

## Chapter 199

# Tests for Two Proportions using Effect Size

## Introduction

This procedure provides sample size and power calculations for one- or two-sided hypothesis tests of the difference between two independent proportions using the effect size. The details of procedure are given in Cohen (1988). The design corresponding to this test procedure is sometimes referred to as a *parallel-groups* design. In this design, two proportions from independent populations are compared by considering their difference. The difference is formed between transformed values of the proportions, formed to create variables that are more normally distributed than the raw proportions and that have a variance not related to the values of the proportions.

## Test Procedure

If we assume that  $P_1$  and  $P_2$  represent the two proportions. The effect size is represented by the difference  $h$  formed as follows

$$h = \varphi_1 - \varphi_2$$

where

$$\varphi_i = 2 \arcsine(\sqrt{P_i})$$

This is referred to as the arcsine, the arcsine root, or the angular transformation.

The null hypothesis is  $H_0: h = 0$  and the alternative hypothesis depends on the number of “sides” of the test:

$$\text{Two-Sided:} \quad H_1: h \neq 0 \text{ or } H_1: \varphi_1 - \varphi_2 \neq 0$$

$$\text{Upper One-Sided:} \quad H_1: h > 0 \text{ or } H_1: \varphi_1 - \varphi_2 > 0$$

$$\text{Lower One-Sided:} \quad H_1: h < 0 \text{ or } H_1: \varphi_1 - \varphi_2 < 0$$

A suitable Type I error probability ( $\alpha$ ) is chosen for the test, the data is collected, and a  $z$ -statistic is generated using the formula

$$z = \frac{\hat{\varphi}_1 - \hat{\varphi}_2}{\sqrt{\frac{N_1 + N_2}{2N_1N_2}}}$$

## Tests for Two Proportions using Effect Size

This  $z$ -statistic follows a standard normal distribution. The null hypothesis is rejected in favor of the alternative if,

for  $H_1: h \neq 0$ ,  $z < z_{\alpha/2}$  or  $z > z_{1-\alpha/2}$

for  $H_1: h > 0$ ,  $z > z_{1-\alpha}$

for  $H_1: h < 0$ ,  $z < z_{\alpha}$

Comparing the  $z$ -statistic to the cut-off  $z$ -value is equivalent to comparing the  $p$ -value to  $\alpha$ .

## Power Calculation

The power of a one-sided test is calculated using the formulation of Cohen (1988):

$$z_{1-\beta} = h \sqrt{\frac{N'}{2}} - z_{1-\alpha}$$

where

$$N' = \frac{2N_1N_2}{N_1+N_2}$$

## The Effect Size

As stated above, the effect size  $h$  is given by  $h = \varphi_1 - \varphi_2$ . Cohen (1988) proposed the following interpretation of the  $h$  values. An  $h$  near 0.2 is a *small* effect, an  $h$  near 0.5 is a *medium* effect, and an  $h$  near 0.8 is a *large* effect. These values for small, medium, and large effects are popular in the social sciences.

Cohen (1988) remarks that the value of  $h$  does not match directly with the value of  $P_1 - P_2$ , so care must be taken when using it. For example, all of the following pairs of values of  $P_1$  and  $P_2$  result in an  $h$  of about 0.30, even though the actual differences  $P_1 - P_2$  are quite different.

$P_1$	$P_2$	$P_1 - P_2$	$h$
0.21	0.10	0.11	0.3
0.39	0.25	0.14	0.3
0.55	0.40	0.15	0.3
0.65	0.50	0.15	0.3
0.78	0.60	0.18	0.3
0.87	0.75	0.13	0.3
0.97	0.90	0.07	0.3

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## Procedure Options

This section describes the options that are specific to this procedure. These are located on the Design tab. For more information about the options of other tabs, go to the Procedure Window chapter.

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### Design Tab

The Design tab contains most of the parameters and options that you will be concerned with.

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#### Solve For

##### Solve For

This option specifies the parameter to be solved for from the other parameters. The parameters that may be selected are *Power*, *Sample Size*, *Effect Size*, and *Alpha*. In most situations, you will likely select either *Power* or *Sample Size*.

The 'Solve For' parameter is the parameter that will be displayed on the vertical axis of any plots that are shown.

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#### Test Direction

##### Alternative Hypothesis

Specify whether the alternative hypothesis of the test is one-sided or two-sided. If a one-sided test is chosen, the hypothesis test direction is chosen based on whether the effect size is greater than or less than zero.

##### Two-Sided Hypothesis Test

$H_0: h = 0$  vs.  $H_1: h \neq 0$

##### One-Sided Hypothesis Tests

Upper:  $H_0: h \leq 0$  vs.  $H_1: h > 0$

Lower:  $H_0: h \geq 0$  vs.  $H_1: h < 0$

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#### Power and Alpha

##### Power

Power is the probability of rejecting the null hypothesis when it is false. Power is equal to  $1 - \text{Beta}$ , so specifying power implicitly specifies beta. Beta is the probability of obtaining a false negative with the statistical test. That is, it is the probability of accepting a false null hypothesis.

The valid range is 0 to 1. Different disciplines have different standards for setting power. The most common choice is 0.90, but 0.80 is also popular.

You can enter a single value, such as 0.90, or a series of values, such as .70 .80 .90, or .70 to .90 by .1.

When a series of values is entered, PASS will generate a separate calculation result for each value of the series.

##### Alpha

Alpha is the probability of obtaining a false positive with the statistical test. That is, it is the probability of rejecting a true null hypothesis. The null hypothesis is usually that the parameters of interest (means, proportions, etc.) are equal.

Since Alpha is a probability, it is bounded by 0 and 1. Commonly, it is between 0.001 and 0.10. Alpha is often set to 0.05 for two-sided tests and to 0.025 for one-sided tests. You can enter a single value, such as 0.05, or a series of values, such as .05 .10 .15, or .05 to .15 by .01. When a series of values is entered, **PASS** will generate a separate calculation result for each value of the series.

## Tests for Two Proportions using Effect Size

**Sample Size (When Solving for Sample Size)****Group Allocation**

Select the option that describes the constraints on  $N1$  or  $N2$  or both.

The options are

- **Equal ( $N1 = N2$ )**  
This selection is used when you wish to have equal sample sizes in each group. Since you are solving for both sample sizes at once, no additional sample size parameters need to be entered.
- **Enter  $N1$ , solve for  $N2$**   
Select this option when you wish to fix  $N1$  at some value (or values), and then solve only for  $N2$ . Please note that for some values of  $N1$ , there may not be a value of  $N2$  that is large enough to obtain the desired power.
- **Enter  $N2$ , solve for  $N1$**   
Select this option when you wish to fix  $N2$  at some value (or values), and then solve only for  $N1$ . Please note that for some values of  $N2$ , there may not be a value of  $N1$  that is large enough to obtain the desired power.
- **Enter  $R = N2/N1$ , solve for  $N1$  and  $N2$**   
For this choice, you set a value for the ratio of  $N2$  to  $N1$ , and then **PASS** determines the needed  $N1$  and  $N2$ , with this ratio, to obtain the desired power. An equivalent representation of the ratio,  $R$ , is
 
$$N2 = R * N1.$$
- **Enter percentage in Group 1, solve for  $N1$  and  $N2$**   
For this choice, you set a value for the percentage of the total sample size that is in Group 1, and then **PASS** determines the needed  $N1$  and  $N2$  with this percentage to obtain the desired power.

 **$N1$  (Sample Size, Group 1)**

*This option is displayed if Group Allocation = "Enter  $N1$ , solve for  $N2$ "*

$N1$  is the number of items or individuals sampled from the Group 1 population.

$N1$  must be  $\geq 2$ . You can enter a single value or a series of values.

 **$N2$  (Sample Size, Group 2)**

*This option is displayed if Group Allocation = "Enter  $N2$ , solve for  $N1$ "*

$N2$  is the number of items or individuals sampled from the Group 2 population.

$N2$  must be  $\geq 2$ . You can enter a single value or a series of values.

 **$R$  (Group Sample Size Ratio)**

*This option is displayed only if Group Allocation = "Enter  $R = N2/N1$ , solve for  $N1$  and  $N2$ ."*

$R$  is the ratio of  $N2$  to  $N1$ . That is,

$$R = N2 / N1.$$

Use this value to fix the ratio of  $N2$  to  $N1$  while solving for  $N1$  and  $N2$ . Only sample size combinations with this ratio are considered.

$N2$  is related to  $N1$  by the formula:

$$N2 = [R \times N1],$$

where the value  $[Y]$  is the next integer  $\geq Y$ .

## Tests for Two Proportions using Effect Size

For example, setting  $R = 2.0$  results in a Group 2 sample size that is double the sample size in Group 1 (e.g.,  $N1 = 10$  and  $N2 = 20$ , or  $N1 = 50$  and  $N2 = 100$ ).

$R$  must be greater than 0. If  $R < 1$ , then  $N2$  will be less than  $N1$ ; if  $R > 1$ , then  $N2$  will be greater than  $N1$ . You can enter a single or a series of values.

### Percent in Group 1

*This option is displayed only if Group Allocation = "Enter percentage in Group 1, solve for  $N1$  and  $N2$ ."*

Use this value to fix the percentage of the total sample size allocated to Group 1 while solving for  $N1$  and  $N2$ . Only sample size combinations with this Group 1 percentage are considered. Small variations from the specified percentage may occur due to the discrete nature of sample sizes.

The Percent in Group 1 must be greater than 0 and less than 100. You can enter a single or a series of values.

## Sample Size (When Not Solving for Sample Size)

### Group Allocation

Select the option that describes how individuals in the study will be allocated to Group 1 and to Group 2.

The options are

- **Equal ( $N1 = N2$ )**  
This selection is used when you wish to have equal sample sizes in each group. A single per group sample size will be entered.
- **Enter  $N1$  and  $N2$  individually**  
This choice permits you to enter different values for  $N1$  and  $N2$ .
- **Enter  $N1$  and  $R$ , where  $N2 = R * N1$**   
Choose this option to specify a value (or values) for  $N1$ , and obtain  $N2$  as a ratio (multiple) of  $N1$ .
- **Enter total sample size and percentage in Group 1**  
Choose this option to specify a value (or values) for the total sample size ( $N$ ), obtain  $N1$  as a percentage of  $N$ , and then  $N2$  as  $N - N1$ .

### Sample Size Per Group

*This option is displayed only if Group Allocation = "Equal ( $N1 = N2$ )."*

The Sample Size Per Group is the number of items or individuals sampled from each of the Group 1 and Group 2 populations. Since the sample sizes are the same in each group, this value is the value for  $N1$ , and also the value for  $N2$ .

The Sample Size Per Group must be  $\geq 2$ . You can enter a single value or a series of values.

### $N1$ (Sample Size, Group 1)

*This option is displayed if Group Allocation = "Enter  $N1$  and  $N2$  individually" or "Enter  $N1$  and  $R$ , where  $N2 = R * N1$ ."*

$N1$  is the number of items or individuals sampled from the Group 1 population.

$N1$  must be  $\geq 2$ . You can enter a single value or a series of values.

## Tests for Two Proportions using Effect Size

### N2 (Sample Size, Group 2)

*This option is displayed only if Group Allocation = "Enter N1 and N2 individually."*

$N2$  is the number of items or individuals sampled from the Group 2 population.

$N2$  must be  $\geq 2$ . You can enter a single value or a series of values.

### R (Group Sample Size Ratio)

*This option is displayed only if Group Allocation = "Enter N1 and R, where  $N2 = R * N1$ ."*

$R$  is the ratio of  $N2$  to  $N1$ . That is,

$$R = N2/N1$$

Use this value to obtain  $N2$  as a multiple (or proportion) of  $N1$ .

$N2$  is calculated from  $N1$  using the formula:

$$N2 = [R \times N1],$$

where the value  $[Y]$  is the next integer  $\geq Y$ .

For example, setting  $R = 2.0$  results in a Group 2 sample size that is double the sample size in Group 1.

$R$  must be greater than 0. If  $R < 1$ , then  $N2$  will be less than  $N1$ ; if  $R > 1$ , then  $N2$  will be greater than  $N1$ . You can enter a single value or a series of values.

### Total Sample Size (N)

*This option is displayed only if Group Allocation = "Enter total sample size and percentage in Group 1."*

This is the total sample size, or the sum of the two group sample sizes. This value, along with the percentage of the total sample size in Group 1, implicitly defines  $N1$  and  $N2$ .

The total sample size must be greater than one, but practically, must be greater than 3, since each group sample size needs to be at least 2.

You can enter a single value or a series of values.

### Percent in Group 1

*This option is displayed only if Group Allocation = "Enter total sample size and percentage in Group 1."*

This value fixes the percentage of the total sample size allocated to Group 1. Small variations from the specified percentage may occur due to the discrete nature of sample sizes.

The Percent in Group 1 must be greater than 0 and less than 100. You can enter a single value or a series of values.

## Tests for Two Proportions using Effect Size

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**Effect Size****h**

Enter one or more values for h, the effect size, that you wish to detect. This is a standardized difference between the group proportions. The effect size is calculated using

$$h = \varphi_1 - \varphi_2,$$

where  $\varphi_i = 2 \arcsin(\sqrt{P_i})$  and P1 and P2 are the group proportions assumed by the alternative hypothesis. Note that  $0 \leq \varphi_i \leq 3.14159$ . Thus, the physical interpretation of these values may be difficult to grasp.

The value of h must be between -6.28318 and 6.28318.

You can enter a single value such as 0.5 or a series of values such as 0.2 0.5 0.8 or 0.2 to 0.8 by 0.1. When a series of values is entered, **PASS** will generate a separate calculation result for each value of the series.

**Cohen's Effect Size Table**

Cohen (1988) gave the following interpretation of h values that is still popular.

- **Small**  
h = 0.2
- **Medium**  
h = 0.5
- **Large**  
h = 0.8

**Interpretation**

A value of h can have several interpretations. So care must be taken when using it. For example, all of the following pairs of values of P1 and P2 result in an h of about 0.30, even though the actual differences P1 - P2 are quite different.

P1=0.21 and P2 = 0.10 Diff = 0.11

P1=0.39 and P2 = 0.25 Diff = 0.14

P1=0.55 and P2 = 0.40 Diff = 0.15

P1=0.65 and P2 = 0.50 Diff = 0.15

P1=0.78 and P2 = 0.60 Diff = 0.18

P1=0.87 and P2 = 0.75 Diff = 0.13

P1=0.97 and P2 = 0.90 Diff = 0.07

## Tests for Two Proportions using Effect Size

**Example 1 – Finding the Sample Size**

Researchers wish to compare two types of local anesthesia using a balanced, parallel-group design. Subjects in pain will be randomized to one of two treatment groups, the treatment will be administered, and the subject's evaluation of pain intensity will be measured on a binary scale (acceptable, unacceptable).

The researchers would like to determine the sample sizes required to detect a small, medium, and large effect size with a two-sided t-test when the power is 80% or 90% and the significance level is 0.05.

**Setup**

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Tests for Two Proportions using Effect Size** procedure. You may then make the appropriate entries as listed below, or open **Example 1** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
<b>Design Tab</b>	
Solve For .....	<b>Sample Size</b>
Alternative Hypothesis .....	<b>Two-Sided</b>
Power .....	<b>0.80 0.90</b>
Alpha .....	<b>0.05</b>
Group Allocation .....	<b>Equal (N1 = N2)</b>
h .....	<b>0.2 0.5 0.8</b>

**Output**

Click the Calculate button to perform the calculations and generate the following output.

**Numeric Results****Numeric Results for Z Test**

Alternative Hypothesis:  $H_1: h \neq 0$

Target Power	Actual Power	N1	N2	N	Effect Size	
					h	Alpha
0.80	0.8006	393	393	786	0.20	0.050
0.90	0.9003	526	526	1052	0.20	0.050
0.80	0.8013	63	63	126	0.50	0.050
0.90	0.9031	85	85	170	0.50	0.050
0.80	0.8074	25	25	50	0.80	0.050
0.90	0.9014	33	33	66	0.80	0.050

**References**

Cohen, Jacob. 1988. *Statistical Power Analysis for the Behavioral Sciences*. Lawrence Erlbaum Associates. Hillsdale, New Jersey

**Report Definitions**

Target Power is the desired power. May not be achieved because of integer N1 and N2.

Actual Power is the achieved power. Because N1 and N2 are integers, this value is often (slightly) larger than the target power.

N1 and N2 are the number of items sampled from each population.

N is the total sample size, N1 + N2.

Effect Size:  $h = \phi_1 - \phi_2$  where  $\phi_i = 2 \text{ ArcSine}(\sqrt{P_i})$ . Cohen recommended Low = 0.2, Medium = 0.5, and High = 0.8.

Alpha is the probability of rejecting a true null hypothesis.

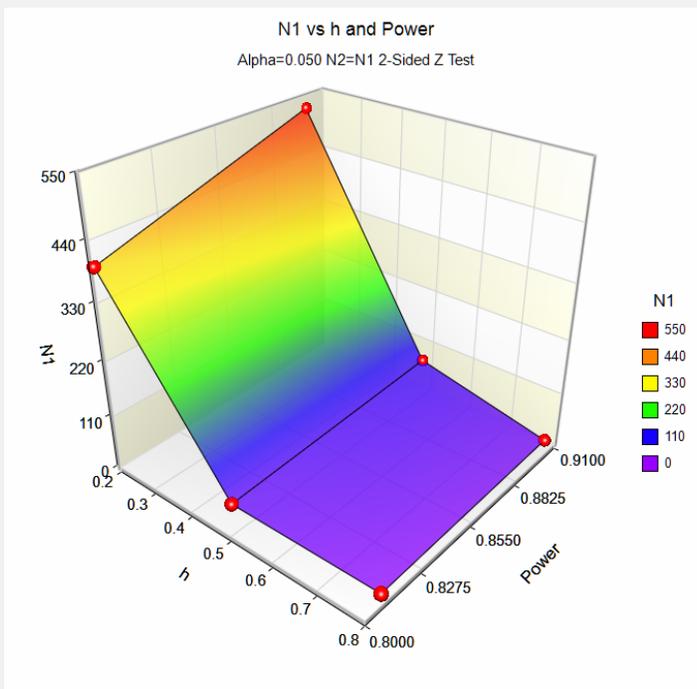
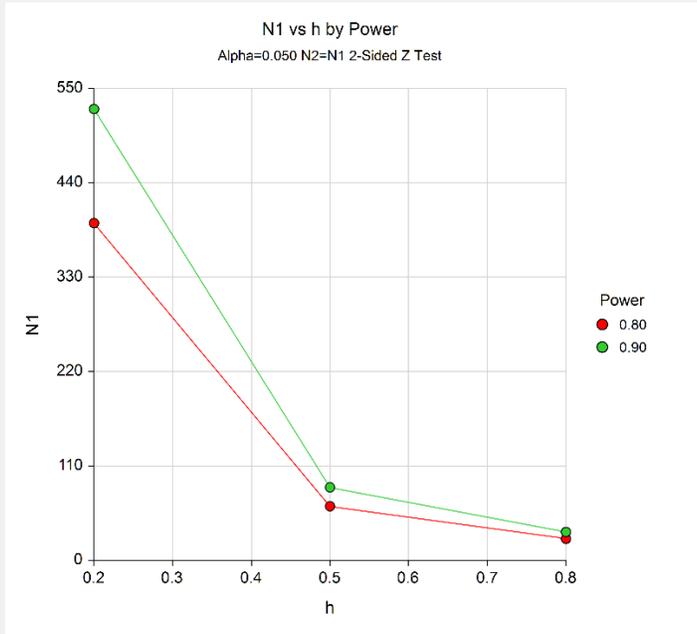
## Tests for Two Proportions using Effect Size

### Summary Statements

Group sample sizes of 393 and 393 achieve 80.06% power to reject the null hypothesis of zero effect size when the population effect size is 0.20 and the significance level (alpha) is 0.050 using a two-sided z test.

These reports show the values of each of the parameters, one scenario per row.

### Plots Section



These plots show the relationship between effect size, power, and sample size.

## Tests for Two Proportions using Effect Size

**Example 2 – Validation using Cohen (1988)**

Cohen (1988) gives an example on page 199 of a one-sided test in which  $\alpha = 0.05$ ,  $h = 0.3$ , and  $N_1 = N_2 = 80$ . He finds the power to be 0.60.

**Setup**

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Tests for Two Proportions using Effect Size** procedure. You may then make the appropriate entries as listed below, or open **Example 2** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
<b>Design Tab</b>	
Solve For .....	<b>Power</b>
Alternative Hypothesis .....	<b>One-Sided</b>
Alpha .....	<b>0.05</b>
Group Allocation .....	<b>Equal (N1 = N2)</b>
Sample Size Per Group .....	<b>80</b>
h .....	<b>0.30</b>

**Output**

Click the Calculate button to perform the calculations and generate the following output.

**Numeric Results**

<b>Numeric Results for Z Test</b>					
Alternative Hypothesis: $H_1: h \neq 0$					
				<b>Effect Size h</b>	
<b>Power</b>	<b>N1</b>	<b>N2</b>	<b>N</b>		<b>Alpha</b>
0.5997	80	80	160	0.30	0.050

PASS also calculated the power as 0.60 which validates the procedure.