

## Chapter 428

# Two-Sample Z-Tests Allowing Unequal Variance

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### Introduction

This procedure provides sample size and power calculations for one- or two-sided two-sample z-tests when the variances are assumed to be known and unequal. The assumed difference between means can be specified by entering the means for the two groups and letting the software calculate the difference or by entering the difference directly.

The design corresponding to this test procedure is sometimes referred to as a *parallel-groups* design. This design is used in situations such as the comparison of the income level of two regions, the nitrogen content of two lakes, or the effectiveness of two drugs.

There are several statistical tests available for the comparison of the center of two populations. This procedure is specific to the two-sample z-test assuming known and unequal variance. You can examine the sections below to identify whether the assumptions and test statistic you intend to use in your study match those of this procedure, or if one of the other PASS procedures may be more suited to your situation.

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### Other PASS Procedures for Comparing Two Means or Medians

Procedures in PASS are primarily built upon the testing methods, test statistic, and test assumptions that will be used when the analysis of the data is performed. You should check to identify that the test procedure described below in the Test Procedure section matches your intended procedure. If your assumptions or testing method are different, you may wish to use one of the other two-sample procedures available in PASS. These procedures are Two-Sample T-Tests Allowing Unequal Variance, Two-Sample T-Tests Assuming Equal Variance, Two-Sample Z-Tests Assuming Equal Variance, and the nonparametric Mann-Whitney-Wilcoxon (also known as the Mann-Whitney U or Wilcoxon rank-sum test) procedure. The methods, statistics, and assumptions for those procedures are described in the associated chapters.

If you wish to show that the mean of one population is larger (or smaller) than the mean of another population by a specified amount, you should use one of the clinical superiority procedures for comparing means. Non-inferiority, equivalence, and confidence interval procedures are also available.

## Two-Sample Z-Tests Allowing Unequal Variance

## Test Assumptions

When running a two-sample unequal-variance z-test, the basic assumptions are that the distributions of the two populations are normal, and that the variances of the two distributions are known and are different. If those assumptions are not likely to be met, another testing procedure could be used, and the corresponding procedure in PASS should be used for sample size or power calculations.

## Test Procedure

If we assume that  $\mu_1$  and  $\mu_2$  represent the means of the two populations of interest, and that  $\delta = \mu_1 - \mu_2$ , the null hypothesis for comparing the two means is  $H_0 : \mu_1 = \mu_2$  (or  $H_0 : \delta = 0$ ). The alternative hypothesis can be any one of

$$\text{Two-Sided:} \quad H_1 : \mu_1 \neq \mu_2 \quad (\text{or } H_1 : \delta \neq 0)$$

$$\text{Upper One-Sided:} \quad H_1 : \mu_1 > \mu_2 \quad (\text{or } H_1 : \delta > 0)$$

$$\text{Lower One-Sided:} \quad H_1 : \mu_1 < \mu_2 \quad (\text{or } H_1 : \delta < 0)$$

depending upon the desire of the researcher or the protocol instructions. A suitable Type I error probability ( $\alpha$ ) is chosen for the test, the data is collected, and a z-statistic is generated using the formula:

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

The null hypothesis is rejected in favor of the alternative if,

for  $H_1 : \mu_1 \neq \mu_2$  (or  $H_1 : \delta \neq 0$ ),

$$z < z_{\alpha/2} \quad \text{or} \quad z > z_{1-\alpha/2},$$

for  $H_1 : \mu_1 > \mu_2$  (or  $H_1 : \delta > 0$ ),

$$z > z_{1-\alpha},$$

or, for  $H_1 : \mu_1 < \mu_2$  (or  $H_1 : \delta < 0$ ),

$$z < z_{\alpha}.$$

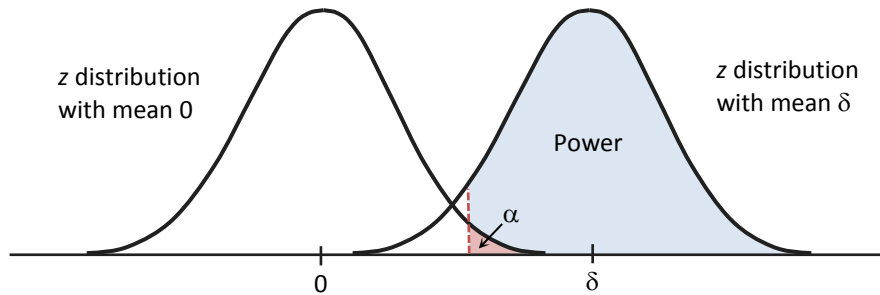
Comparing the z-statistic to the cut-off z-value (as shown here) is equivalent to comparing the p-value to  $\alpha$ .

## Two-Sample Z-Tests Allowing Unequal Variance

## Power Calculation

This section describes the procedure for computing the power from  $n_1$  and  $n_2$ ,  $\alpha$ , the assumed  $\mu_1$  and  $\mu_2$ , and the known standard deviations,  $\sigma_1$  and  $\sigma_2$ . Two good references for these general methods are Julious (2010) and Chow, Shao, and Wang (2008), although these texts do not specifically cover the unequal variance z-test methods.

The figure below gives a visual representation for the calculation of power for a one-sided test.



If we call the assumed difference between the means,  $\delta = \mu_1 - \mu_2$ , the steps for calculating the power are as follows:

1. Find  $z_{1-\alpha}$  based on the  $z$  distribution.
2. Calculate the power as the probability that the test statistic  $z$  is greater than  $z_{1-\alpha}$  under the  $z$  distribution with mean  $\delta$ :

$$Power = \Pr(z > z_{1-\alpha} | \delta).$$

The algorithms for calculating power for the opposite direction and the two-sided hypotheses are analogous to this method.

When solving for something other than power, PASS uses this same power calculation formulation, but performs a search to determine that parameter.

## A Note on Specifying the Means or Difference in Means

When means are specified in this procedure, they are used to determine the assumed difference in means for power or sample size calculations. When the difference in means is specified in this procedure, it is the assumed difference in means for power or sample size calculations. It does not mean that the study will be powered to show that the mean difference is this amount, but rather that the design is powered to reject the null hypothesis of equal means if this were the true difference in means. If your purpose is to show that one mean is greater than another by a specific amount, you should use one of the clinical superiority procedures for comparing means.

## Two-Sample Z-Tests Allowing Unequal Variance

### Procedure Options

This section describes the options that are specific to this procedure. These are located on the Design tab. For more information about the options of other tabs, go to the Procedure Window chapter.

#### Design Tab

The Design tab contains most of the parameters and options that you will be concerned with.

#### Solve For

##### Solve For

This option specifies the parameter to be solved for from the other parameters. The parameters that may be selected are *Power*, *Sample Size*, *Effect Size (Means or Difference)*, and *Alpha*. In most situations, you will likely select either *Power* or *Sample Size*.

The 'Solve For' parameter is the parameter that will be displayed on the vertical axis of any plots that are shown.

#### Test Direction

##### Alternative Hypothesis

Specify whether the alternative hypothesis of the test is one-sided or two-sided. If a one-sided test is chosen, the hypothesis test direction is chosen based on whether the difference ( $\delta = \mu_1 - \mu_2$ ) is greater than or less than zero.

##### Two-Sided Hypothesis Test

$H_0: \delta = 0$  vs.  $H_1: \delta \neq 0$

##### One-Sided Hypothesis Tests

Upper:  $H_0: \delta \leq 0$  vs.  $H_1: \delta > 0$

Lower:  $H_0: \delta \geq 0$  vs.  $H_1: \delta < 0$

#### Power and Alpha

##### Power

Power is the probability of rejecting the null hypothesis when it is false. Power is equal to  $1 - \text{Beta}$ , so specifying power implicitly specifies beta. Beta is the probability of obtaining a false negative with the statistical test. That is, it is the probability of accepting a false null hypothesis.

The valid range is 0 to 1. Different disciplines have different standards for setting power. The most common choice is 0.90, but 0.80 is also popular.

You can enter a single value, such as 0.90, or a series of values, such as .70 .80 .90, or .70 to .90 by .1.

When a series of values is entered, PASS will generate a separate calculation result for each value of the series.

##### Alpha

Alpha is the probability of obtaining a false positive with the statistical test. That is, it is the probability of rejecting a true null hypothesis. The null hypothesis is usually that the parameters of interest (means, proportions, etc.) are equal.

Since Alpha is a probability, it is bounded by 0 and 1. Commonly, it is between 0.001 and 0.10.

Alpha is often set to 0.05 for two-sided tests and to 0.025 for one-sided tests.

## Two-Sample Z-Tests Allowing Unequal Variance

You can enter a single value, such as 0.05, or a series of values, such as .05 .10 .15, or .05 to .15 by .01. When a series of values is entered, PASS will generate a separate calculation result for each value of the series.

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### Sample Size (When Solving for Sample Size)

#### Group Allocation

Select the option that describes the constraints on  $N1$  or  $N2$  or both.

The options are

- **Equal ( $N1 = N2$ )**  
This selection is used when you wish to have equal sample sizes in each group. Since you are solving for both sample sizes at once, no additional sample size parameters need to be entered.
- **Enter  $N1$ , solve for  $N2$**   
Select this option when you wish to fix  $N1$  at some value (or values), and then solve only for  $N2$ . Please note that for some values of  $N1$ , there may not be a value of  $N2$  that is large enough to obtain the desired power.
- **Enter  $N2$ , solve for  $N1$**   
Select this option when you wish to fix  $N2$  at some value (or values), and then solve only for  $N1$ . Please note that for some values of  $N2$ , there may not be a value of  $N1$  that is large enough to obtain the desired power.
- **Enter  $R = N2/N1$ , solve for  $N1$  and  $N2$**   
For this choice, you set a value for the ratio of  $N2$  to  $N1$ , and then PASS determines the needed  $N1$  and  $N2$ , with this ratio, to obtain the desired power. An equivalent representation of the ratio,  $R$ , is
 
$$N2 = R * N1.$$
- **Enter percentage in Group 1, solve for  $N1$  and  $N2$**   
For this choice, you set a value for the percentage of the total sample size that is in Group 1, and then PASS determines the needed  $N1$  and  $N2$  with this percentage to obtain the desired power.

#### $N1$ (Sample Size, Group 1)

*This option is displayed if Group Allocation = "Enter  $N1$ , solve for  $N2$ "*

$N1$  is the number of items or individuals sampled from the Group 1 population.

$N1$  must be  $\geq 2$ . You can enter a single value or a series of values.

#### $N2$ (Sample Size, Group 2)

*This option is displayed if Group Allocation = "Enter  $N2$ , solve for  $N1$ "*

$N2$  is the number of items or individuals sampled from the Group 2 population.

$N2$  must be  $\geq 2$ . You can enter a single value or a series of values.

#### $R$ (Group Sample Size Ratio)

*This option is displayed only if Group Allocation = "Enter  $R = N2/N1$ , solve for  $N1$  and  $N2$ ."*

$R$  is the ratio of  $N2$  to  $N1$ . That is,

$$R = N2 / N1.$$

Use this value to fix the ratio of  $N2$  to  $N1$  while solving for  $N1$  and  $N2$ . Only sample size combinations with this ratio are considered.

## Two-Sample Z-Tests Allowing Unequal Variance

$N_2$  is related to  $N_1$  by the formula:

$$N_2 = [R \times N_1],$$

where the value  $[Y]$  is the next integer  $\geq Y$ .

For example, setting  $R = 2.0$  results in a Group 2 sample size that is double the sample size in Group 1 (e.g.,  $N_1 = 10$  and  $N_2 = 20$ , or  $N_1 = 50$  and  $N_2 = 100$ ).

$R$  must be greater than 0. If  $R < 1$ , then  $N_2$  will be less than  $N_1$ ; if  $R > 1$ , then  $N_2$  will be greater than  $N_1$ . You can enter a single or a series of values.

### Percent in Group 1

*This option is displayed only if Group Allocation = "Enter percentage in Group 1, solve for  $N_1$  and  $N_2$ ."*

Use this value to fix the percentage of the total sample size allocated to Group 1 while solving for  $N_1$  and  $N_2$ . Only sample size combinations with this Group 1 percentage are considered. Small variations from the specified percentage may occur due to the discrete nature of sample sizes.

The Percent in Group 1 must be greater than 0 and less than 100. You can enter a single or a series of values.

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## Sample Size (When Not Solving for Sample Size)

### Group Allocation

Select the option that describes how individuals in the study will be allocated to Group 1 and to Group 2.

The options are

- **Equal ( $N_1 = N_2$ )**  
This selection is used when you wish to have equal sample sizes in each group. A single per group sample size will be entered.
- **Enter  $N_1$  and  $N_2$  individually**  
This choice permits you to enter different values for  $N_1$  and  $N_2$ .
- **Enter  $N_1$  and  $R$ , where  $N_2 = R * N_1$**   
Choose this option to specify a value (or values) for  $N_1$ , and obtain  $N_2$  as a ratio (multiple) of  $N_1$ .
- **Enter total sample size and percentage in Group 1**  
Choose this option to specify a value (or values) for the total sample size ( $N$ ), obtain  $N_1$  as a percentage of  $N$ , and then  $N_2$  as  $N - N_1$ .

### Sample Size Per Group

*This option is displayed only if Group Allocation = "Equal ( $N_1 = N_2$ )."*

The Sample Size Per Group is the number of items or individuals sampled from each of the Group 1 and Group 2 populations. Since the sample sizes are the same in each group, this value is the value for  $N_1$ , and also the value for  $N_2$ .

The Sample Size Per Group must be  $\geq 2$ . You can enter a single value or a series of values.

### $N_1$ (Sample Size, Group 1)

*This option is displayed if Group Allocation = "Enter  $N_1$  and  $N_2$  individually" or "Enter  $N_1$  and  $R$ , where  $N_2 = R * N_1$ ."*

$N_1$  is the number of items or individuals sampled from the Group 1 population.

$N_1$  must be  $\geq 2$ . You can enter a single value or a series of values.

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### N2 (Sample Size, Group 2)

*This option is displayed only if Group Allocation = "Enter N1 and N2 individually."*

$N2$  is the number of items or individuals sampled from the Group 2 population.

$N2$  must be  $\geq 2$ . You can enter a single value or a series of values.

### R (Group Sample Size Ratio)

*This option is displayed only if Group Allocation = "Enter N1 and R, where  $N2 = R * N1$ ."*

$R$  is the ratio of  $N2$  to  $N1$ . That is,

$$R = N2/N1$$

Use this value to obtain  $N2$  as a multiple (or proportion) of  $N1$ .

$N2$  is calculated from  $N1$  using the formula:

$$N2 = [R \times N1],$$

where the value  $[Y]$  is the next integer  $\geq Y$ .

For example, setting  $R = 2.0$  results in a Group 2 sample size that is double the sample size in Group 1.

$R$  must be greater than 0. If  $R < 1$ , then  $N2$  will be less than  $N1$ ; if  $R > 1$ , then  $N2$  will be greater than  $N1$ . You can enter a single value or a series of values.

### Total Sample Size (N)

*This option is displayed only if Group Allocation = "Enter total sample size and percentage in Group 1."*

This is the total sample size, or the sum of the two group sample sizes. This value, along with the percentage of the total sample size in Group 1, implicitly defines  $N1$  and  $N2$ .

The total sample size must be greater than one, but practically, must be greater than 3, since each group sample size needs to be at least 2.

You can enter a single value or a series of values.

### Percent in Group 1

*This option is displayed only if Group Allocation = "Enter total sample size and percentage in Group 1."*

This value fixes the percentage of the total sample size allocated to Group 1. Small variations from the specified percentage may occur due to the discrete nature of sample sizes.

The Percent in Group 1 must be greater than 0 and less than 100. You can enter a single value or a series of values.

## Two-Sample Z-Tests Allowing Unequal Variance

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### Effect Size

#### Input Type

Indicate what type of values to enter to specify the effect size. Regardless of the entry type chosen, the test statistics used in the power and sample size calculations are the same. This option is simply given for convenience in specifying the effect size.

The difference is calculated using the formula

$$\delta = \mu_1 - \mu_2$$

The choices are

- **Means**  
Enter  $\mu_1$  (Group 1 Mean) and  $\mu_2$  (Group 2 Mean).
- **Difference in Means**  
Enter the difference,  $\delta$ , directly.

Both choices also require you to enter the standard deviations,  $\sigma_1$  and  $\sigma_2$ .

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### Effect Size – Means

#### $\mu_1$ (Mean of Group 1)

Enter a value for the assumed mean of Group 1. The calculations of this procedure are based on difference between the two means,  $\mu_1 - \mu_2$ . This difference is the difference at which the design is powered to reject equal means.

$\mu_1$  can be any value (positive, negative, or zero).

You can enter a single value, such as 10, or a series of values, such as 10 20 30, or 5 to 50 by 5.

When a series of values is entered, PASS will generate a separate calculation result for each value of the series.

#### $\mu_2$ (Mean of Group 2)

Enter a value for the assumed mean of Group 2. The calculations of this procedure are based on difference between the two means,  $\mu_1 - \mu_2$ . This difference is the difference at which the design is powered to reject equal means.

$\mu_2$  can be any value (positive, negative, or zero).

You can enter a single value, such as 10, or a series of values, such as 10 20 30, or 5 to 50 by 5.

When a series of values is entered, PASS will generate a separate calculation result for each value of the series.

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### Effect Size – Difference in Means

#### $\delta$

Enter a value for the assumed difference between the means of Groups 1 and 2. This difference is the difference at which the design is powered to reject equal means.

$$\delta = \mu_1 - \mu_2$$

$\delta$  can be any non-zero value (positive or negative).

You can enter a single value, such as 10, or a series of values, such as 10 20 30, or 5 to 50 by 5.

When a series of values is entered, PASS will generate a separate calculation result for each value of the series.



## Two-Sample Z-Tests Allowing Unequal Variance

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### Standard Deviations

#### $\sigma_1$ (Standard Deviation 1)

The standard deviation entered here is the assumed, known standard deviation for the Group 1 population.

$\sigma_1$  must be a positive number.

You can enter a single value, such as 5, or a series of values, such as 1 3 5 7 9, or 1 to 10 by 1.

When a series of values is entered, PASS will generate a separate calculation result for each value of the series.

Press the small ' $\sigma$ ' button to the right to obtain calculation options for estimating the standard deviation.

#### $\sigma_2$ (Standard Deviation 2)

The standard deviation entered here is the assumed, known standard deviation for the Group 2 population.

$\sigma_2$  must be a positive number.

You can enter a single value, such as 5, or a series of values, such as 1 3 5 7 9, or 1 to 10 by 1.

When a series of values is entered, PASS will generate a separate calculation result for each value of the series.

Press the small ' $\sigma$ ' button to the right to obtain calculation options for estimating the standard deviation.

## Two-Sample Z-Tests Allowing Unequal Variance

## Example 1 – Finding the Sample Size

Researchers wish to compare two types of local anesthesia to determine whether there is a difference in time to loss of pain. Subjects will be randomized to treatment, the treatment will be administered, and the time to loss of pain measured. The anticipated time to loss of pain for one of the types of anesthesia is 9 minutes. The researchers would like to generate a sample size for the study with 90% power to reject the null hypothesis of equal loss-of-pain time if the true difference is between 2 and 10 minutes. How many participants are needed to achieve 90% power at significance levels of 0.01 and 0.05?

It is known that the standard deviation is 4.1 minutes for Group 1, and 5.3 for Group 2. It is unknown which treatment has lower time to loss of pain, so a two-sided test will be used.

### Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Two-Sample Z-Tests Allowing Unequal Variance** procedure window by expanding **Means**, then **Two Independent Means**, then clicking on **Test (Inequality)**, and then clicking on **Two-Sample Z-Tests Allowing Unequal Variance**. You may then make the appropriate entries as listed below, or open **Example 1** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
<b>Design Tab</b>	
Solve For .....	<b>Sample Size</b>
Alternative Hypothesis .....	<b>Two-Sided</b>
Power .....	<b>0.90</b>
Alpha .....	<b>0.01 0.05</b>
Group Allocation .....	<b>Equal (N1 = N2)</b>
Input Type .....	<b>Means</b>
$\mu_1$ .....	<b>11 to 19 by 1</b>
$\mu_2$ .....	<b>9</b>
$\sigma_1$ .....	<b>4.1</b>
$\sigma_2$ .....	<b>5.3</b>

### Output

Click the Calculate button to perform the calculations and generate the following output.

### Numeric Results

#### Numeric Results for Two-Sample Z-Test Allowing Unequal Variance

Alternative Hypothesis:  $H_1: \delta = \mu_1 - \mu_2 \neq 0$

Target Power	Actual Power	N1	N2	N	$\mu_1$	$\mu_2$	$\delta$	$\sigma_1$	$\sigma_2$	Alpha
0.90	0.90197	168	168	336	11.0	9.0	2.0	4.1	5.3	0.010
0.90	0.90013	118	118	236	11.0	9.0	2.0	4.1	5.3	0.050
0.90	0.90345	75	75	150	12.0	9.0	3.0	4.1	5.3	0.010
0.90	0.90310	53	53	106	12.0	9.0	3.0	4.1	5.3	0.050
0.90	0.90197	42	42	84	13.0	9.0	4.0	4.1	5.3	0.010
0.90	0.90484	30	30	60	13.0	9.0	4.0	4.1	5.3	0.050
0.90	0.90345	27	27	54	14.0	9.0	5.0	4.1	5.3	0.010
0.90	0.90192	19	19	38	14.0	9.0	5.0	4.1	5.3	0.050
0.90	0.90778	19	19	38	15.0	9.0	6.0	4.1	5.3	0.010
0.90	0.91780	14	14	28	15.0	9.0	6.0	4.1	5.3	0.050

### Two-Sample Z-Tests Allowing Unequal Variance

0.90	0.90872	14	14	28	16.0	9.0	7.0	4.1	5.3	0.010
0.90	0.91045	10	10	20	16.0	9.0	7.0	4.1	5.3	0.050
0.90	0.91680	11	11	22	17.0	9.0	8.0	4.1	5.3	0.010
0.90	0.92174	8	8	16	17.0	9.0	8.0	4.1	5.3	0.050
0.90	0.92697	9	9	18	18.0	9.0	9.0	4.1	5.3	0.010
0.90	0.90825	6	6	12	18.0	9.0	9.0	4.1	5.3	0.050
0.90	0.91506	7	7	14	19.0	9.0	10.0	4.1	5.3	0.010
0.90	0.91576	5	5	10	19.0	9.0	10.0	4.1	5.3	0.050

**References**

Julious, S. A. 2010. Sample Sizes for Clinical Trials. Chapman & Hall/CRC. Boca Raton, FL.  
 Chow, S. C., Shao, J., and Wang, H. 2008. Sample Size Calculations in Clinical Research (Second Edition). Chapman & Hall/CRC. Boca Raton, FL.  
 Machin, D., Campbell, M., Fayers, P., and Pinol, A. 1997. Sample Size Tables for Clinical Studies, 2nd Edition. Blackwell Science. Malden, MA.  
 Zar, Jerrold H. 1984. Biostatistical Analysis (Second Edition). Prentice-Hall. Englewood Cliffs, New Jersey.

**Report Definitions**

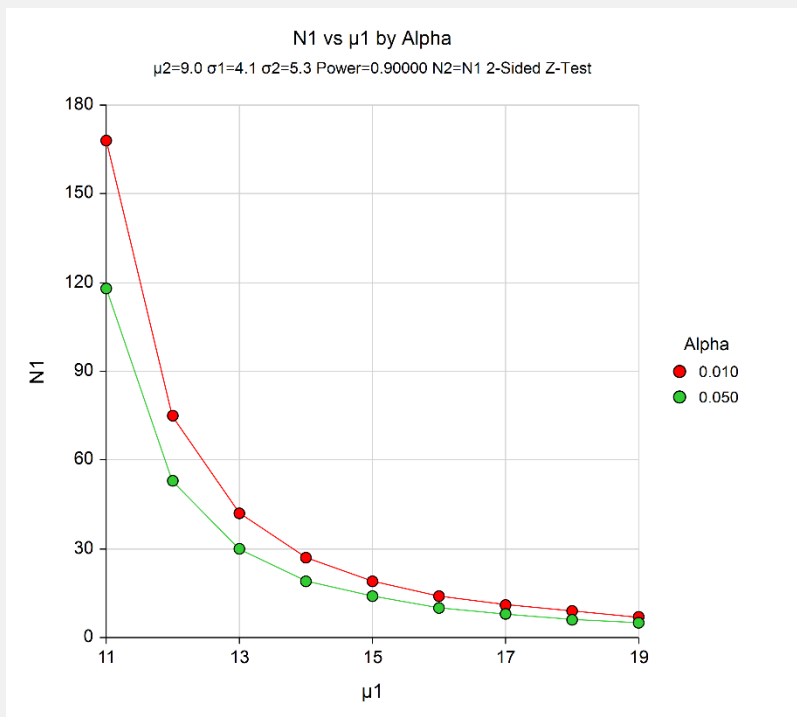
Target Power is the desired power value (or values) entered in the procedure. Power is the probability of rejecting a false null hypothesis.  
 Actual Power is the power obtained in this scenario. Because N1 and N2 are discrete, this value is often (slightly) larger than the target power.  
 N1 and N2 are the number of items sampled from each population.  
 N is the total sample size, N1 + N2.  
 $\mu_1$  and  $\mu_2$  are the assumed population means.  
 $\delta = \mu_1 - \mu_2$  is the difference between population means at which power and sample size calculations are made.  
 $\sigma_1$  and  $\sigma_2$  are the assumed population standard deviations for groups 1 and 2, respectively.  
 Alpha is the probability of rejecting a true null hypothesis.

**Summary Statements**

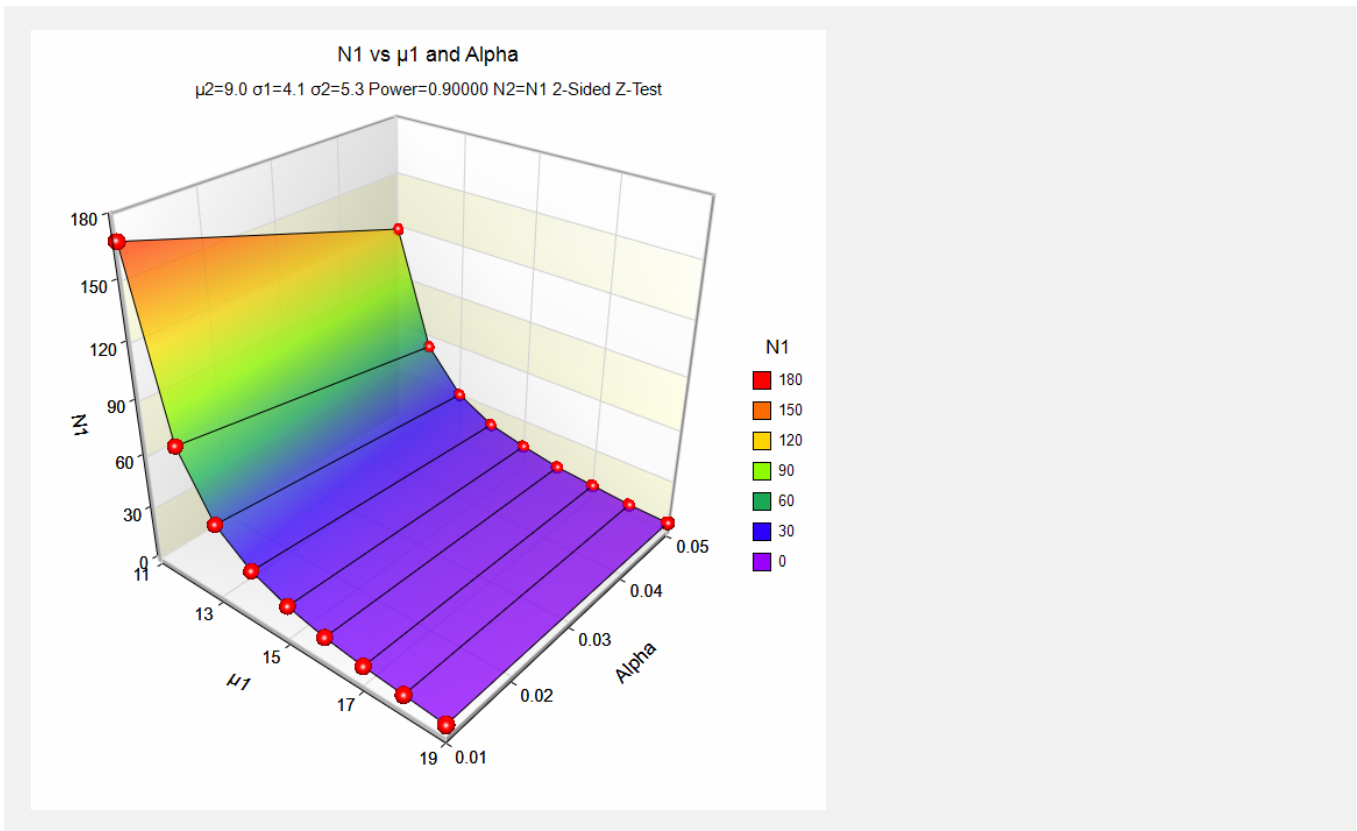
Group sample sizes of 168 and 168 achieve 90.197% power to reject the null hypothesis of equal means when the population mean difference is  $\mu_1 - \mu_2 = 11.0 - 9.0 = 2.0$  with standard deviations of 4.1 for group 1 and 5.3 for group 2, and with a significance level (alpha) of 0.010 using a two-sided two-sample unequal-variance z-test.

These reports show the values of each of the parameters, one scenario per row.

**Plots Section**



## Two-Sample Z-Tests Allowing Unequal Variance



These plots show the relationship between the sample size and mean for Group 2 for the two alpha levels.

To specify the difference directly to get the same results, change **Input Type** to **Difference** and enter **2 to 10 by 1** for  $\delta$ , leaving everything else the same. You may then make the appropriate entries as listed below, or open **Example 1b** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
<b>Design Tab</b>	
Input Type.....	Difference
$\delta$ .....	2 to 10 by 1

## Output

Click the Calculate button to perform the calculations and generate the following output.

### Numeric Results

**Numeric Results for Two-Sample T-Test Assuming Equal Variance**  
 Alternative Hypothesis:  $H1: \delta = \mu_1 - \mu_2 \neq 0$

Target Power	Actual Power	N1	N2	N	$\delta$	$\sigma_1$	$\sigma_2$	Alpha
0.90	0.90197	168	168	336	2.0	4.1	5.3	0.010
0.90	0.90013	118	118	236	2.0	4.1	5.3	0.050
0.90	0.90345	75	75	150	3.0	4.1	5.3	0.010
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.

The results are the same as those obtained when the means are entered.

## Two-Sample Z-Tests Allowing Unequal Variance

## Example 2 – Finding the Power

Suppose a new corn fertilizer is to be compared to a current fertilizer. The current fertilizer produces an average of about 74 lbs. per plot. The researchers need only show that there is difference in yield with the new fertilizer. They would like to consider the effect of the number of plots used on the power of the test if the improvement in yield is at least 10 lbs.

Researchers plan to use a one-sided two-sample z-test with alpha equal to 0.05. The standard deviation is known to be 24 lbs. for Group 1, and 26 lbs. for Group 2. The plot group sizes of interest are 10 to 100 plots per group.

### Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Two-Sample Z-Tests Allowing Unequal Variance** procedure window by expanding **Means**, then **Two Independent Means**, then clicking on **Test (Inequality)**, and then clicking on **Two-Sample Z-Tests Allowing Unequal Variance**. You may then make the appropriate entries as listed below, or open **Example 2** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
<b>Design Tab</b>	
Solve For .....	<b>Power</b>
Alternative Hypothesis .....	<b>One-Sided</b>
Alpha .....	<b>0.05</b>
Group Allocation .....	<b>Equal (N1 = N2)</b>
Sample Size Per Group .....	<b>10 to 100 by 10</b>
Input Type .....	<b>Means</b>
$\mu_1$ .....	<b>84</b>
$\mu_2$ .....	<b>74</b>
$\sigma_1$ .....	<b>24</b>
$\sigma_2$ .....	<b>26</b>

### Output

Click the Calculate button to perform the calculations and generate the following output.

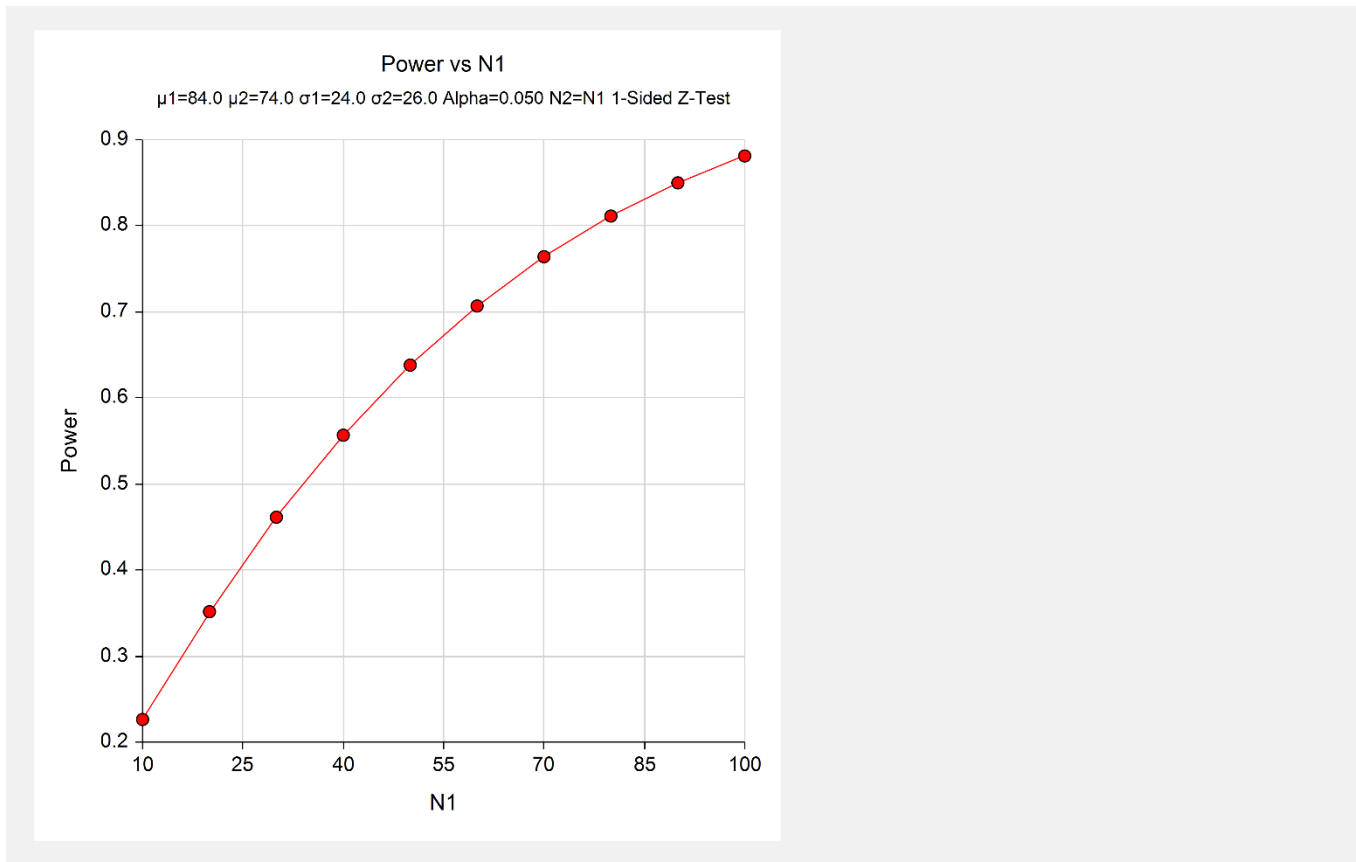
### Numeric Results

Numeric Results for Two-Sample Z-Test Allowing Unequal Variance									
Alternative Hypothesis: $H_1: \delta = \mu_1 - \mu_2 > 0$									
Power	N1	N2	N	$\mu_1$	$\mu_2$	$\delta$	$\sigma_1$	$\sigma_2$	Alpha
0.22628	10	10	20	84.0	74.0	10.0	24.0	26.0	0.050
0.35162	20	20	40	84.0	74.0	10.0	24.0	26.0	0.050
0.46140	30	30	60	84.0	74.0	10.0	24.0	26.0	0.050
0.55669	40	40	80	84.0	74.0	10.0	24.0	26.0	0.050
0.63816	50	50	100	84.0	74.0	10.0	24.0	26.0	0.050
0.70688	60	60	120	84.0	74.0	10.0	24.0	26.0	0.050
0.76414	70	70	140	84.0	74.0	10.0	24.0	26.0	0.050
0.81137	80	80	160	84.0	74.0	10.0	24.0	26.0	0.050
0.84997	90	90	180	84.0	74.0	10.0	24.0	26.0	0.050
0.88126	100	100	200	84.0	74.0	10.0	24.0	26.0	0.050

These reports show the values of each of the parameters, one scenario per row.

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### Plots Section



This plot shows the relationship between the power and sample size.

Again, to specify the difference directly to get the same results, change **Input Type** to **Difference** and enter **10** for  $\delta$ , leaving everything else the same. You'll get the same results as those above.

<u>Option</u>	<u>Value</u>
<b>Design Tab</b>	
Input Type .....	<b>Difference</b>
$\delta$ .....	<b>10</b>

## Two-Sample Z-Tests Allowing Unequal Variance

## Example 3 – Finding the Difference

In some cases it may be useful to determine how different the means of two populations would need to be to achieve the desired power with a specific constrained sample size.

Suppose, for example, that 80 subjects are available for a study to compare weight loss regimens. Researchers would like to determine which of the two regimens is better. The anticipated weight loss after two months is anticipated to be about 20 lbs. The researchers would like to know how different the mean weight loss must be to have a study with 90% power when alpha is 0.05 and the standard deviation is known to be 6 lbs. for Group 1 and 8 lbs. for Group 2.

### Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Two-Sample Z-Tests Allowing Unequal Variance** procedure window by expanding **Means**, then **Two Independent Means**, then clicking on **Test (Inequality)**, and then clicking on **Two-Sample Z-Tests Allowing Unequal Variance**. You may then make the appropriate entries as listed below, or open **Example 3** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
<b>Design Tab</b>	
Solve For .....	<b>Effect Size (Means or Difference)</b>
Alternative Hypothesis .....	<b>Two-Sided</b>
Power .....	<b>0.90</b>
Alpha .....	<b>0.05</b>
Group Allocation .....	<b>Equal (N1 = N2)</b>
Sample Size Per Group .....	<b>40</b>
Input Type .....	<b>Difference</b>
$\delta$ .....	<b>Search &gt; 0</b>
$\sigma_1$ .....	<b>6</b>
$\sigma_2$ .....	<b>8</b>
<b>Reports Tab</b>	
Means, Difference Decimals .....	<b>3</b>

### Output

Click the Calculate button to perform the calculations and generate the following output.

#### Numeric Results

##### Numeric Results for Two-Sample Z-Test Allowing Unequal Variance

Alternative Hypothesis:  $H_1: \delta = \mu_1 - \mu_2 \neq 0$

Power	N1	N2	N	$\delta$	$\sigma_1$	$\sigma_2$	Alpha
0.90000	40	40	80	5.125	6.0	8.0	0.050

If the true population mean weight loss for one group is 5.125 lbs. more or less than the other, the researchers would have 90% power to show a difference between the groups.

## Two-Sample Z-Tests Allowing Unequal Variance

## Example 4 – Validation of Sample Size using Chow, Shao, and Wang (2008)

Chow, Show, and Wang (2008) pages 63 and 64 present an example for a two-sided test in which the mean difference is 0.05, the standard deviation for each group is known to be 0.10, the power is 80%, and the significance level is 0.05. They calculate the per group sample size as 63.

### Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Two-Sample Z-Tests Allowing Unequal Variance** procedure window by expanding **Means**, then **Two Independent Means**, then clicking on **Test (Inequality)**, and then clicking on **Two-Sample Z-Tests Allowing Unequal Variance**. You may then make the appropriate entries as listed below, or open **Example 4** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
<b>Design Tab</b>	
Solve For .....	<b>Sample Size</b>
Alternative Hypothesis .....	<b>Two-Sided</b>
Power .....	<b>0.80</b>
Alpha .....	<b>0.05</b>
Group Allocation .....	<b>Equal (N1 = N2)</b>
Input Type .....	<b>Difference</b>
$\delta$ .....	<b>0.05</b>
$\sigma_1$ .....	<b>0.1</b>
$\sigma_2$ .....	<b>0.1</b>
<b>Reports Tab</b>	
Means, Difference Decimals .....	<b>2</b>

### Output

Click the Calculate button to perform the calculations and generate the following output.

### Numeric Results

Numeric Results for Two-Sample Z-Test Allowing Unequal Variance								
Alternative Hypothesis: $H_1: \delta = \mu_1 - \mu_2 \neq 0$								
Target Power	Actual Power	N1	N2	N	$\delta$	$\sigma_1$	$\sigma_2$	Alpha
0.80	0.80130	63	63	126	0.05	0.1	0.1	0.050

The sample size of 63 per group matches Chow, Show, and Wang's result exactly.