

## Chapter 820

# Tests for Two Coefficient Alphas

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## Introduction

*Coefficient alpha*, or *Cronbach's alpha*, is a popular measure of the reliability of a scale consisting of  $k$  parts. The  $k$  parts often represent  $k$  items on a questionnaire (scale) or  $k$  raters. This module calculates power and sample size for testing whether two coefficient alphas are different when the two samples are either dependent or independent.

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## Technical Details

Feldt et al. (1999) presents methods for testing one-, or two-, sided hypotheses about two coefficient alphas, which we label  $\rho_1$  and  $\rho_2$ . The results assume that  $N_1$  observations for each of  $k_1$  items are available for one scale and  $N_2$  observations for each of  $k_2$  items are available for another scale. These sets of observations may either be from two independent groups of subjects (independent case) or two sets of observations on each subject (dependent case). In the dependent case,  $N_1 = N_2$  and the correlation coefficient between the overall scores of each scale is represented by  $\phi$ . For the independent case  $\phi = 0$ .

Suppose  $\hat{\rho}_1$  and  $\hat{\rho}_2$  are the sample estimates of  $\rho_1$  and  $\rho_2$ , respectively. Hypothesis tests are based on the result that the test statistic,

$$W = \left( \frac{1 - \hat{\rho}_2}{1 - \hat{\rho}_1} \right) \left( \frac{1 - \rho_1}{1 - \rho_2} \right) = \hat{\delta} \left( \frac{1 - \rho_1}{1 - \rho_2} \right),$$

is approximately distributed as a central  $F$  variable with degrees of freedom  $\nu_1$  and  $\nu_2$ . The values of  $\nu_1$  and  $\nu_2$  depend on  $N_1$ ,  $N_2$ ,  $k_1$ ,  $k_2$ , and  $\phi$ .

Also define

$$c_i = (N_i - 1)(k_i - 1), \quad i = 1, 2$$

## Tests for Two Coefficient Alphas

**Independent Case**

When the two scales are independent, there are two situations that must be considered separately. If  $c_i > 1000$  and  $k_i > 25$ , the values of  $\nu_1$  and  $\nu_2$  are computed using

$$\nu_1 = N_1 - 1$$

$$\nu_2 = N_2 - 1$$

otherwise, they are computed using

$$\nu_1 = \frac{2A^2}{2B - AB - A^2}$$

$$\nu_2 = \frac{2A}{A - 1}$$

where

$$A = \frac{c_1(N_2 - 1)}{(c_1 - 2)(N_2 - 3)}$$

$$B = \frac{(N_1 + 1)(N_2 - 1)^2(c_2 + 2)c_1^2}{(N_2 - 3)(N_2 - 5)(N_1 - 1)(c_1 - 2)(c_1 - 4)c_2}$$

**Dependent Case**

When the two scales are dependent, it follows that  $N_1 = N_2 = N$ . There are two situations that must be considered separately.

If  $c_i > 1000$  and  $k_i > 25$ , the values of  $\nu_1$  and  $\nu_2$  are computed using

$$\nu_1 = \nu_2 = \frac{N - 1 - 7\phi^2}{1 - \phi^2}$$

otherwise, they are computed using

$$\nu_1 = \frac{2M^2}{V(2 - M) - M^2(M - 1)}$$

$$\nu_2 = \frac{2M}{M - 1}$$

where

$$M = A - \frac{2\phi^2}{N - 1}$$

$$V = B - A^2 - \frac{4\phi^2}{N - 1}$$

## Calculating the Power

Let  $\rho_{20}$  be the value of coefficient alpha in the second set under  $H_0$ ,  $\rho_{21}$  be the value of coefficient alpha in the second set at which the power is calculated, and  $\rho_1$  be the value of coefficient alpha in the first set. The power of the one-sided hypothesis that  $H_0: \rho_{20} \leq \rho_1$  versus the alternative that  $H_1: \rho_{20} > \rho_1$  is calculated as follows:

1. Find  $F_\alpha$  such that  $\text{Prob}(F < F_{\alpha, v_1, v_2}) = \alpha$
2. Compute  $\delta' = \frac{1}{F_\alpha} \left( \frac{1 - \rho_1}{1 - \rho_{20}} \right)$
3. Compute  $W_1 = \left( \frac{1 - \rho_1}{1 - \rho_{21}} \right) \delta'$
4. Compute the power =  $1 - \text{Pr}(W_1 > F_{v_1, v_2})$

## Procedure Options

This section describes the options that are specific to this procedure. These are located on the Design tab. For more information about the options of other tabs, go to the Procedure Window chapter.

### Design Tab

The Design tab contains most of the parameters and options of interest for this procedure.

#### Solve For

##### Solve For

This option specifies the parameter to be calculated from the values of the other parameters. Under most conditions, you would either select *Power* or *Sample Size (NI)*.

Select *Sample Size (NI)* when you want to determine the sample size needed to achieve a given power and alpha error level.

Select *Power* when you want to calculate the power of an experiment.

#### Test

##### Alternative Hypothesis

This option specifies whether the alternative hypothesis is one-sided or two-sided. It also specifies the direction of the hypothesis test. The null hypothesis is  $H_0: \rho_1 = \rho_2$ . The alternative hypothesis enters into power calculations by specifying the rejection region of the hypothesis test. Its accuracy is critical.

## Tests for Two Coefficient Alphas

Possible selections are:

- **H1: CA1  $\neq$  CA2**  
This is the most common selection. It yields the *two-tailed* test. Use this option when you are testing whether values are different, but you do not want to specify beforehand which is larger.
- **H1: CA1 < CA2**  
This option yields a *one-tailed* test.
- **H1: CA1 > CA2**  
This option also yields a *one-tailed* test.

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## Power and Alpha

### Power

This option specifies one or more values for power. Power is the probability of rejecting a false null hypothesis, and is equal to one minus Beta. Beta is the probability of a type-II error, which occurs when a false null hypothesis is not rejected.

Values must be between zero and one. Historically, the value of 0.80 (Beta = 0.20) was used for power. Now, 0.90 (Beta = 0.10) is also commonly used.

A single value may be entered here or a range of values such as *0.8 to 0.95 by 0.05* may be entered.

### Alpha

This option specifies one or more values for the probability of a type-I error. A type-I error occurs when you reject the null hypothesis when in fact it is true.

Values of alpha must be between zero and one. Historically, the value of 0.05 has been used for alpha. This means that about one test in twenty will falsely reject the null hypothesis. You should pick a value for alpha that represents the risk of a type-I error you are willing to take in your experimental situation.

You may enter a range of values such as *0.01 0.05 0.10* or *0.01 to 0.10 by 0.01*.

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## Sample Size – Set 1

### N1 (Sample Size in Set 1)

N1 is the sample size (number of observations or subjects) in dataset one. Feldt (1999) states that it is ill-advised to use sample sizes less than 30.

You may enter a list of values separated with blanks or a range of values.

### K1 (Items/Scale in Set 1)

K1 is the number of items or raters in dataset one. Since it is a count, it must be an integer. K1 must be greater than or equal to two.

You may enter a list of values separated with blanks or a range of values.

## Tests for Two Coefficient Alphas

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### Sample Size – Set 1

#### N2 (Sample Size in Set 2)

N2 is the sample size (number of observations or subjects) in dataset two. Feldt (1999) states that it is ill-advised to use sample sizes less than 30. Note that if phi is non-zero, this value will be forced equal to N1 regardless of what is entered here.

For ease of input, when phi is non-zero, you can enter multiples of N1. For example, all of the following entries are allowed:

N1 2N1 0.5N1

You may enter a list of values separated with blanks or a range of values.

#### K2 (Items/Scale in Set 2)

K2 is the number of items or raters in dataset two. Since it is a count, it must be an integer greater than or equal to two.

You may enter a list of values separated with blanks or a range of values.

For ease of input, you can enter multiples of K1. For example, all of the following entries are allowed:

K1 2K1 0.5K1

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### Effect Size – Coefficient Alpha Set 1

#### CA1 (Coefficient Alpha Set 1)

Specify the value of  $\rho_1$ , the value of coefficient alpha, for dataset one. Often, this value will be zero, but any value between -1 and 1 is valid as long as it is not equal to CA21.

You may enter a list of values separated with blanks or a range of values.

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### Effect Size – Coefficient Alpha Set 2

#### CA20 (Coefficient Alpha Set 2|H0)

Enter the value of coefficient alpha in dataset two under the null hypothesis. The null hypothesis is  $H_0: CA1=CA20$ , so often you will set this value equal to CA1. Any value between -1 and 1 (non-inclusive) is valid as long as it is not equal to CA21.

You may enter CA1 to indicate that you want the value of CA1 copied here. You may also enter a multiple of CA1 such as  $1.5CA1$ .

You may enter a list of values separated with blanks or a range of values.

#### CA21 (Coefficient Alpha Set 2|H1)

Enter the value of coefficient alpha in dataset two at which the power is computed. Any value between -1 and 1 (non-inclusive) is valid as long as it is not equal to CA20. The values of CA20 and CA21 should match the direction set by the 'Alternative Hypothesis' option.

You may enter a list of values separated with blanks or a range of values.

## Tests for Two Coefficient Alphas

### Effect Size – Correlation

#### Phi (Correlation Between Sets)

This option implicitly specifies the two datasets as being either independent (datasets on different subjects) or dependent (both datasets on the same subjects). Suppose you calculate the average score for each subject for both dataset one and dataset two. This parameter is the correlation between those two averages over all subjects. If the correlation is zero, the two datasets are assumed to be independent. That is, it is assumed that they come from different sets of subjects.

If the correlation is non-zero, the two datasets are assumed to be dependent. That is, it is assumed that both sets of items were measured on the same subjects. Typical values in this case are between 0.2 and 0.7. When the datasets are dependent, it is assumed that  $N_1=N_2$ .

Since this is a correlation, the theoretical range is from -1 to 1. Typical values are 0.0 for independent designs and between 0.2 and 0.7 for dependent designs.

### Example 1 – Finding the Power

Suppose a study is being designed to compare the coefficient alphas of two scales. The researchers are going to use a two-sided F-test at a significance level of 0.05. Past experience has shown that  $CA_1$  is approximately 0.4. The researchers will use different subjects in each dataset. Find the power when  $K_1 = K_2 = 10$ ,  $CA_2 = CA_1$ ,  $N_1 = 50, 100, 150, 200, 250, \text{ and } 300$ ,  $N_2 = N_1$ , and  $CA_2 = 0.6 \text{ and } 0.7$ .

### Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Tests for Two Coefficient Alphas** procedure window by expanding **Correlation**, then clicking on **Coefficient (Cronbach's) Alpha**, and then clicking on **Tests for Two Coefficient Alphas**. You may then make the appropriate entries as listed below, or open **Example 1** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
<b>Design Tab</b>	
Solve For .....	<b>Power</b>
H1 (Alternative Hypothesis) .....	<b>H1: CA1 ≠ CA2</b>
Alpha .....	<b>0.05</b>
N1 (Sample Size in Set 1) .....	<b>50 to 300 by 50</b>
K1 (Items/Scale in Set 1) .....	<b>10</b>
N2 (Sample Size in Set 2) .....	<b>N1</b>
K2 (Items/Scale in Set 2) .....	<b>K1</b>
CA1 (Coefficient Alpha Set 1) .....	<b>0.4</b>
CA20 (Coefficient Alpha Set 2 H0) .....	<b>CA1</b>
CA21 (Coefficient Alpha Set 2 H1) .....	<b>0.60 0.70</b>
Phi (Correlation Between Sets) .....	<b>0</b>

## Tests for Two Coefficient Alphas

## Annotated Output

Click the Calculate button to perform the calculations and generate the following output.

## Numeric Results

**Numeric Results for Comparing Two Coefficient Alphas**  
**H1: CA1 ≠ CA2**

	Sample Sizes	Number of Items	Coef. Alpha Set 2 H1 (CA21)	Coef. Alpha Set 2 H0 (CA20)	Coef. Alpha Set 1 (CA1)	Signif. Level (Alpha)	Corr. Between Datasets (Phi)	Beta
Power	N1/N2	K1/K2						
0.2642	50/50	10/10	0.6000	0.4000	0.4000	0.0500	0.0000	0.7358
0.4775	100/100	10/10	0.6000	0.4000	0.4000	0.0500	0.0000	0.5225
0.6481	150/150	10/10	0.6000	0.4000	0.4000	0.0500	0.0000	0.3519
0.7725	200/200	10/10	0.6000	0.4000	0.4000	0.0500	0.0000	0.2275
0.8576	250/250	10/10	0.6000	0.4000	0.4000	0.0500	0.0000	0.1424
0.9132	300/300	10/10	0.6000	0.4000	0.4000	0.0500	0.0000	0.0868
0.6253	50/50	10/10	0.7000	0.4000	0.4000	0.0500	0.0000	0.3747
0.9026	100/100	10/10	0.7000	0.4000	0.4000	0.0500	0.0000	0.0974
0.9793	150/150	10/10	0.7000	0.4000	0.4000	0.0500	0.0000	0.0207
0.9961	200/200	10/10	0.7000	0.4000	0.4000	0.0500	0.0000	0.0039
0.9993	250/250	10/10	0.7000	0.4000	0.4000	0.0500	0.0000	0.0007
0.9999	300/300	10/10	0.7000	0.4000	0.4000	0.0500	0.0000	0.0001

**Report Definitions**

H0, H1 abbreviate the null and alternative hypotheses, respectively.

Power is the probability of rejecting H0 when it is false. It should be close to one.

N1 & N2 are the sample sizes of datasets one and two, respectively.

K1 & K2 are the number of items in datasets one and two, respectively.

CA21 is the coefficient alpha in dataset two at which the power is calculated.

CA20 is the coefficient alpha in dataset two under H0.

CA1 is the coefficient alpha in dataset one.

Phi is the correlation between the average scores of each of the two datasets.

Alpha is the probability of a type-I error: rejecting H0 when it is true.

Beta is the probability of a type-II error: accepting H0 when it is false. Power = 1 - Beta.

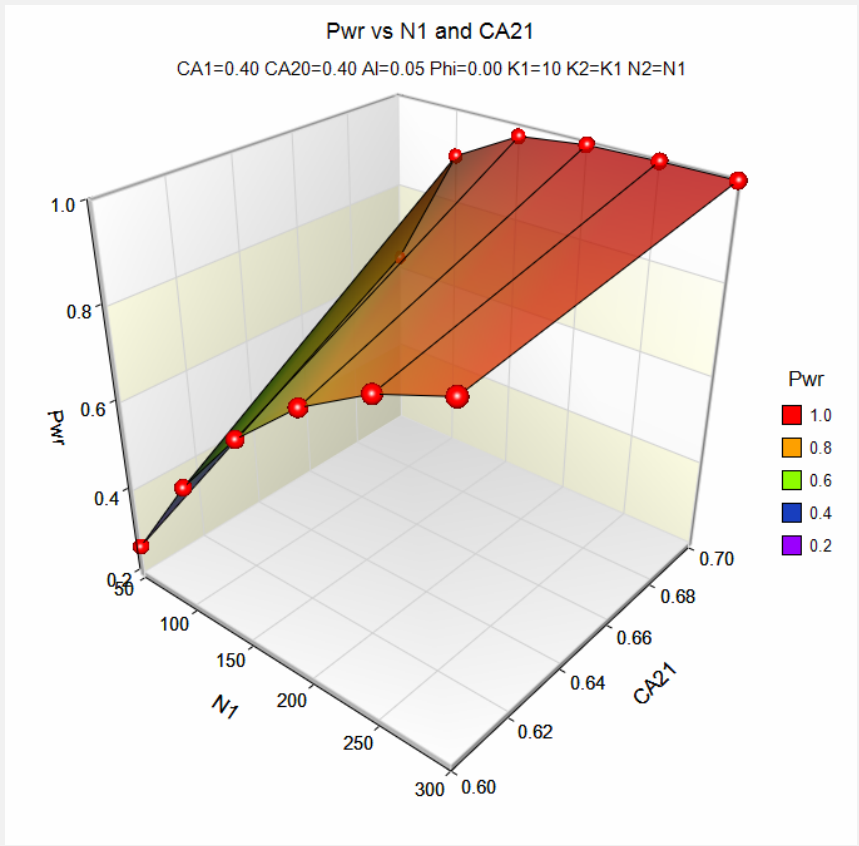
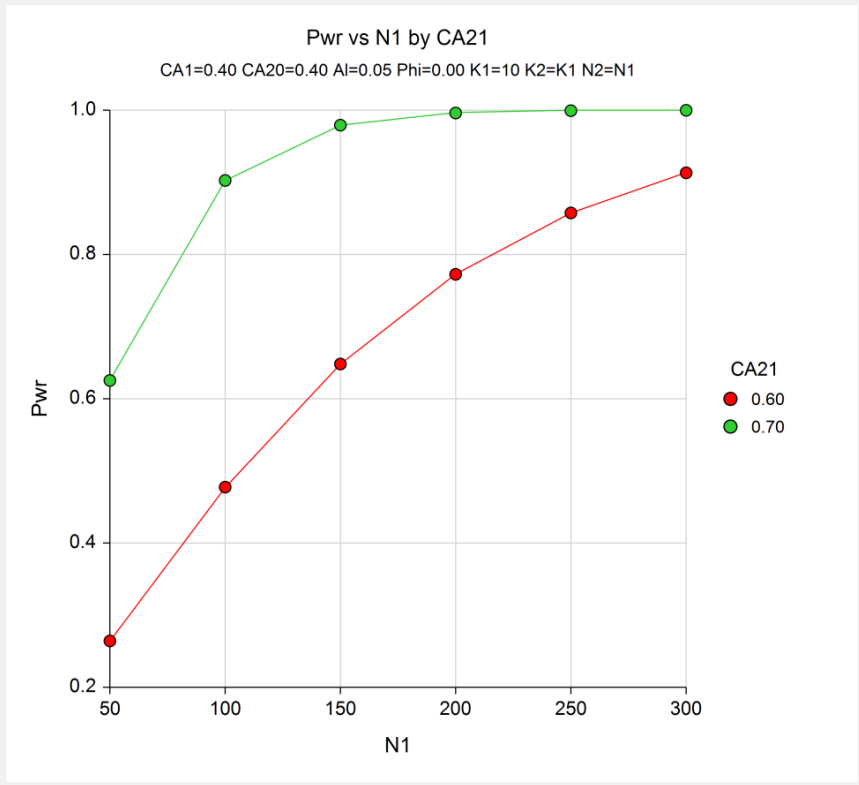
**Summary Statements**

Samples of 10 items on 50 subjects in dataset one and 10 items on 50 subjects in dataset two achieve 26% power to detect the difference between the coefficient alphas in the two datasets. Under the null hypothesis, the coefficient alphas in datasets one and two are 0.4000 and 0.4000, respectively. The power is computed assuming that the coefficient alpha of dataset two is actually 0.6000. The test statistic used is the two-sided F-test. The significance level of the test was 0.0500.

This report shows the values of each of the parameters, one scenario per row. The values from this table are plotted in the chart below.

Tests for Two Coefficient Alphas

Plots Section



This plot shows the relationship between CA21, N1, and power.



## Tests for Two Coefficient Alphas

**Example 2 – Finding the Sample Size**

Continuing with the last example, find the sample size necessary to achieve a power of 90% at the 0.05 significance level.

**Setup**

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Tests for Two Coefficient Alphas** procedure window by expanding **Correlation**, then clicking on **Coefficient (Cronbach's) Alpha**, and then clicking on **Tests for Two Coefficient Alphas**. You may then make the appropriate entries as listed below, or open **Example 2** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
<b>Design Tab</b>	
Solve For .....	<b>Sample Size (N1)</b>
Alternative Hypothesis (H1) .....	<b>H1: CA1 ≠ CA2</b>
Power .....	<b>0.90</b>
Alpha .....	<b>0.05</b>
K1 (Items/Scale in Set 1) .....	<b>10</b>
N2 (Sample Size in Set 2) .....	<b>N1</b>
K2 (Items/Scale in Set 2) .....	<b>K1</b>
CA1 (Coefficient Alpha Set 1) .....	<b>0.4</b>
CA20 (Coefficient Alpha Set 2 H0) .....	<b>CA1</b>
CA21 (Coefficient Alpha Set 2 H1) .....	<b>0.60 0.70</b>
Phi (Correlation Between Sets) .....	<b>0</b>

**Output**

Click the Run button to perform the calculations and generate the following output.

Numeric Results for Comparing Two Coefficient Alphas								
H1: CA1 ≠ CA2								
	Sample Sizes	Number of Items	Coef. Alpha Set 2 H1 (CA21)	Coef. Alpha Set 2 H0 (CA20)	Coef. Alpha Set 1 (CA1)	Signif. Level (Alpha)	Corr. Between Datasets (Phi)	Beta
Power	N1/N2	K1/K2						
0.9000	286/286	10/10	0.6000	0.4000	0.4000	0.0500	0.0000	0.1000
0.9026	100/100	10/10	0.7000	0.4000	0.4000	0.0500	0.0000	0.0974

This report shows that 286 subjects per dataset are needed when CA21 is 0.60 and 100 subjects per dataset are needed when CA21 is 0.70.

## Tests for Two Coefficient Alphas

**Example 3 – Validation using Feldt**

Feldt et al. (1999) presents an example in which  $CA1 = 0$ ,  $CA20 = 0$ ,  $CA21 = 0.5$ ,  $\alpha = 0.05$ ,  $\Phi = 0$ ,  $N1 = N2 = 60$ , and  $k = 5$ . They find the power of a one-sided test to be 0.761.

**Setup**

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Tests for Two Coefficient Alphas** procedure window by expanding **Correlation**, then clicking on **Coefficient (Cronbach's) Alpha**, and then clicking on **Tests for Two Coefficient Alphas**. You may then make the appropriate entries as listed below, or open **Example 3** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
<b>Design Tab</b>	
Solve For .....	<b>Power</b>
Alternative Hypothesis (H1) .....	<b>H1: CA1 &lt; CA2</b>
Alpha.....	<b>0.05</b>
N1 (Sample Size in Set 1) .....	<b>60</b>
K1 (Items/Scale in Set 1) .....	<b>5</b>
N2 (Sample Size in Set 2) .....	<b>N1</b>
K2 (Items/Scale in Set 2) .....	<b>K1</b>
CA1 (Coefficient Alpha Set 1).....	<b>0.0</b>
CA20 (Coefficient Alpha Set 2 H0) .....	<b>CA1</b>
CA21 (Coefficient Alpha Set 2 H1) .....	<b>0.5</b>
Phi (Correlation Between Sets) .....	<b>0</b>

**Output**

Click the Calculate button to perform the calculations and generate the following output.

**Numeric Results**

Numeric Results for Comparing Two Coefficient Alphas								
H1: CA1 < CA2								
	Sample Sizes N1/N2	Number of Items K1/K2	Coef. Alpha Set 2 H1 (CA21)	Coef. Alpha Set 2 H0 (CA20)	Coef. Alpha Set 1 (CA1)	Signif. Level (Alpha)	Corr. Between Datasets (Phi)	Beta
Power	60/60	5/5	0.5000	0.0000	0.0000	0.0500	0.0000	0.2345

Note that **PASS's** result is slightly different from Feldt's because **PASS** uses fractional degrees of freedom and Feldt rounds to the closest integer. Although the difference in power is small, allowing fractional degrees of freedom is more accurate.