tab. For more information about the options of other tabs, go to the Procedure Window chapter.

Design Tab

The Design tab contains most of the parameters and options that you will be concerned with.

Solve For

Solve For

This option specifies the parameter to be calculated from the values of the other parameters. Under most conditions, you would select either *Power* or *Sample Size*.

Select Sample Size when you want to determine the sample size needed to achieve a given power and alpha level.

Select *Power* when you want to calculate the power of an experiment that has already been run.

Select *Effect Size (OR1)* when you want to calculate the minimum effect size that can be detected for a particular design.

Test

Higher Proportions Are

Use this option to specify the direction of the test.

If Higher Proportions are "Better", the alternative hypothesis is H1: OR > OR0.

If Higher Proportions are "Worse", the alternative hypothesis is H1: OR < OR0.

Power and Alpha

Power

This option specifies one or more values for power. Power is the probability of rejecting a false null hypothesis, and is equal to one minus Beta. Beta is the probability of a type-II error, which occurs when a false null hypothesis is not rejected. In this procedure, a type-II error occurs when you fail to reject the null hypothesis of equal means when in fact the means are different.

Values must be between zero and one. Historically, the value of 0.80 (Beta = 0.20) was used for power. Now, 0.90 (Beta = 0.10) is also commonly used.

A single value may be entered here or a range of values such as 0.8 to 0.95 by 0.05 may be entered.

Alpha

This option specifies one or more values for the probability of a type-I error. A type-I error occurs when a true null hypothesis is rejected. In this procedure, a type-I error occurs when you reject the null hypothesis of equal means when in fact the means are equal.

Values must be between zero and one. Historically, the value of 0.05 has been used for alpha. This means that about one test in twenty will falsely reject the null hypothesis. You should pick a value for alpha that represents the risk of a type-I error you are willing to take in your experimental situation.

You may enter a range of values such as 0.01 0.05 0.10 or 0.01 to 0.10 by 0.01.

Sample Size

n (Sample Size per Sequence)

This is the sample size of each sequence or group (AB and BA) in the cross-over design. The individual sequence sample sizes are assumed to be equal such that the total sample size is equal to N = 2n.

You can enter a single value such as 50 or a list of values using the syntax 50 100 150 200 250 or 50 to 250 by 50.

Effect Size - Odds Ratio

OR0 (Non-Inferiority Odds Ratio)

Specify the non-inferiority odds ratio.

When higher proportions are "Better", the non-inferiority odds ratio is the smallest odds ratio (Ot/Oc) for which the treatment will still be considered non-inferior to the standard or control.

When higher proportions are "Worse", the non-inferiority odds ratio is the largest odds ratio (Ot/Oc) for which the treatment will still be considered non-inferior to the standard or control.

You can enter a single value such as 0.8 or a series of values such as $0.8 \ 0.85 \ 0.9$ or $0.8 \ to \ 0.9 \ by \ 0.05$ in the range OR0 > 0, $OR0 \neq OR1$. When higher proportions are "Better", OR0 should be less than one. When higher proportions are "Worse", OR0 should be greater than one.

OR1 (Actual Odds Ratio)

Specify the actual odds ratio at which power is calculated.

The odds ratio is the ratio of the odds of the treatment group showing a response to the odds of the control group showing a response.

OR1 = Ot/Oc =
$$[Pt/(1-Pt)] / [Pc/(1-Pc)]$$
 or $\sqrt{[P01(1)P10(2) / P10(1)P01(2)]}$

For example, an odds ratio of 2 means that the odds of a response in the treatment group is twice the odds of a positive response in the control group.

You can enter a single value such as 1 or a series of values such as 1 1.5 2 or 1 to 2 by 0.5 in the range OR1 > 0, $OR1 \neq OR0$. When higher proportions are "Better", OR1 should be greater than OR0. When higher proportions are "Worse", OR1 should be less than OR0.

Effect Size – Standard Deviation of Log(Odds Ratio)

Standard Deviation Estimation Method

Choose how the standard deviation of the log odds ratio (SD) will be estimated.

The options are

• Enter SD Directly

Input log odds ratio SD values directly. This option allows you to input any value for SD, regardless of how it was estimated.

Use Estimated Discordant Cell Proportions

Input discordant cell proportions from 2 tables from a previous 2x2 cross-over study. This is the method described in Lui (2016). These estimates are used only to calculate the standard deviation and are not necessarily related to the value used for OR1.

Non-Inferiority Tests for the Odds Ratio of Two Proportions in a 2x2 Cross-Over Design

Standard Deviation (SD)

Enter a value for the standard deviation of the log odds ratio, SD.

Estimating SD using Previously-Estimated Discordant Cell Proportions

The standard deviation may be estimated using discordant cell proportions from a previous study using the method of Lui (2016) on page 42, where

$$SD = \sqrt{[(1/4)*(1/p01(1) + 1/p10(1) + 1/p01(2) + 1/p10(2))]}.$$

PASS will calculate the SD for you using this method if you set Estimation Method to "Use Estimated Discordant Cell Proportions" and enter discordant cell proportions from 2 tables from a previous 2x2 cross-over study.

Discordant Cell Proportions

Enter the corresponding discordant cell proportion from a previous 2x2 cross-over study.

The estimated proportions are defined as follows:

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SEQUENCE 1 (C \rightarrow T)
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p01(1): Control = "No", Treatment = "Yes"

p10(1): Control = "Yes", Treatment = "No"

SEQUENCE 2 ($T\rightarrow C$)

p01(2): Control = "Yes", Treatment = "No"

p10(2): Control = "No", Treatment = "Yes"

The standard deviation is calculated by PASS as described in Lui (2016) on page 42 as

$$SD = \sqrt{[(1/4)*(1/p01(1) + 1/p10(1) + 1/p01(2) + 1/p10(2))]}.$$

These estimates are used only to calculate the standard deviation and are not necessarily related to the value used for OR1.

Range: 0 < p01(g) < 1, 0 < p10(g) < 1, $p01(g) + p10(g) \le 1$

Example 1 – Power Analysis

Suppose you want to consider the power of a non-inferiority test of the hypotheses $H_0: OR \le 0.8$ versus $H_A: OR > 0.8$ in a balanced cross-over design with a binary endpoint where the test is computed based on the odds ratio for sample sizes between 25 and 125. Let's assume that the actual odds ratio is 2 and the estimated standard deviation of the log odds ratio is 2.5. The significance level is 0.05.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the Non-Inferiority Tests for the Odds Ratio of Two Proportions in a 2x2 Cross-Over Design procedure window by expanding Proportions, then Cross-Over (2x2) Design, then clicking on Non-Inferiority, and then clicking on Non-Inferiority Tests for the Odds Ratio of Two Proportions in a 2x2 Cross-Over Design. You may then make the appropriate entries as listed below, or open Example 1 by going to the File menu and choosing Open Example Template.

<u>Option</u>	<u>Value</u>
Design Tab	
Solve For	Power
Higher Proportions Are	Better
Alpha	0.05
n (Sample Size per Sequence)	25 to 125 by 25
OR0 (Non-Inferiority Odds Ratio)	0.8
OR1 (Actual Odds Ratio)	2
Estimation Method	Enter SD Directly
Standard Deviation (SD)	2.5

Annotated Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results for a Non-Inferiority Test

H0: OR ≤ OR0 vs. H1: OR > OR0

	Sequence Sample Size	Total Sample Size	Non-Inf. Odds Ratio	Actual Odds Ratio	Standard Deviation	
Power	n	N	OR0	OR1	SD	Alpha
0.57445	25	50	0.800	2.000	2.500	0.050
0.82813	50	100	0.800	2.000	2.500	0.050
0.93690	75	150	0.800	2.000	2.500	0.050
0.97832	100	200	0.800	2.000	2.500	0.050
0.99291	125	250	0.800	2.000	2.500	0.050

References

Lui, Kung-Jong. 2016. Crossover Designs: Testing, Estimation, and Sample Size. John Wiley & Sons Ltd. Chichester, West Sussex, England.

Report Definitions

Power is the probability of rejecting a false null hypothesis. It should be close to one.

n is the sample size in each sequence (or group).

N is the total combined sample size from both sequences.

OR0 is the non-inferiority odds ratio used to specify the hypothesis test.

OR1 is the actual odds ratio at which power is calculated.

SD is the user-entered standard deviation. This is estimated from a previous study.

Alpha is the probability of rejecting a true null hypothesis. It should be small.

Non-Inferiority Tests for the Odds Ratio of Two Proportions in a 2x2 Cross-Over Design

Summary Statements

For a 2x2 cross-over design, a sample size of 25 in each sequence for a total of 50 achieves 57.445% power to detect an odds ratio of 2.000 using a one-sided non-inferiority test against a bound of 0.800 with a significance level of 0.050 when the standard deviation is 2.500.

Dropout-Inflated Sample Size -

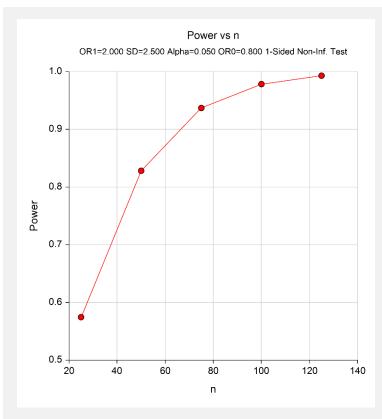
	— Samı	ole Size —	Enro	t-Inflated Ilment ole Size —	Num	ected ber of pouts —
Dropout Rate	n	N	n'	N'	d	D
20%	25	50	32	64	7	14
20%	50	100	63	126	13	26
20%	75	150	94	188	19	38
20%	100	200	125	250	25	50
20%	125	250	157	314	32	64

Definitions

Dropout Rate (DR) is the percentage of subjects (or items) that are expected to be lost at random during the course of the study and for whom no response data will be collected (i.e. will be treated as "missing").

- n and N are the evaluable group and total sample sizes, respectively, at which power is computed (as entered by the user). If n subjects from each group are evaluated out of the n' subjects that are enrolled in the study, the design will achieve the stated power. N = 2n.
- n' and N' are the number of subjects that should be enrolled in the study in order to end up with n and N evaluable subjects, based on the assumed dropout rate. n' is calculated by inflating n using the formula n' = n / (1 DR), with n' always rounded up. (See Julious, S.A. (2010) pages 52-53, or Chow, S.C., Shao, J., and Wang, H. (2008) pages 39-40.). N' = 2n'.
- d and D are the expected number of group and total dropouts, respectively. d = n' n and D = 2d.

Charts Section



This report shows the values of each of the parameters, one scenario per row. This plot shows the relationship between sample size and power. We see that a sample size of just under 50 per sequence is required to detect an odds ratio of 2 with 80% power.

Example 2 – Calculating Sample Size when Estimating the Standard Deviation from a Previous Study (Validation using Lui (2016))

This example demonstrates how to calculate the sample size when estimating the standard deviation of the log odds ratio from data in a previous study using the method in Lui (2016) on page 42. In this example we'll find the sample size required to detect an odds ratio of 2 with 80% power at a significance level of 0.05 in a non-inferiority test against a lower odds ratio bound of 0.8. The SD is estimated from discordant proportions in a previous study.

Table 3.2 of Lui (2016) on page 36 presents the results below from 279 subjects a simple 2x2 cross-over trial comparing two inhalation devices, A and B. Lui (2016) presents this sample size calculation in Example 3.5 on page 43 and finds a required sample size of 48 subjects per sequence.

SEQUENCE 1 (Control (A) \rightarrow **Treatment (B))**

SEQUENCE 2 (Treatment (B) \rightarrow **Control (A))**

			Period 2 (B)	2
		Yes	No	Total
Period 1	Yes	26	41	67
(A)	No	15	57	72
	Total	41	98	139

			Period 2 (A)	2
		Yes	No	Total
Period 1	Yes	38	16	54
(B)	No	32	54	86
	Total	70	70	140

The discordant proportions are estimated as

$$\hat{P}_{01}^{(1)} = \frac{15}{139} = 0.1079$$

$$\hat{P}_{10}^{(1)} = \frac{41}{139} = 0.2950$$

$$\hat{P}_{01}^{(2)} = \frac{32}{140} = 0.2286$$

$$\hat{P}_{10}^{(2)} = \frac{16}{140} = 0.1143$$

Using the method of Lui (2016) on page 42, SD is estimated as

$$\widehat{SD} = \sqrt{\widehat{\sigma}^2}$$

$$= \sqrt{\frac{1}{4} \left(\frac{1}{\widehat{p}_{01}^{(1)}} + \frac{1}{\widehat{p}_{10}^{(1)}} + \frac{1}{\widehat{p}_{01}^{(2)}} + \frac{1}{\widehat{p}_{10}^{(2)}} \right)}$$

$$= \sqrt{\frac{1}{4} \left(\frac{1}{0.1079} + \frac{1}{0.2950} + \frac{1}{0.2286} + \frac{1}{0.1143} \right)}$$

$$= 2.5388$$

PASS will calculate this SD value for you automatically when you input the discordant cell proportions.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the Non-Inferiority Tests for the Odds Ratio of Two Proportions in a 2x2 Cross-Over Design procedure window by expanding Proportions, then Cross-Over (2x2) Design, then clicking on Non-Inferiority, and then clicking on Non-Inferiority Tests for the Odds Ratio of Two Proportions in a 2x2 Cross-Over Design. You may then make the appropriate entries as listed below, or open Example 2 by going to the File menu and choosing Open Example Template.

Value
Sample Size
Better
0.80
0.05
0.8
. 2
Use Estimated Discordant Cell Proportions
0.1079
0.2950
0.2286
0.1143

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

	Sequence Sample Size	Total Sample Size	Non-Inf. Odds Ratio	Actual Odds Ratio	Standard Deviation	
Power	n	N	OR0	OR1	SD*	Alpha
0.80391	48	96	0.800	2.000	2.539	0.050

This report indicates that the estimated standard deviation using the method of Lui (2016) is 2.539 and the required sample size is 48 per sequence, which matches the published result in Lui (2016) exactly. The discordant cell proportions are also listed. The calculated value for SD matches our hand calculations above.