Chapter 207

Non-Unity Null Tests for the Odds Ratio of Two Proportions

Introduction

This module computes power and sample size for hypothesis tests of the odds ratio of two independent proportions where the null-hypothesized value is not equal to one. The *non-offset* case is available in another procedure. This procedure compares the power achieved by each of several test statistics.

The power calculations assume that independent, random samples are drawn from two populations.

Technical Details

Suppose you have two populations from which dichotomous (binary) responses will be recorded. The probability (or risk) of obtaining the event of interest in population 1 (the treatment group) is p_1 and in population 2 (the control group) is p_2. The corresponding failure proportions are given by $q_1 = 1 - p_1$ and $q_2 = 1 - p_2$.

An assumption is made that the responses from each group follow a binomial distribution. This means that the event probability, p_i, is the same for all subjects within the group and that the response from one subject is independent of that of any other subject.

Random samples of *m* and *n* individuals are obtained from these two populations. The data from these samples can be displayed in a 2-by-2 contingency table as follows

Group	Success	Failure	Total
Treatment	*a*	*c*	*m*
Control	*b*	*d*	*n*
Total	*s*	*f*	*N*

The following alternative notation is sometimes used.

Group	Success	Failure	Total
Treatment	x_{11}	x_{12}	n_1
Control	x_{21}	x_{22}	n_2
Total	m_1	m_2	N

The binomial proportions, p_1 and p_2, are estimated from these data using the formulae

$$\hat{p}_1 = \frac{a}{m} = \frac{x_{11}}{n_1} \text{ and } \hat{p}_2 = \frac{b}{n} = \frac{x_{21}}{n_2}$$

Comparing Two Proportions

When analyzing studies such as this, you usually want to compare the two binomial probabilities, p_1 and p_2 . The most direct method of comparing these quantities is to calculate their difference or their ratio. If the binomial probability is expressed in terms of odds rather than probability, another measure is the odds ratio. Mathematically, these comparison parameters are

Parameter	Computation
Difference	$\delta = p_1 - p_2$
Risk Ratio	$\phi = p_1 / p_2$
Odds Ratio	$\psi = \frac{p_1 / (1 - p_1)}{p_2 / (1 - p_2)} = \frac{p_1 q_2}{p_2 q_1}$

The choice of which of these measures is used might seem arbitrary, but it is not. Not only will the interpretation be different, but, for small sample sizes, the powers of tests based on different parameters will be different. The non-null case is commonly used in equivalence and non-inferiority testing.

Odds Ratio

Chances are usually communicated as long-term proportions or probabilities. In betting, chances are often given as odds. For example, the odds of a horse winning a race might be set at 10-to-1 or 3-to-2. Odds can easily be translated into probability. An odds of 3-to-2 means that the event is expected to occur three out of five times. That is, an odds of 3-to-2 (1.5) translates to a probability of winning of 0.60.

The odds of an event are calculated by dividing the event risk by the non-event risk. Thus, in our case of two populations, the odds are

$$o_1 = \frac{p_1}{1 - p_1} \text{ and } o_2 = \frac{p_2}{1 - p_2}$$

For example, if p_1 is 0.60, the odds are 0.60/0.40 = 1.5. Rather than represent the odds as a decimal amount, it is re-scaled into whole numbers. Instead of saying the odds are 1.5-to-1, we say they are 3-to-2.

Another way to compare proportions is to compute the ratio of their odds. The odds ratio of two events is

$$\psi = \frac{o_1}{o_2} = \frac{p_1 / (1 - p_1)}{p_2 / (1 - p_2)} = \frac{p_1 q_2}{p_2 q_1}$$

Although the odds ratio is more complicated to interpret than the risk ratio, it is often the parameter of choice. One reason for this is the fact that the odds ratio can be accurately estimated from case-control studies, while the risk ratio cannot. Also, the odds ratio is the basis of logistic regression (used to study the influence of risk factors). Furthermore, the odds ratio is the natural parameter in the conditional likelihood of the two-group, binomial-response design. Finally, when the baseline event rates are rare, the odds ratio provides a close approximation to the risk ratio since, in this case, $1 - p_1 \approx 1 - p_2$, so that

$$\psi = \frac{o_1}{o_2} = \frac{p_1 / (1 - p_1)}{p_2 / (1 - p_2)} \approx \frac{p_1}{p_2} = \phi$$

Hypothesis Tests

Although several statistical tests are available for testing the inequality of two proportions, only a few can be generalized to the non-null case. No single test is the champion in every situation, so one should compare the powers of the various tests to determine which to use.

Odds Ratio

The odds ratio, $\psi = \left[p_1/(1-p_1)\right]/\left[p_2/(1-p_2)\right]$, is sometimes used as the comparison because of its statistical properties and because some convenient experimental designs only allow it to be estimated. Three sets of statistical hypotheses can be formulated:

1. $H_0\colon \psi = \psi_0$ versus $H_1\colon \psi \neq \psi_0$; this is often called the *two-tailed test*.
2. $H_0\colon \psi \leq \psi_0$ versus $H_1\colon \psi > \psi_0$; this is often called the *upper-tailed test*.
3. $H_0\colon \psi \geq \psi_0$ versus $H_1\colon \psi < \psi_0$; this is often called the *lower-tailed test*.

Power Calculation

The power for a test statistic that is based on the normal approximation can be computed exactly using two binomial distributions. The following steps are taken to compute the power of such a test.

1. Find the critical value (or values in the case of a two-sided test) using the standard normal distribution. The critical value, $z_{critical}$, is that value of *z* that leaves exactly the target value of alpha in the appropriate tail of the normal distribution. For example, for an upper-tailed test with a target alpha of 0.05, the critical value is 1.645.
2. Compute the value of the test statistic, z_t, for every combination of x_{11} and x_{21}. Note that x_{11} ranges from 0 to n_1, and x_{21} ranges from 0 to n_2. A small value (around 0.0001) can be added to the zero cell counts to avoid numerical problems that occur when the cell value is zero.
3. If $z_t > z_{critical}$, the combination is in the rejection region. Call all combinations of x_{11} and x_{21} that lead to a rejection the set *A*.
4. Compute the power for given values of $p_{1.1}$ (p_1 under the alternative) and p_2 as
$$1-\beta = \sum_A \binom{n_1}{x_{11}} p_{1.1}^{x_{11}} q_{1.1}^{n_1-x_{11}} \binom{n_2}{x_{21}} p_2^{x_{21}} q_2^{n_2-x_{21}}.$$
5. Compute the actual value of alpha achieved by the design by substituting $p_{1.0}$ (p_1 under the null) for $p_{1.1}$ to obtain
$$\alpha^* = \sum_A \binom{n_1}{x_{11}} p_{1.0}^{x_{11}} q_{1.0}^{n_1-x_{11}} \binom{n_2}{x_{21}} p_2^{x_{21}} q_2^{n_2-x_{21}}.$$

Asymptotic Approximations

When the values of n_1 and n_2 are large (say over 200), these formulas often take a long time to evaluate. In this case, a large sample approximation is used. The large sample approximation is made by replacing the values of $\hat{p}_1$ and $\hat{p}_2$ in the *z* values with the corresponding values of p_1 and p_2 under the alternative hypothesis and then computing the results based on the normal distribution. Note that in large samples, the results for the odds ratio have not (to our knowledge) been published. In this case, we substitute the calculations based on the ratio.

Test Statistics

Several test statistics have been proposed for testing whether the difference, ratio, or odds ratio are different from a specified value. The main difference among the several test statistics is in the formula used to compute the standard error used in the denominator. These tests are based on the following *z*-test

$$z_t = \frac{\hat{p}_1 - \hat{p}_2 - \delta_0 - c}{\hat{\sigma}}$$

The constant, *c*, represents a continuity correction that is applied in some cases. When the continuity correction is not used, *c* is zero. In power calculations, the values of $\hat{p}_1$ and $\hat{p}_2$ are not known. The corresponding values of p_1 and p_2 under the alternative hypothesis are reasonable substitutes.

Following is a list of the test statistics available in **PASS**. The availability of several test statistics begs the question of which test statistic you should use. The answer is simple: you should use the test statistic that you will use to analyze your data. You may choose a method because it is a standard in your industry, because it seems to have better statistical properties, or because your statistical package calculates it. Whatever your reasons for selecting a certain test statistic, you should use the same test statistic during power or sample calculations.

Miettinen and Nurminen's Likelihood Score Test

Miettinen and Nurminen (1985) proposed a test statistic for testing whether the odds ratio is equal to a specified value, ψ_0. Because the approach they used with the difference and ratio does not easily extend to the odds ratio, they used a score statistic approach for the odds ratio. The regular MLE's are $\hat{p}_1$ and $\hat{p}_2$. The constrained MLE's are $\tilde{p}_1$ and $\tilde{p}_2$. These estimates are constrained so that $\tilde{\psi} = \psi_0$. A correction factor of N/(N-1) is applied to make the variance estimate less biased. The significance level of the test statistic is based on the asymptotic normality of the score statistic.

The formula for computing the test statistic is

$$z_{MNO} = \frac{\dfrac{(\hat{p}_1 - \tilde{p}_1)}{\tilde{p}_1\tilde{q}_1} - \dfrac{(\hat{p}_2 - \tilde{p}_2)}{\tilde{p}_2\tilde{q}_2}}{\sqrt{\left(\dfrac{1}{n_1\tilde{p}_1\tilde{q}_1} + \dfrac{1}{n_2\tilde{p}_2\tilde{q}_2}\right)\left(\dfrac{N}{N-1}\right)}}$$

where

$$\tilde{p}_1 = \frac{\tilde{p}_2\psi_0}{1 + \tilde{p}_2(\psi_0 - 1)}, \quad \tilde{p}_2 = \frac{-B + \sqrt{B^2 - 4AC}}{2A},$$

$$A = n_2(\psi_0 - 1), B = n_1\psi_0 + n_2 - m_1(\psi_0 - 1), C = -m_1$$

Farrington and Manning's Likelihood Score Test of the Odds Ratio

Farrington and Manning (1990) indicate that the Miettinen and Nurminen statistic may be modified by removing the factor $N/(N\text{-}1)$.

The formula for computing this test statistic is

$$z_{FMO} = \frac{\dfrac{(\hat{p}_1 - \tilde{p}_1)}{\tilde{p}_1\tilde{q}_1} - \dfrac{(\hat{p}_2 - \tilde{p}_2)}{\tilde{p}_2\tilde{q}_2}}{\sqrt{\left(\dfrac{1}{n_1\tilde{p}_1\tilde{q}_1} + \dfrac{1}{n_2\tilde{p}_2\tilde{q}_2}\right)}}$$

where the estimates, $\tilde{p}_1$ and $\tilde{p}_2$, are computed as in the corresponding test of Miettinen and Nurminen (1985) given above.

Procedure Options

This section describes the options that are specific to this procedure. These are located on the Design tab. For more information about the options of other tabs, go to the Procedure Window chapter.

Design Tab

The Design tab contains the parameters associated with this test such as the proportions, odds ratios, sample sizes, alpha, and power.

Solve For

Solve For

This option specifies the parameter to be solved for using the other parameters. The parameters that may be selected are *Power*, *Sample Size*, and *Effect Size*.

Select *Power* when you want to calculate the power of an experiment.

Select *Sample Size* when you want to calculate the sample size needed to achieve a given power and alpha level.

Power Calculation

Power Calculation Method

Select the method to be used to calculate power. When the sample sizes are reasonably large (i.e. greater than 50) and the proportions are between 0.2 and 0.8 the two methods will give similar results. For smaller sample sizes and more extreme proportions (less than 0.2 or greater than 0.8), the normal approximation is not as accurate so the binomial calculations may be more appropriate.

The choices are

- **Binomial Enumeration**

  Power for each test is computed using binomial enumeration of all possible outcomes when N1 and N2 ≤ Maximum N1 or N2 for Binomial Enumeration (otherwise, the normal approximation is used). Binomial enumeration of all outcomes is possible because of the discrete nature of the data.

- **Normal Approximation**
  Approximate power for each test is computed using the normal approximation to the binomial distribution.

Actual alpha values are only computed when “Binomial Enumeration” is selected.

Power Calculation – Binomial Enumeration Options

Only shown when Power Calculation Method = “Binomial Enumeration”

Maximum N1 or N2 for Binomial Enumeration

When both N1 and N2 are less than or equal to this amount, power calculations using the binomial distribution are made. The value of the “Actual Alpha” is only calculated when binomial power calculations are made.

When either N1 or N2 is larger than this amount, the normal approximation to the binomial is used for power calculations.

Zero Count Adjustment Method

Zero cell counts cause many calculation problems when enumerating binomial probabilities. To compensate for this, a small value (called the “Zero Count Adjustment Value”) may be added either to all cells or to all cells with zero counts. This option specifies which type of adjustment you want to use.

Adding a small value is controversial, but may be necessary. Some statisticians recommend adding 0.5 while others recommend 0.25. We have found that adding values as small as 0.0001 seems to work well.

Zero Count Adjustment Value

Zero cell counts cause many calculation problems when enumerating binomial probabilities. To compensate for this, a small value may be added either to all cells or to all zero cells. This is the amount that is added. We have found that 0.0001 works well.

Be warned that the value of the odds ratio will be affected by the amount specified here!

Test

Alternative Hypothesis

Specify the direction of the alternative hypothesis, H1. Note that OR = O1/O2.

- **One-Sided (H1: OR < OR0)**
  Refers to a lower-tailed, one-sided test in which the alternative hypothesis is H1: OR < OR0.

- **One-Sided (H1: OR > OR0)**
  Refers to an upper-tailed, one-sided test in which the alternative hypothesis is H1: OR > OR0.

- **Two-Sided (H1: OR ≠ OR0)**
  Refers to a two-sided test in which the alternative hypothesis is H1: OR ≠ OR0.

For one-sided tests, the direction you select must match the values entered for OR0 and OR1. For example, if you select H1: OR < OR0, then the value(s) for OR1 must be less than the value(s) for OR0.

Test Type

Specify which test statistic is used in searching and reporting.

Power and Alpha

Power

This option specifies one or more values for power. Power is the probability of rejecting a false null hypothesis, and is equal to one minus Beta. Beta is the probability of a type-II error, which occurs when a false null hypothesis is not rejected. In this procedure, a type-II error occurs when you fail to reject the null hypothesis of equal proportions when in fact they are different.

Values must be between zero and one. Historically, the value of 0.80 (Beta = 0.20) was used for power. Now, 0.90 (Beta = 0.10) is also commonly used.

A single value may be entered here or a range of values such as *0.8 to 0.95 by 0.05* may be entered.

Alpha

This option specifies one or more values for the probability of a type-I error. A type-I error occurs when a true null hypothesis is rejected. For this procedure, a type-I error occurs when you reject the null hypothesis of equal proportions when in fact they are equal.

Values must be between zero and one. Historically, the value of 0.05 has been used for alpha. This means that about one test in twenty will falsely reject the null hypothesis. You should pick a value for alpha that represents the risk of a type-I error you are willing to take in your experimental situation.

You may enter a range of values such as *0.01 0.05 0.10* or *0.01 to 0.10 by 0.01*.

Sample Size (When Solving for Sample Size)

Group Allocation

Select the option that describes the constraints on *N1* or *N2* or both.

The options are

- **Equal (N1 = N2)**
  This selection is used when you wish to have equal sample sizes in each group. Since you are solving for both sample sizes at once, no additional sample size parameters need to be entered.

- **Enter N1, solve for N2**
  Select this option when you wish to fix *N1* at some value (or values), and then solve only for *N2*. Please note that for some values of *N1*, there may not be a value of *N2* that is large enough to obtain the desired power.

- **Enter N2, solve for N1**
  Select this option when you wish to fix *N2* at some value (or values), and then solve only for *N1*. Please note that for some values of *N2*, there may not be a value of *N1* that is large enough to obtain the desired power.

- **Enter R = N2/N1, solve for N1 and N2**
  For this choice, you set a value for the ratio of *N2* to *N1*, and then PASS determines the needed *N1* and *N2*, with this ratio, to obtain the desired power. An equivalent representation of the ratio, *R*, is

  $$N2 = R * N1.$$

- **Enter percentage in Group 1, solve for N1 and N2**
  For this choice, you set a value for the percentage of the total sample size that is in Group 1, and then PASS determines the needed *N1* and *N2* with this percentage to obtain the desired power.

N1 (Sample Size, Group 1)

This option is displayed if Group Allocation = "Enter N1, solve for N2"

N1 is the number of items or individuals sampled from the Group 1 population.

N1 must be ≥ 2. You can enter a single value or a series of values.

N2 (Sample Size, Group 2)

This option is displayed if Group Allocation = "Enter N2, solve for N1"

N2 is the number of items or individuals sampled from the Group 2 population.

N2 must be ≥ 2. You can enter a single value or a series of values.

R (Group Sample Size Ratio)

This option is displayed only if Group Allocation = "Enter R = N2/N1, solve for N1 and N2."

R is the ratio of *N2* to *N1*. That is,

$$R = N2 / N1.$$

Use this value to fix the ratio of *N2* to *N1* while solving for *N1* and *N2*. Only sample size combinations with this ratio are considered.

N2 is related to *N1* by the formula:

$$N2 = [R \times N1],$$

where the value *[Y]* is the next integer ≥ *Y*.

For example, setting *R* = 2.0 results in a Group 2 sample size that is double the sample size in Group 1 (e.g., *N1* = 10 and *N2* = 20, or *N1* = 50 and *N2* = 100).

R must be greater than 0. If *R* < 1, then *N2* will be less than *N1*; if *R* > 1, then *N2* will be greater than *N1*. You can enter a single or a series of values.

Percent in Group 1

This option is displayed only if Group Allocation = "Enter percentage in Group 1, solve for N1 and N2."

Use this value to fix the percentage of the total sample size allocated to Group 1 while solving for *N1* and *N2*. Only sample size combinations with this Group 1 percentage are considered. Small variations from the specified percentage may occur due to the discrete nature of sample sizes.

The Percent in Group 1 must be greater than 0 and less than 100. You can enter a single or a series of values.

Sample Size (When Not Solving for Sample Size)

Group Allocation

Select the option that describes how individuals in the study will be allocated to Group 1 and to Group 2.

The options are

- **Equal (N1 = N2)**
  This selection is used when you wish to have equal sample sizes in each group. A single per group sample size will be entered.

- **Enter N1 and N2 individually**
  This choice permits you to enter different values for *N1* and *N2*.

- **Enter N1 and R, where N2 = R * N1**
  Choose this option to specify a value (or values) for *N1*, and obtain *N2* as a ratio (multiple) of *N1*.

- **Enter total sample size and percentage in Group 1**
  Choose this option to specify a value (or values) for the total sample size (*N*), obtain *N1* as a percentage of *N*, and then *N2* as *N* - *N1*.

Sample Size Per Group

This option is displayed only if Group Allocation = "Equal (N1 = N2)."

The Sample Size Per Group is the number of items or individuals sampled from each of the Group 1 and Group 2 populations. Since the sample sizes are the same in each group, this value is the value for *N1*, and also the value for *N2*.

The Sample Size Per Group must be ≥ 2. You can enter a single value or a series of values.

N1 (Sample Size, Group 1)

*This option is displayed if Group Allocation = "Enter N1 and N2 individually" or "Enter N1 and R, where N2 = R * N1."*

N1 is the number of items or individuals sampled from the Group 1 population.

N1 must be ≥ 2. You can enter a single value or a series of values.

N2 (Sample Size, Group 2)

This option is displayed only if Group Allocation = "Enter N1 and N2 individually."

N2 is the number of items or individuals sampled from the Group 2 population.

N2 must be ≥ 2. You can enter a single value or a series of values.

R (Group Sample Size Ratio)

*This option is displayed only if Group Allocation = "Enter N1 and R, where N2 = R * N1."*

R is the ratio of *N2* to *N1*. That is,

$$R = N2/N1$$

Use this value to obtain *N2* as a multiple (or proportion) of *N1*.

N2 is calculated from *N1* using the formula:

$$N2=[R \times N1],$$

where the value *[Y]* is the next integer ≥ *Y*.

For example, setting *R* = 2.0 results in a Group 2 sample size that is double the sample size in Group 1.

R must be greater than 0. If *R* < 1, then *N2* will be less than N1; if *R* > 1, then *N2* will be greater than *N1*. You can enter a single value or a series of values.

Total Sample Size (N)

This option is displayed only if Group Allocation = "Enter total sample size and percentage in Group 1."

This is the total sample size, or the sum of the two group sample sizes. This value, along with the percentage of the total sample size in Group 1, implicitly defines *N1* and *N2*.

The total sample size must be greater than one, but practically, must be greater than 3, since each group sample size needs to be at least 2.

You can enter a single value or a series of values.

Percent in Group 1

This option is displayed only if Group Allocation = "Enter total sample size and percentage in Group 1."

This value fixes the percentage of the total sample size allocated to Group 1. Small variations from the specified percentage may occur due to the discrete nature of sample sizes.

The Percent in Group 1 must be greater than 0 and less than 100. You can enter a single value or a series of values.

Effect Size – Odds Ratios

OR0 (Odds Ratio|H0 = O1.0/O2)

This option specifies the odds ratio between the group 1 proportion under the null hypothesis and the proportion in group 2. This value is used with P2 to calculate the value of P1.0. The power calculations assume that P1.0 is the value of the P1 under the null hypothesis. In this non-null case, the value of P1.0 is not equal to P2 as it is in the null case.

You may enter a range of values such as *0.5 0.6 0.7 0.8* or *1.25 to 2.0 by 0.25*.

Odds ratios must greater than zero.

OR1 (Odds Ratio|H1 = O1.1/O2)

This option specifies the odds ratio of the two proportions P1.1 and P2. This odds ratio is used with P2 to calculate the value of P1.1. The power calculations assume that P1.1 is the actual value of the proportion in group 1, which is the experimental, or treatment, group.

You may enter a range of values such as *0.5 0.6 0.7 0.8* or *1.25 to 2.0 by 0.25*. Odds ratios must greater than zero.

Effect Size – Group 2 (Reference)

P2 (Group 2 Proportion)

Specify the value of P2, the control, baseline, or standard group's proportion. The null hypothesis is that the two proportions differ by a specified amount. Since P2 is a proportion, these values must be between zero and one.

You may enter a range of values such as *0.1,0.2,0.3* or *0.1 to 0.9 by 0.1*.

Example 1 – Finding Power

A study is being designed to determine the effectiveness of a new treatment. The standard treatment has a success rate of 65%. They would like to show that the odds ratio of success rates for the new treatment to the old treatment is at least 1.4.

The researchers plan to use the Farrington and Manning likelihood score test statistic to analyze the data that will be (or has been) obtained. They want to study the power of the Farrington and Manning test at group sample sizes ranging from 50 to 200 for detecting an odds ratio of 1.4 when the actual odds ratio ranges from 2 to 3. The significance level will be 0.025.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Non-Unity Null Tests for the Odds Ratio of Two Proportions** procedure window by expanding **Proportions**, then **Two Independent Proportions**, then clicking on **Test (Non-Zero Null)**, and then clicking on **Non-Unity Null Tests for the Odds Ratio of Two Proportions**. You may then make the appropriate entries as listed below, or open **Example 1** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Power**
Power Calculation Method	**Normal Approximation**
Alternative Hypothesis	**One-Sided (H1: OR > OR0)**
Test Type	**Likelihood Score (Farr. & Mann.)**
Alpha	**0.025**
Group Allocation	**Equal (N1 = N2)**
Sample Size Per Group	**50 100 150 200**
OR0 (Odds Ratio|H0 = O1.0/O2)	**1.4**
OR1 (Odds Ratio|H1 = O1.1/O2)	**2 2.5 3**
P2 (Group 2 Proportion)	**0.65**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results
Test Statistic: Farrington & Manning Likelihood Score Test
Hypotheses: H0: OR ≤ OR0 vs. H1: OR > OR0

Power*	N1	N2	N	Ref. P2	P1|H0 P1.0	P1|H1 P1.1	OR|H0 OR0	OR|H1 OR1	Alpha
0.12420	50	50	100	0.6500	0.7222	0.7879	1.400	2.000	0.0250
0.20182	100	100	200	0.6500	0.7222	0.7879	1.400	2.000	0.0250
0.27751	150	150	300	0.6500	0.7222	0.7879	1.400	2.000	0.0250
0.35055	200	200	400	0.6500	0.7222	0.7879	1.400	2.000	0.0250
0.24109	50	50	100	0.6500	0.7222	0.8228	1.400	2.500	0.0250
0.41585	100	100	200	0.6500	0.7222	0.8228	1.400	2.500	0.0250
0.56501	150	150	300	0.6500	0.7222	0.8228	1.400	2.500	0.0250
0.68469	200	200	400	0.6500	0.7222	0.8228	1.400	2.500	0.0250

(Report continues)

* Power was computed using the normal approximation method.

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Report Definitions

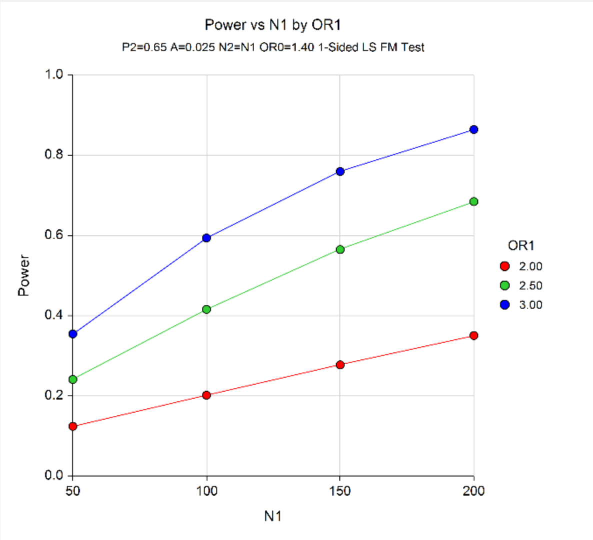
Power is the probability of rejecting a false null hypothesis.
N1 and N2 are the number of items sampled from each population.
N is the total sample size, N1 + N2.
P2 is the proportion for Group 2, which is the standard, reference, or control group.
P1 is the proportion for Group 1, which is the treatment or experimental group. P1.0 is the Group 1 proportion under the null hypothesis. P1.1 is the proportion for Group 1 under the alternative hypothesis at which power and sample size calculations are made.
OR0 is the odds ratio (O1/O2) under the null hypothesis. OR1 is the odds ratio under the alternative hypothesis used for power and sample size calculations.
Alpha is the probability of rejecting a true null hypothesis.

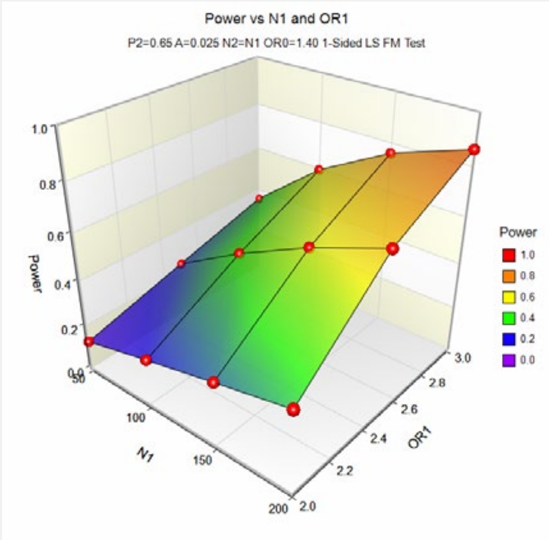
Summary Statements

Group sample sizes of 50 in Group 1 and 50 in Group 2 achieve 12.420% power to detect an odds ratio of 2.0000 when the null-hypothesized odds ratio is 1.4000. The Group 2 proportion is 0.6500. The Group 1 proportion is assumed to be 0.7222 under the null hypothesis and 0.7879 under the alternative hypothesis. The test statistic used is the one-sided Score test (Farrington & Manning). The significance level of the test is 0.0250.

This report shows the values of each of the parameters, one scenario per row.

Plots Section





The values from the table are displayed in the above charts. These charts give us a quick look at the sample size that will be required for various values of OR1.

Example 2 – Finding the Sample Size

Continuing with the scenario given in Example 1, the researchers want to determine the sample size necessary for each value of OR1 to achieve a power of 0.80.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Non-Unity Null Tests for the Odds Ratio of Two Proportions** procedure window by expanding **Proportions**, then **Two Independent Proportions**, then clicking on **Test (Non-Zero Null)**, and then clicking on **Non-Unity Null Tests for the Odds Ratio of Two Proportions**. You may then make the appropriate entries as listed below, or open **Example 2** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Sample Size**
Power Calculation Method	**Normal Approximation**
Alternative Hypothesis	**One-Sided (H1: OR > OR0)**
Test Type	**Likelihood Score (Farr. & Mann.)**
Power	**0.80**
Alpha	**0.025**
Group Allocation	**Equal (N1 = N2)**
OR0 (Odds Ratio|H0 = O1.0/O2)	**1.4**
OR1 (Odds Ratio|H1 = O1.1/O2)	**2 2.5 3**
P2 (Group 2 Proportion)	**0.65**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results
Test Statistic: Farrington & Manning Likelihood Score Test
Hypotheses: H0: OR ≤ OR0 vs. H1: OR > OR0

Target Power	Actual Power*	N1	N2	N	Ref. P2	P1|H0 P1.0	P1|H1 P1.1	OR|H0 OR0	OR|H1 OR1	Alpha
0.80	0.80022	645	645	1290	0.6500	0.7222	0.7879	1.400	2.000	0.0250
0.80	0.80057	266	266	532	0.6500	0.7222	0.8228	1.400	2.500	0.0250
0.80	0.80122	167	167	334	0.6500	0.7222	0.8478	1.400	3.000	0.0250

* Power was computed using the normal approximation method.

The required sample size will depend a great deal on the value of OR1. Any effort spent determining an accurate value for OR1 will be worthwhile.

Example 3 – Comparing the Power of the Two Test Statistics

Continuing with Example 2, the researchers want to determine which of the eight possible test statistics to adopt by using the comparative reports and charts that **PASS** produces. They decide to compare the powers from binomial enumeration and actual alphas for various sample sizes between 600 and 800 when OR1 is 2.0.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Non-Unity Null Tests for the Odds Ratio of Two Proportions** procedure window by expanding **Proportions**, then **Two Independent Proportions**, then clicking on **Test (Non-Zero Null)**, and then clicking on **Non-Unity Null Tests for the Odds Ratio of Two Proportions**. You may then make the appropriate entries as listed below, or open **Example 3** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Power**
Power Calculation Method	**Binomial Enumeration**
Maximum N1 or N2 for Binomial Enumeration	**5000**
Zero Count Adjustment Method	**Add to zero cells only**
Zero Count Adjustment Value	**0.0001**
Alternative Hypothesis	**One-Sided (H1: OR > OR0)**
Test Type	**Likelihood Score (Farr. & Mann.)**
Alpha	**0.025**
Group Allocation	**Equal (N1 = N2)**
Sample Size Per Group	**600 700 800**
OR0 (Odds Ratio|H0 = O1.0/O2)	**1.4**
OR1 (Odds Ratio|H1 = O1.1/O2)	**2**
P2 (Group 2 Proportion)	**0.65**
Reports Tab	
Show Comparative Reports	**Checked**
Comparative Plots Tab	
Show Comparative Plots	**Checked**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results and Plots

Power Comparison of Two Different Tests
Hypotheses: H0: OR ≤ OR0 vs. H1: OR > OR0

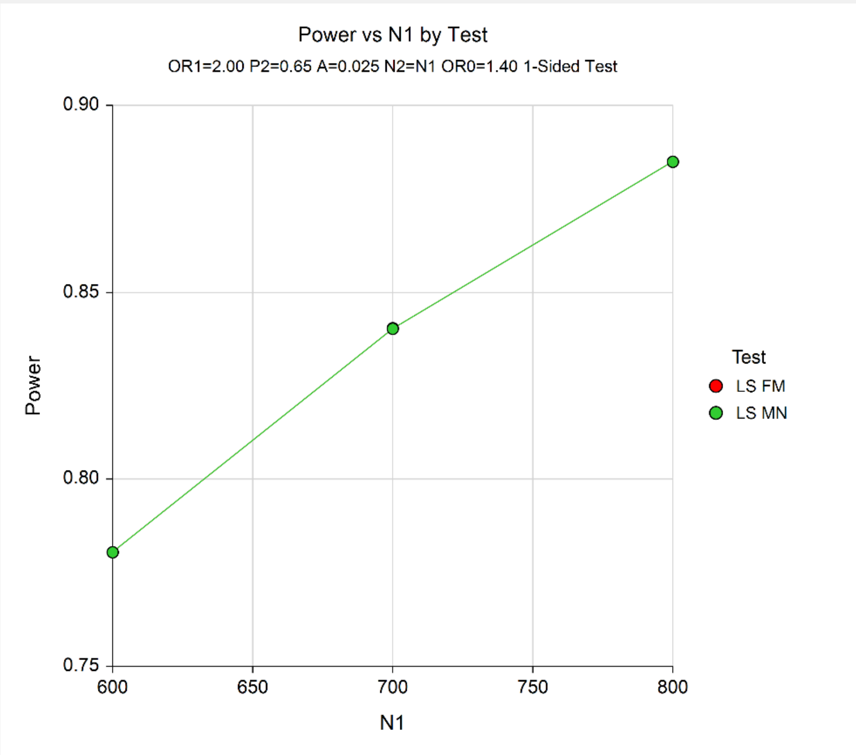
N1/N2	P2	P1.1	Target Alpha	F.M. Score Power	M.N. Score Power
600/600	0.6500	0.7879	0.0250	0.7805	0.7805
700/700	0.6500	0.7879	0.0250	0.8404	0.8402
800/800	0.6500	0.7879	0.0250	0.8849	0.8849

Note: Power was computed using binomial enumeration of all possible outcomes.

Actual Alpha Comparison of Two Different Tests
Hypotheses: H0: OR ≤ OR0 vs. H1: OR > OR0

N1/N2	P2	P1.1	Target Alpha	F.M. Score Alpha	M.N. Score Alpha
600/600	0.6500	0.7879	0.0250	0.0250	0.0250
700/700	0.6500	0.7879	0.0250	0.0250	0.0249
800/800	0.6500	0.7879	0.0250	0.0249	0.0249

Note: Actual alpha was computed using binomial enumeration of all possible outcomes.



Both test statistics have almost identical power and actual alpha values for all sample sizes studied.

Example 4 – Comparing Power Calculation Methods

Continuing with Example 3, let's see how the results compare if we were to use approximate power calculations instead of power calculations based on binomial enumeration.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Non-Unity Null Tests for the Odds Ratio of Two Proportions** procedure window by expanding **Proportions**, then **Two Independent Proportions**, then clicking on **Test (Non-Zero Null)**, and then clicking on **Non-Unity Null Tests for the Odds Ratio of Two Proportions**. You may then make the appropriate entries as listed below, or open **Example 4** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Power**
Power Calculation Method	**Normal Approximation**
Alternative Hypothesis	**One-Sided (H1: OR > OR0)**
Test Type	**Likelihood Score (Farr. & Mann.)**
Alpha	**0.025**
Group Allocation	**Equal (N1 = N2)**
Sample Size Per Group	**600 700 800**
OR0 (Odds Ratio|H0 = O1.0/O2)	**1.4**
OR1 (Odds Ratio|H1 = O1.1/O2)	**2**
P2 (Group 2 Proportion)	**0.65**
Reports Tab	
Show Power Detail Report	**Checked**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results and Plots

Power Detail Report
Test Statistic: Farrington & Manning Likelihood Score Test
Hypotheses: H0: OR ≤ OR0 vs. H1: OR > OR0

				Normal Approximation		Binomial Enumeration	
N1/N2	**P2**	**OR0**	**OR1**	**Power**	**Alpha**	**Power**	**Alpha**
600/600	0.6500	1.400	2.000	0.77161	0.0250	0.78049	0.0250
700/700	0.6500	1.400	2.000	0.83097	0.0250	0.84041	0.0250
800/800	0.6500	1.400	2.000	0.87637	0.0250	0.88489	0.0249

Notice that the approximate power values are very close to the binomial enumeration values for all sample sizes.

Example 5 – Validation of Power Calculations using Blackwelder (1993)

We could not find a validation example for a test for the odds ratio of two proportions with a non-unity null hypothesis. The calculations are basically the same as those for a test of the ratio of two proportions with a non-unity null hypothesis, which has been validated using Blackwelder (1993). We refer you to Example 5 of Chapter 206, "Non-Unity Null Tests for the Ratio of Two Proportions," for a validation example.