Chapter 650

Tests for One Variance

Introduction

Occasionally, researchers are interested in the estimation of the variance (or standard deviation) rather than the mean. This module calculates the sample size and performs power analysis for hypothesis tests concerning a single variance.

Technical Details

Assuming that a variable *X* is normally distributed with mean μ and variance σ^2, the sample variance is distributed as a Chi-square random variable with *N* - 1 degrees of freedom, where *N* is the sample size. That is,

$$X^2 = \frac{(N-1)s^2}{\sigma^2}$$

is distributed as a Chi-square random variable. The sample statistic, s^2, is calculated as follows

$$s^2 = \frac{\sum_{i=1}^{N}\left(X_i - \overline{X}\right)^2}{N-1}.$$

The power or sample size of a hypothesis test about the variance can be calculated using the appropriate one of the following three formulas from Ostle and Malone (1988) page 130.

Case 1: $H_0: \sigma_1^2 = \sigma_0^2 \quad versus \quad H_a: \sigma_1^2 \neq \sigma_0^2$

$$\beta = P\left(\frac{\sigma_0^2}{\sigma_1^2}\chi^2_{\alpha/2,N-1} < \chi^2 < \frac{\sigma_0^2}{\sigma_1^2}\chi^2_{1-\alpha/2,N-1}\right)$$

Case 2: $H_0: \sigma_1^2 = \sigma_0^2 \quad versus \quad H_a: \sigma_1^2 > \sigma_0^2$

$$\beta = P\left(\chi^2 < \frac{\sigma_0^2}{\sigma_1^2}\chi^2_{1-\alpha,N-1}\right)$$

Case 3: $H_0: \sigma_1^2 = \sigma_0^2 \quad versus \quad H_a: \sigma_1^2 < \sigma_0^2$

$$\beta = P\left(\chi^2 > \frac{\sigma_0^2}{\sigma_1^2}\chi^2_{\alpha,N-1}\right)$$

Procedure Options

This section describes the options that are specific to this procedure. These are located on the Design tab. For more information about the options of other tabs, go to the Procedure Window chapter.

Design Tab

The Design tab contains most of the parameters and options that you will be concerned with.

Solve For

Solve For

This option specifies the parameter to be solved for from the other parameters.

Test

Alternative Hypothesis

This option specifies the alternative hypothesis. This implicitly specifies the direction of the hypothesis test. The null hypothesis is always $H_0: \sigma_0^2 = \sigma_1^2$.

Note that the alternative hypothesis enters into power calculations by specifying the rejection region of the hypothesis test. Its accuracy is critical.

Possible selections are:

- **Ha: V0 ≠ V1**
  This selection yields a *two-tailed* test. Use this option when you are testing whether the variances are different but you do not want to specify beforehand which variance is larger.

- **Ha: V0 > V1**
  The options yields a *one-tailed* test. Use it when you are only interested in the case in which *V1* is less than *V0*.

- **Ha: V0 < V1**
  This option yields a *one-tailed* test. Use it when you are only interested in the case in which *V1* is greater than *V0*.

Known Mean

The degrees of freedom of the Chi-square test is *N* - 1 if the mean is calculated from the data (this is usually the case) or it is *N* if the mean is known. Check this box if the mean is known. This will cause an increase of the sample size by one.

Power and Alpha

Power

This option specifies one or more values for power. Power is the probability of rejecting a false null hypothesis, and is equal to one minus Beta. Beta is the probability of a type-II error, which occurs when a false null hypothesis is not rejected.

Values must be between zero and one. Historically, the value of 0.80 (Beta = 0.20) was used for power. Now, 0.90 (Beta = 0.10) is also commonly used.

A single value may be entered here or a range of values such as *0.8 to 0.95 by 0.05* may be entered.

If your only interest is in determining the appropriate sample size for a confidence interval, set power or beta to 0.5.

Alpha

This option specifies one or more values for the probability of a type-I error. A type-I error occurs when a true null hypothesis is rejected.

Values must be between zero and one. Historically, the value of 0.05 has been used for alpha. This means that about one test in twenty will falsely reject the null hypothesis. You should pick a value for alpha that represents the risk of a type-I error you are willing to take in your experimental situation.

You may enter a range of values such as *0.01 0.05 0.10* or *0.01 to 0.10 by 0.01*.

Sample Size

N (Sample Size)

This is the number of observations in the study.

Effect Size

Scale

Specify whether *V0* and *V1* are variances or standard deviations.

V0 (Baseline Variance)

Enter one or more value(s) of the baseline variance. This variance will be compared to the alternative variance. It must be greater than zero.

Actually, only the ratio of the two variances (or standard deviations) is used, so you can enter a one here and enter the ratio value in the *V1* box.

If Scale is *Standard Deviation* this value is treated as a standard deviation rather than a variance.

V1 (Alternative Variance)

Enter one or more value(s) of the alternative variance. This variance will be compared to the baseline variance. It must be greater than zero.

Actually, only the ratio of the two variances (or standard deviations) is used, so you can enter a one for *V0* and enter a ratio value here.

If Scale is *Standard Deviation* this value is treated as a standard deviation rather than a variance.

Example 1 – Calculating the Power

A machine used to perform a particular analysis is to be replaced with a new type of machine if the new machine reduces the variation in the output. The current machine has been tested repeatedly and found to have an output variance of 42.5. The new machine will be cost effective if it can reduce the variance by 30% to 29.75. If the significance level is set to 0.05, calculate the power for sample sizes of 10, 50, 90, 130, 170, 210, and 250.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Tests for One Variance** procedure window by expanding **Variances**, then clicking on **One Variance**, and then clicking on **Tests for One Variance**. You may then make the appropriate entries as listed below, or open **Example 1** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Power**
Alternative Hypothesis	**Ha: V0 > V1**
Known Mean	**Not checked**
Alpha	**0.05**
N (Sample Size)	**10 50 90 130 170 210 250**
Scale	**Variance**
V0 (Baseline Variance)	**42.5**
V1 (Alternative Variance)	**29.75**

Annotated Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results when H0: V0 = V1 versus Ha: V0 > V1

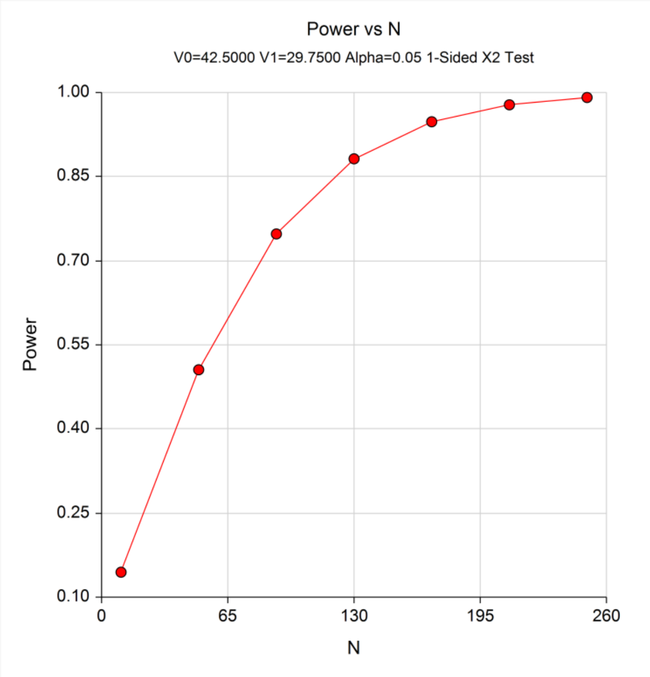
Power	N	V0	V1	Alpha	Beta
0.14448	10	42.5000	29.7500	0.05000	0.85552
0.50556	50	42.5000	29.7500	0.05000	0.49444
0.74775	90	42.5000	29.7500	0.05000	0.25225
0.88174	130	42.5000	29.7500	0.05000	0.11826
0.94785	170	42.5000	29.7500	0.05000	0.05215
0.97806	210	42.5000	29.7500	0.05000	0.02194
0.99111	250	42.5000	29.7500	0.05000	0.00889

Report Definitions
Power is the probability of rejecting a false null hypothesis. It should be close to one.
N is the size of the sample drawn from the population.
V0 is the value of the population variance under the null hypothesis.
V1 is the value of the population variance under the alternative hypothesis.
Alpha is the probability of rejecting a true null hypothesis. It should be small.
Beta is the probability of accepting a false null hypothesis. It should be small.

Summary Statements
A sample size of 10 achieves 14% power to detect a difference of 12.7500 between the null hypothesis variance of 42.5000 and the alternative hypothesis variance of 29.7500 using a one-sided, Chi-square hypothesis test with a significance level (alpha) of 0.050000.

This report shows the calculated power for each scenario.

Plots Section



This plot shows the power versus the sample size. We see that a sample size of about 150 is necessary to achieve a power of 0.90.

Example 2 – Calculating Sample Size

Continuing with the previous example, the analyst wants to find the necessary sample sizes to achieve a power of 0.9, for two significance levels, 0.01 and 0.05, and for several variance values. To make interpreting the output easier, the analyst decides to switch from the absolute scale to a ratio scale. To accomplish this, the baseline variance is set at 1.0 and the alternative variances of 0.2, 0.3, 0.4, 0.5, 0.6, and 0.7 are tried.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Tests for One Variance** procedure window by expanding **Variances**, then clicking on **One Variance**, and then clicking on **Tests for One Variance**. You may then make the appropriate entries as listed below, or open **Example 2** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Sample Size**
Alternative Hypothesis	**Ha: V0 > V1**
Known Mean	**Not checked**
Power	**0.90**
Alpha	**0.01 0.05**
Scale	**Variance**
V0 (Baseline Variance)	**1.0**
V1 (Alternative Variance)	**0.2 to 0.7 by 0.1**

Output

Click the Calculate button to perform the calculations and generate the following output.

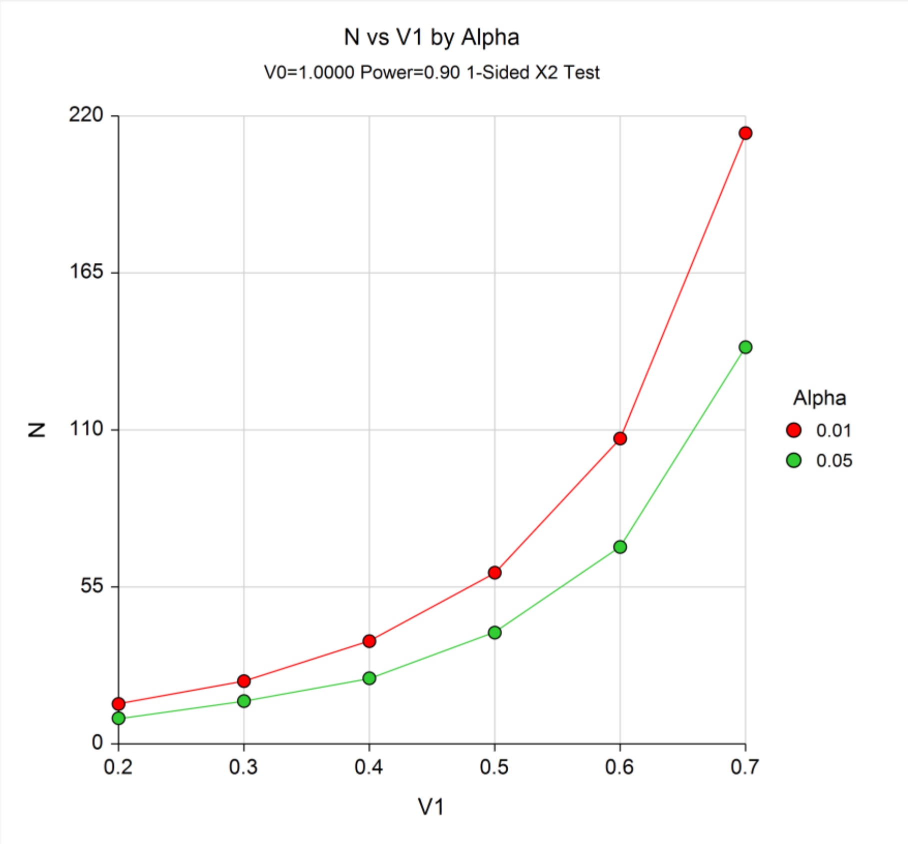
Numeric Results

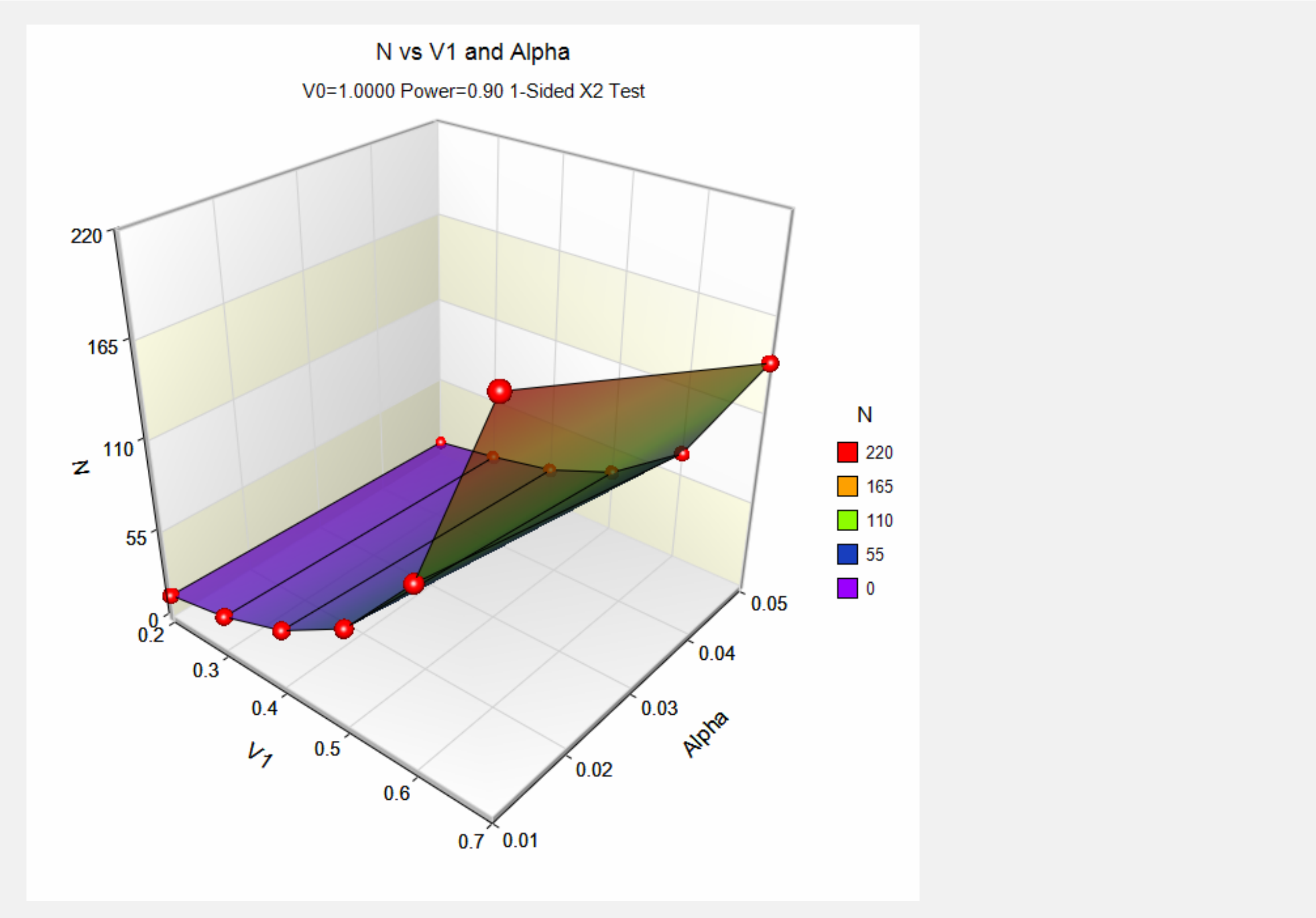
Numeric Results when H0: V0 = V1 versus Ha: V0 > V1

Power	N	V0	V1	Alpha	Beta
0.91734	14	1.0000	0.2000	0.01000	0.08266
0.90902	9	1.0000	0.2000	0.05000	0.09098
0.90091	22	1.0000	0.3000	0.01000	0.09909
0.91935	15	1.0000	0.3000	0.05000	0.08065
0.90368	36	1.0000	0.4000	0.01000	0.09632
0.90067	23	1.0000	0.4000	0.05000	0.09933
0.90163	60	1.0000	0.5000	0.01000	0.09837
0.90423	39	1.0000	0.5000	0.05000	0.09577
0.90126	107	1.0000	0.6000	0.01000	0.09874
0.90078	69	1.0000	0.6000	0.05000	0.09922
0.90060	214	1.0000	0.7000	0.01000	0.09940
0.90117	139	1.0000	0.7000	0.05000	0.09883

This report shows the necessary sample size for each scenario.

Plots Section





These plots show the necessary sample size for various values of *V1*. Note that as *V1* gets farther from zero, the required sample size increases.

Example 3 – Validation using Zar

Zar (1984) page 117 presents an example with *V0* = 1.5, *V1* = 2.6898, *N* = 40, *Alpha* = 0.05, and *Power* = 0.84.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Tests for One Variance** procedure window by expanding **Variances**, then clicking on **One Variance**, and then clicking on **Tests for One Variance**. You may then make the appropriate entries as listed below, or open **Example 3** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Power**
Alternative Hypothesis	**Ha: V0 < V1**
Known Mean	**Not checked**
Alpha	**0.05**
N (Sample Size)	**40**
Scale	**Variance**
V0 (Baseline Variance)	**1.5**
V1 (Alternative Variance)	**2.6898**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results when H0: V0 = V1 versus Ha: V0 < V1

Power	N	V0	V1	Alpha	Beta
0.83517	40	1.5000	2.6898	0.05000	0.16483

PASS calculated the power at 0.835167 which matches Zar's result of 0.84 within rounding.

Example 4 – Validation using Davies

Davies (1971) page 40 presents an example of determining *N* when (in the standard deviation scale) *V0* = 0.04, *V1* = 0.10, Alpha = 0.05, and Power = 0.99. Davies calculates *N* to be 13.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Tests for One Variance** procedure window by expanding **Variances**, then clicking on **One Variance**, and then clicking on **Tests for One Variance**. You may then make the appropriate entries as listed below, or open **Example 4** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Sample Size**
Alternative Hypothesis	**Ha: V0 < V1**
Known Mean	**Not checked**
Power	**0.99**
Alpha	**0.05**
Scale	**Standard Deviation**
V0 (Baseline Variance)	**0.04**
V1 (Alternative Variance)	**0.10**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results when H0: S0 = S1 versus Ha: S0 < S1

Power	N	S0	S1	Alpha	Beta
0.99238	13	0.0400	0.1000	0.05000	0.00762

PASS calculated an *N* of 13 which matches Davies' result.