

Chapter 198

Tests for One Proportion using Effect Size

Introduction

This procedure provides sample size and power calculations for one- or two-sided hypothesis tests of the difference between a proportion and a given value (between 0 and 1) using the effect size. The details of procedure are given in Cohen (1988). The design corresponding to this test procedure is sometimes referred to as a *one-arm* design versus a historical control. In this design, two proportions are compared by considering their difference. This difference is formed between transformed values of the proportions, formed to create variables that are more normally distributed than the raw proportions and that have a variance not related to the values of the proportions.

Test Procedure

If we assume that P_1 and P_0 represent the two proportions. P_1 represents the population being studied. P_0 is the historical control proportion. The effect size is represented by the difference h formed as follows

$$h = \varphi_1 - \varphi_0$$

where

$$\varphi_i = 2 \arcsine(\sqrt{P_i})$$

This is referred to as the arcsine, the arcsine root, or the angular transformation.

The null hypothesis is $H_0: h = 0$ and the alternative hypothesis depends on the number of “sides” of the test:

$$\text{Two-Sided:} \quad H_1: h \neq 0 \text{ or } H_1: \varphi_1 - \varphi_0 \neq 0$$

$$\text{Upper One-Sided:} \quad H_1: h > 0 \text{ or } H_1: \varphi_1 - \varphi_0 > 0$$

$$\text{Lower One-Sided:} \quad H_1: h < 0 \text{ or } H_1: \varphi_1 - \varphi_0 < 0$$

A suitable Type I error probability (α) is chosen for the test, the data is collected, and a z-statistic is generated using the formula

$$z = \frac{\hat{\varphi}_1 - \varphi_0}{\sqrt{\frac{1}{N}}}$$

Tests for One Proportion using Effect Size

This z -statistic follows a standard normal distribution. The null hypothesis is rejected in favor of the alternative if,

for $H_1: h \neq 0$, $z < z_{\alpha/2}$ or $z > z_{1-\alpha/2}$

for $H_1: h > 0$, $z > z_{1-\alpha}$

for $H_1: h < 0$, $z < z_{\alpha}$

Comparing the z -statistic to the cut-off z -value is equivalent to comparing the p -value to α .

Power Calculation

The power of a one-sided test is calculated using the formulation of Cohen (1988):

$$z_{1-\beta} = h\sqrt{N} - z_{1-\alpha}$$

The Effect Size

As stated above, the effect size h is given by $h = \varphi_1 - \varphi_0$. Cohen (1988) proposed the following interpretation of the h values. An h near 0.2 is a *small* effect, an h near 0.5 is a *medium* effect, and an h near 0.8 is a *large* effect. These values for small, medium, and large effects are popular in the social sciences.

Cohen (1988) remarks that the value of h does not match directly with the value of $P_1 - P_0$, so care must be taken when using it. For example, all of the following pairs of values of P_1 and P_0 result in an h of about 0.30, even though the actual differences $P_1 - P_0$ are quite different.

P_1	P_0	$P_1 - P_0$	h
0.21	0.10	0.11	0.3
0.39	0.25	0.14	0.3
0.55	0.40	0.15	0.3
0.65	0.50	0.15	0.3
0.78	0.60	0.18	0.3
0.87	0.75	0.13	0.3
0.97	0.90	0.07	0.3

Procedure Options

This section describes the options that are specific to this procedure. These are located on the Design tab. For more information about the options of other tabs, go to the Procedure Window chapter.

Design Tab

The Design tab contains most of the parameters and options that you will be concerned with.

Solve For

Solve For

This option specifies the parameter to be solved for from the other parameters. The parameters that may be selected are *Power*, *Sample Size*, *Effect Size*, and *Alpha*. In most situations, you will likely select either *Power* or *Sample Size*.

The 'Solve For' parameter is the parameter that will be displayed on the vertical axis of any plots that are shown.

Tests for One Proportion using Effect Size

Test Direction

Alternative Hypothesis

Specify whether the alternative hypothesis of the test is one-sided or two-sided. If a one-sided test is chosen, the hypothesis test direction is chosen based on whether the effect size is greater than or less than zero.

Two-Sided Hypothesis Test

H0: $h = 0$ vs. H1: $h \neq 0$

One-Sided Hypothesis Tests

Upper: H0: $h \leq 0$ vs. H1: $h > 0$

Lower: H0: $h \geq 0$ vs. H1: $h < 0$

Power and Alpha

Power

Power is the probability of rejecting the null hypothesis when it is false. Power is equal to $1 - \text{Beta}$, so specifying power implicitly specifies beta. Beta is the probability obtaining a false negative with the statistical test. That is, it is the probability of accepting a false null hypothesis.

The valid range is 0 to 1. Different disciplines have different standards for setting power. The most common choice is 0.90, but 0.80 is also popular.

You can enter a single value, such as 0.90, or a series of values, such as .70 .80 .90, or .70 to .90 by .1.

When a series of values is entered, PASS will generate a separate calculation result for each value of the series.

Alpha

Alpha is the probability of obtaining a false positive with the statistical test. That is, it is the probability of rejecting a true null hypothesis. The null hypothesis is usually that the parameters of interest (means, proportions, etc.) are equal.

Since Alpha is a probability, it is bounded by 0 and 1. Commonly, it is between 0.001 and 0.10.

Alpha is often set to 0.05 for two-sided tests and to 0.025 for one-sided tests.

You can enter a single value, such as 0.05, or a series of values, such as .05 .10 .15, or .05 to .15 by .01.

When a series of values is entered, PASS will generate a separate calculation result for each value of the series.

Sample Size

N (Sample Size)

Enter a value for the sample size (N), the number of individuals in the study. You may enter a single value such as 242, range of values such as 50 to 400 by 50, or a list of values such as 100 300 800 900.

Tests for One Proportion using Effect Size

Effect Size**h**

Enter one or more values for h, the effect size, that you wish to detect. This is a standardized difference between the group proportions. The effect size is calculated using

$$h = \varphi_1 - \varphi_0 \text{ where } \varphi_i = 2 \arcsin(\sqrt{P_i})$$

where P1 and P0 are the group proportions assumed by the alternative hypothesis. Note that $0 \leq \varphi_i \leq 3.14159$. Thus, the physical interpretation of these values may be difficult to grasp.

The value of h must be between -6.28318 and 6.28318.

You can enter a single value such as 0.5 or a series of values such as 0.2 0.5 0.8 or 0.2 to 0.8 by 0.1. When a series of values is entered, PASS will generate a separate calculation result for each value of the series.

Cohen's Effect Size Table

Cohen (1988) gave the following interpretation of h values that is still popular.

- **Small**
h = 0.2
- **Medium**
h = 0.5
- **Large**
h = 0.8

Interpretation

A value of h can have several interpretations. So care must be taken when using it. For example, all of the following pairs of values of P1 and P2 result in an h of about 0.30, even though the actual differences P1 - P2 are quite different.

P1=0.21 and P2 = 0.10 Diff = 0.11

P1=0.39 and P2 = 0.25 Diff = 0.14

P1=0.55 and P2 = 0.40 Diff = 0.15

P1=0.65 and P2 = 0.50 Diff = 0.15

P1=0.78 and P2 = 0.60 Diff = 0.18

P1=0.87 and P2 = 0.75 Diff = 0.13

P1=0.97 and P2 = 0.90 Diff = 0.07

Tests for One Proportion using Effect Size

Example 1 – Finding the Sample Size

Researchers wish to compare a new type of local anesthesia to the commonly used type using a one-sample design. Subjects in pain will be studied. The treatment will be administered, and the subject's evaluation of pain intensity will be measured on a binary scale (acceptable, unacceptable).

The researchers would like to determine the sample sizes required to detect a small, medium, and large effect size with a two-sided z test when the power is 80% or 90% and the significance level is 0.05.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Tests for One Proportion using Effect Size** procedure. You may then make the appropriate entries as listed below, or open **Example 1** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
Design Tab	
Solve For	Sample Size
Alternative Hypothesis	Two-Sided
Power	0.80 0.90
Alpha	0.05
h	0.2 0.5 0.8

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results**Numeric Results for Z Test**

Alternative Hypothesis: $H_1: h \neq 0$

Target Power	Actual Power	N	Effect Size	Alpha
			h	
0.80	0.8016	197	0.20	0.050
0.90	0.9003	263	0.20	0.050
0.80	0.8074	32	0.50	0.050
0.90	0.9064	43	0.50	0.050
0.80	0.8224	13	0.80	0.050
0.90	0.9096	17	0.80	0.050

References

Cohen, Jacob. 1988. *Statistical Power Analysis for the Behavioral Sciences*. Lawrence Erlbaum Associates. Hillsdale, New Jersey

Report Definitions

Target Power is the desired power. May not be achieved because of integer N.

Actual Power is the achieved power. Because N is an integer, this value is often (slightly) larger than the target power.

N is the number of items sampled from the population.

Effect Size: $h = \phi_1 - \phi_0$ where $\phi_1 = 2 \text{ ArcSine}(\sqrt{P_1})$. Cohen recommended Low = 0.2, Medium = 0.5, and High = 0.8.

Alpha is the probability of rejecting a true null hypothesis.

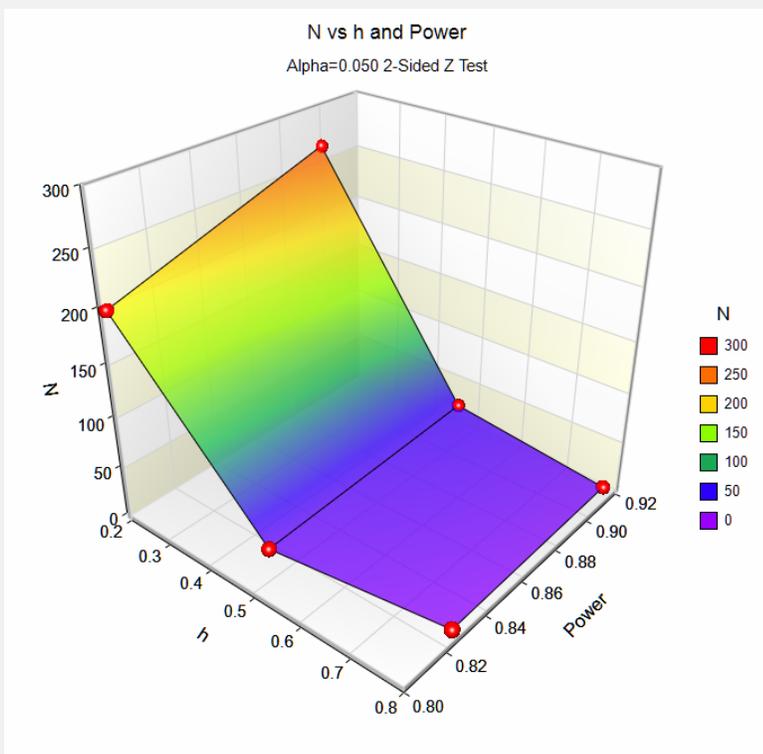
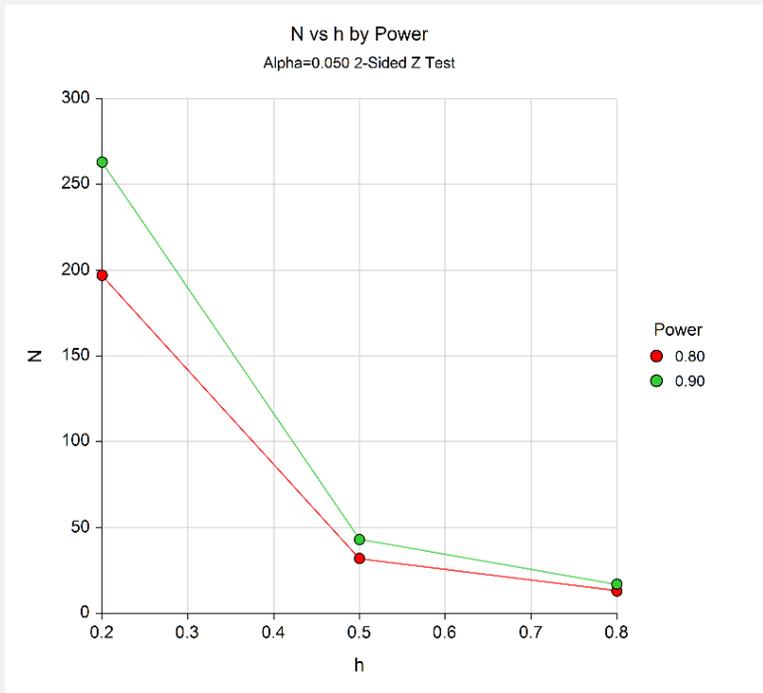
Summary Statements

A sample size of 197 achieves 80.2% power to reject the null hypothesis of zero effect size when the population effect size is 0.20 and the significance level (alpha) is 0.050 using a two-sided one-sample z test.

These reports show the values of each of the parameters, one scenario per row.

Tests for One Proportion using Effect Size

Plots Section



These plots show the relationship between effect size, power, and sample size.

Example 2 – Validation using Cohen (1988)

Cohen (1988) gives an example on page 208 of a two-sided test in which $\alpha = 0.05$, $h = 0.2$, and power = 0.95. He finds the sample size to be 325.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Test for One Proportion using Effect Size** procedure. You may then make the appropriate entries as listed below, or open **Example 2** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
Design Tab	
Solve For	Sample Size
Alternative Hypothesis	Two-Sided
Power	0.95
Alpha	0.05
h	0.2

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results for Z Test

Alternative Hypothesis: $H_1: h \neq 0$

Target Power	Actual Power	N	Effect Size		Alpha
			N	h	
0.95	0.9501	325	0.20	0.050	

PASS also calculated the sample size to be 325 which validates the procedure.