Chapter 411

Wilcoxon Signed-Rank Tests

Introduction

The one-sample *t*-test is used to test whether the mean of a population is greater than, less than, or not equal to a specific value. Because the *t* distribution is used to calculate critical values for the test, this test is often called the one-sample *t*-test. The *t*-test assumes that the population standard deviation is unknown and will be estimated by the data.

The nonparametric analog of the *t*-test is the *Wilcoxon Signed-Rank Test* and may be used when the one-sample *t*-test assumptions are violated.

Other PASS Procedures for Testing One Mean or Median

Procedures in **PASS** are primarily built upon the testing methods, test statistic, and test assumptions that will be used when the analysis of the data is performed. You should check to identify that the test procedure described below in the Test Procedure section matches your intended procedure. If your assumptions or testing method are different, you may wish to use one of the other one-sample procedures available in **PASS**–the One-Sample Z-Tests and the One-Sample T-Tests procedures. The methods, statistics, and assumptions for those procedures are described in the associated chapters.

If you wish to show that the mean of a population is larger (or smaller) than a reference value by a specified amount, you should use one of the clinical superiority procedures for comparing means. Non-inferiority, equivalence, and confidence interval procedures are also available.

Assumptions for One-Sample Tests

This section describes the assumptions that are made when you use one of the one-sample tests. The key assumption relates to normality or non-normality of the data. One of the reasons for the popularity of the *t*-test is its robustness in the face of assumption violation. However, if an assumption is not met even approximately, the significance levels and the power of the *t*-test are invalidated. Unfortunately, in practice it often happens that several assumptions are not met. This makes matters even worse! Hence, take the steps to check the assumptions before you make important decisions based on these tests.

One-Sample Z-Test Assumptions

The assumptions of the one-sample *z*-test are:

1. The data are continuous (not discrete).
2. The data follow the normal probability distribution.
3. The sample is a simple random sample from its population. Each individual in the population has an equal probability of being selected in the sample.
4. The population standard deviation is known.

One-Sample T-Test Assumptions

The assumptions of the one-sample *t*-test are:

1. The data are continuous (not discrete).
2. The data follow the normal probability distribution.
3. The sample is a simple random sample from its population. Each individual in the population has an equal probability of being selected in the sample.

Paired T-Test Assumptions

The assumptions of the paired *t*-test are:

1. The data are continuous (not discrete).
2. The data, i.e., the differences for the matched-pairs, follow a normal probability distribution.
3. The sample of pairs is a simple random sample from its population. Each individual in the population has an equal probability of being selected in the sample.

Wilcoxon Signed-Rank Test Assumptions

The assumptions of the Wilcoxon signed-rank test are as follows (note that the difference is between a data value and the hypothesized median or between the two data values of a pair):

1. The differences are continuous (not discrete).
2. The distribution of each difference is symmetric.
3. The differences are mutually independent.
4. The differences all have the same median.
5. The measurement scale is at least interval.

Limitations

There are few limitations when using these tests. Sample sizes may range from a few to several hundred. If your data are discrete with at least five unique values, you can often ignore the continuous variable assumption. Perhaps the greatest restriction is that your data come from a random sample of the population. If you do not have a random sample, your significance levels will probably be incorrect.

Wilcoxon Signed-Rank Test Statistic

The Wilcoxon signed-rank test is a popular, nonparametric substitute for the *t*-test. It assumes that the data follow a symmetric distribution. The test is computed using the following steps.

1. Subtract the hypothesized mean, μ_0, from each data value. Rank the values according to their absolute values.

2. Compute the sum of the positive ranks Sp and the sum of the negative ranks Sn. The test statistic, W_R, is the minimum of Sp and Sn.

3. Compute the mean and standard deviation of W_R using the formulas

$$\mu_{W_R} = \frac{n(n+1)}{4}$$

$$\sigma_{W_R} = \sqrt{\frac{n(n+1)(2n+1)}{24} - \frac{\sum t^3 - \sum t}{48}}$$

   where *t* represents the number of times the *i*th value occurs.

4. Compute the *z*-value using

$$z_W = \frac{W_R - \mu_{W_R}}{\sigma_{W_R}}$$

The significance of the test statistic is determined by computing the p-value using the standard normal distribution. If this p-value is less than a specified level (usually 0.05), the null hypothesis is rejected in favor of the alternative hypothesis. Otherwise, no conclusion can be reached.

Power Calculation for the Wilcoxon Signed-Rank Test

The power calculation for the Wilcoxon signed-rank test is the same as that for the one-sample *t*-test except that an adjustment is made to the sample size based on an assumed data distribution as described in Al-Sunduqchi and Guenther (1990). The sample size n' used in power calculations is equal to

$$n' = n/W,$$

where W is the Wilcoxon adjustment factor based on the assumed data distribution.

The adjustments are as follows:

Distribution	W
Uniform	1
Double Exponential	2/3
Logistic	$9/\pi^2$
Normal	$\pi/3$

The power is calculated as follows for a directional alternative (one-tailed test) in which $\mu_1 > \mu_0$.

1. Find t_α such that $1 - T_{df}(t_\alpha) = \alpha$, where $T_{df}(t_\alpha)$ is the area under a central-*t* curve to the left of *x* and $df = n' - 1$.

2. Calculate: $X_1 = \mu_0 + t_\alpha \frac{\sigma}{\sqrt{n'}}$.

3. Calculate the noncentrality parameter: $\lambda = \frac{\mu_1 - \mu_0}{\frac{\sigma}{\sqrt{n'}}} = \frac{\delta_1}{\frac{\sigma}{\sqrt{n'}}}$.

4. Calculate: $t_1 = \frac{X_1 - \mu_1}{\frac{\sigma}{\sqrt{n'}}} + \lambda$.

5. Power $= 1 - T'_{df,\lambda}(t_1)$, where $T'_{df,\lambda}(x)$ is the area to the left of x under a noncentral-t curve with degrees of freedom df and noncentrality parameter λ.

Procedure Options

This section describes the options that are specific to this procedure. These are located on the Design tab. For more information about the options of other tabs, go to the Procedure Window chapter.

Design Tab

The Design tab contains most of the parameters and options that you will be concerned with.

Solve For

Solve For

This option specifies the parameter to be calculated from the values of the other parameters. Under most conditions, you would select either *Power* or *Sample Size*.

Select *Sample Size* when you want to determine the sample size needed to achieve a given power and alpha error level.

Select *Power* when you want to calculate the power of an experiment that has already been run.

Test

Alternative Hypothesis

Specify the alternative hypothesis of the test. Since the null hypothesis is the opposite of the alternative, specifying the alternative is all that is needed. Usually, the two-tailed ($\neq$) option is selected.

The options containing only $<$ or $>$ are one-tailed tests. When you choose one of these options, you must be sure that the input parameters match this selection.

Possible selections are:

- **Two-Sided (H1: μ ≠ μ0)**

  This is the most common selection. It yields the *two-tailed t-test*. Use this option when you are testing whether the means are different but you do not want to specify beforehand which mean is larger. Many scientific journals require two-tailed tests.

- **One-Sided (H1: μ < μ0)**

  This option yields a *one-tailed t-test*. Use it when you are only interested in the case in which μ is less than μ0.

- **One-Sided (H1: μ > μ0)**

  This option yields a *one-tailed t-test*. Use it when you are only interested in the case in which μ is greater than μ0.

Wilcoxon Signed-Rank Tests

Data Distribution

This option makes appropriate sample size adjustments for the Wilcoxon Signed-Rank test. Results by Al-Sunduqchi and Guenther (1990) indicate that power calculations for the Wilcoxon Signed-Rank test may be made using the standard *t*-test formulations with a simple adjustment to the sample size. The size of the adjustment depends upon the actual distribution of the data. They give sample size adjustment factors for four distributions.

The options are as follows:

- **Uniform**

  The sample size adjustment factor, W, is equal to "1". This selection gives the same result as the one-sample t-test.

- **Double Exponential**

  The sample size adjustment factor, W, is equal to "2/3".

- **Logistic**

  The sample size adjustment factor, W, is equal to "$9/\pi^2$".

- **Normal**

  The sample size adjustment factor, W, is equal to "$\pi/3$".

Population Size

This is the number of subjects in the population. Usually, you assume that samples are drawn from a very large (infinite) population. Occasionally, however, situations arise in which the population of interest is of limited size. In these cases, appropriate adjustments must be made.

When a finite population size is specified, the standard deviation is reduced according to the formula:

$$\sigma_1^2 = \left(1 - \frac{n}{N}\right)\sigma^2$$

where *n* is the sample size, *N* is the population size, σ is the original standard deviation, and σ_1 is the new standard deviation.

The quantity *n/N* is often called the sampling fraction. The quantity $\left(1 - \frac{n}{N}\right)$ is called the *finite population correction factor*.

Power and Alpha

Power

This option specifies one or more values for power. Power is the probability of rejecting a false null hypothesis, and is equal to one minus Beta. Beta is the probability of a type-II error, which occurs when a false null hypothesis is not rejected.

Values must be between zero and one. Historically, the value of 0.80 (Beta = 0.20) was used for power. Now, 0.90 (Beta = 0.10) is also commonly used.

A single value may be entered here or a range of values such as *0.8 to 0.95 by 0.05* may be entered.

Alpha

This option specifies one or more values for the probability of a type-I error. A type-I error occurs when a true null hypothesis is rejected.

Values must be between zero and one. Historically, the value of 0.05 has been used for alpha. This means that about one test in twenty will falsely reject the null hypothesis. You should pick a value for alpha that represents the risk of a type-I error you are willing to take in your experimental situation.

You may enter a range of values such as *0.01 0.05 0.10* or *0.01 to 0.10 by 0.01*.

Sample Size

N (Sample Size)

This option specifies one or more values of the sample size, the number of individuals in the study. This value must be an integer greater than one. Note that you may enter a list of values using the syntax *50,100,150,200,250* or *50 to 250 by 50*.

Effect Size – Means

μ0 (Null or Baseline Mean)

Enter a value for the population mean under the null hypothesis. This is the reference or baseline mean. If you are analyzing a paired *t*-test, this value should be zero.

Only the difference between μ1 and μ0 is used in the calculations.

μ1 (Actual Mean)

Enter a value (or range of values) for the actual population mean at which power and sample size are calculated.

Only the difference between μ1 and μ0 is used in the calculations.

Effect Size – Standard Deviation

σ (Standard Deviation)

This option specifies one or more values of the standard deviation. This must be a positive value. Be sure to use the standard deviation of *X* and not the standard deviation of the mean (the standard error). If you are doing a paired test, this is the standard deviation of the differences.

When this value is not known, you must supply an estimate of it. **PASS** includes a special module for estimating the standard deviation. This module may be loaded by pressing the *SD* button. Refer to the Standard Deviation Estimator chapter for further details.

Example 1 – Power after a Study

This example will cover the situation in which you are calculating the power of a Wilcoxon Signed-Rank test on data that have already been collected and analyzed. For example, you might be playing the role of a reviewer, looking at the power of a Wilcoxon Signed-Rank test from a study you are reviewing. In this case, you would not vary the means, standard deviation, or sample size since they are given by the experiment. Instead, you investigate the power of the significance tests. You might look at the impact of different alpha values on the power.

Suppose an experiment involving 100 individuals yields the following summary statistics:

Hypothesized mean (μ_0)	100.0
Sample mean (μ_1)	110.0
Sample standard deviation	40.0
Sample size	100

Given the above data, analyze the power of a Wilcoxon Signed-Rank test which tests the hypothesis that the population median is 100 versus the alternative hypothesis that the population median is 110. Consider the power at significance levels 0.01, 0.05, 0.10 and sample sizes 20 to 120 by 20 when the assumed data distribution is Logistic.

Note that we have set μ_1 equal to the sample mean. In this case, we are studying the power of the Wilcoxon Signed-Rank test for a mean difference the size of that found in the experimental data.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Wilcoxon Signed-Rank Tests** procedure window by expanding **Means**, then **One Mean**, then clicking on **Nonparametric**, and then clicking on **Wilcoxon Signed-Rank Tests**. You may then make the appropriate entries as listed below, or open **Example 1** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Power**
Alternative Hypothesis............................	**Two-Sided (H1: μ ≠ μ0)**
Data Distribution	**Logistic**
Population Size...	**Infinite**
Alpha...	**0.01 0.05 0.10**
N (Sample Size).......................................	**20 to 120 by 20**
μ0 (Null or Baseline Mean).....................	**100**
μ1 (Actual mean)	**110**
σ (Standard Deviation)	**40**

Annotated Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results ──
Hypotheses: H0: μ = μ0   vs.   H1: μ ≠ μ0
Data Distribution: Logistic

Power	N	μ0	μ1	Diff μ1 - μ0	σ	Effect Size	Alpha	Beta
0.06416	20	100.0	110.0	10.0	40.0	0.250	0.010	0.93584
0.15855	40	100.0	110.0	10.0	40.0	0.250	0.010	0.84145
0.27019	60	100.0	110.0	10.0	40.0	0.250	0.010	0.72981
0.38634	80	100.0	110.0	10.0	40.0	0.250	0.010	0.61366
0.49768	100	100.0	110.0	10.0	40.0	0.250	0.010	0.50232
0.59833	120	100.0	110.0	10.0	40.0	0.250	0.010	0.40167

(report continues)

Report Definitions
Power is the probability of rejecting a false null hypothesis. It should be close to one.
N is the size of the sample drawn from the population. To conserve resources, it should be small.
μ0 is the value of the population mean under the null hypothesis.
μ1 is the actual value of the population mean at which power and sample size are calculated.
μ1 - μ0 is the difference between the actual and null means.
σ is the standard deviation of the population. It measures the variability in the population.
Effect Size = |μ1 - μ0|/σ is the relative magnitude of the effect.
Alpha is the probability of rejecting a true null hypothesis. It should be small.
Beta is the probability of accepting a false null hypothesis. It should be small.

Summary Statements ───
A sample size of 20 achieves 6% power to detect a difference of 10.0 between the actual mean of
110.0 and the null hypothesized mean of 100.0 with an estimated standard deviation of 40.0 and
with a significance level (alpha) of 0.010 using a two-sided Wilcoxon Signed-Rank test assuming
that the actual data distribution is logistic.

This report shows the values of each of the parameters, one scenario per row. The values of power and beta were calculated from the other parameters.

The definitions of each column are given in the Report Definitions section.

Plots Section
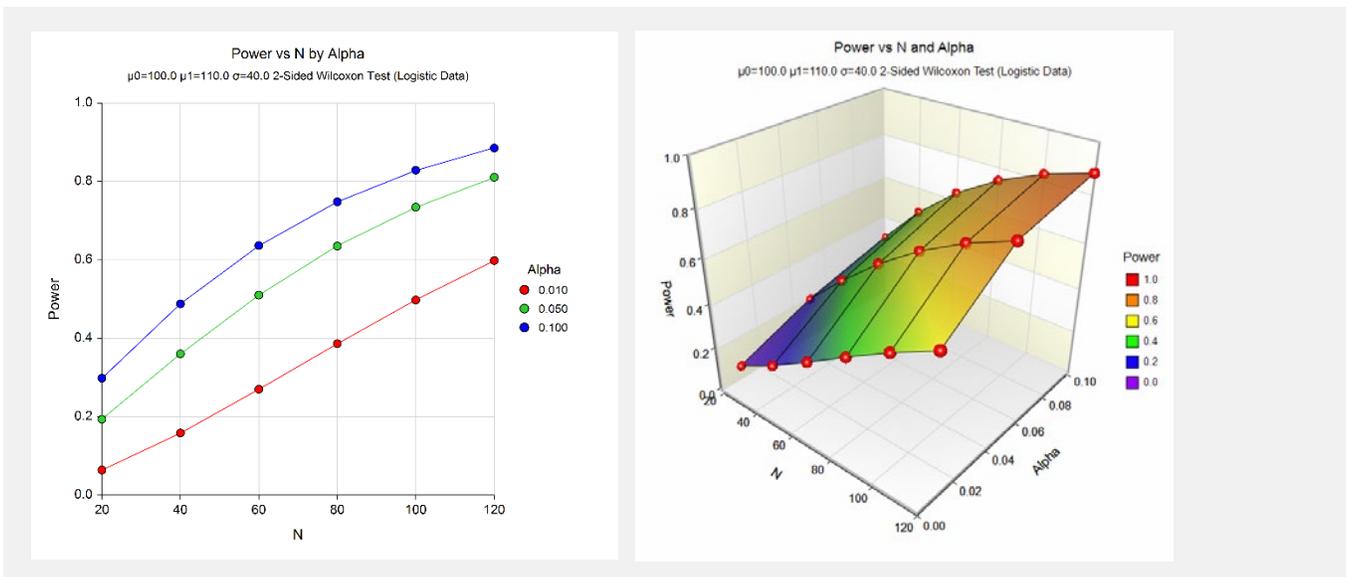
These plots show the relationship between sample size and power for various values of alpha.

Example 2 – Finding the Sample Size

This example will consider the situation in which you are planning a study that will use the Wilcoxon Signed-Rank test and want to determine an appropriate sample size. This example is more subjective than the first because you now have to obtain estimates of all the parameters. In the first example, these estimates were provided by the data.

In studying deaths from SIDS (Sudden Infant Death Syndrome), one hypothesis put forward is that infants dying of SIDS weigh less than normal at birth. Suppose the average birth weight of infants is 3300 grams with a standard deviation of 663 grams. Use an alpha of 0.05 and power of both 0.80 and 0.90. How large a sample of SIDS infants will be needed to detect a drop in average weight of 25%? Of 10%? Of 5%? Note that applying these percentages to the average weight of 3300 yields 2475, 2970, and 3135.

Although a one-sided hypothesis is being considered, sample size estimates will assume a two-sided alternative to keep the research design in line with other studies. The assumed data distribution will be normal.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Wilcoxon Signed-Rank Tests** procedure window by expanding **Means**, then **One Mean**, then clicking on **Nonparametric**, and then clicking on **Wilcoxon Signed-Rank Tests**. You may then make the appropriate entries as listed below, or open **Example 2** by going to the **File** menu and choosing **Open Example Template**.

Option	**Value**
Design Tab	
Solve For	**Sample Size**
Alternative Hypothesis	**Two-Sided (H1: μ ≠ μ0)**
Data Distribution	**Normal**
Population Size	**Infinite**
Power	**0.80 0.90**
Alpha	**0.05**
μ0 (Null or Baseline Mean)	**3300**
μ1 (Actual mean)	**2475 2970 3135**
σ (Standard Deviation)	**663**

Output

Click the Calculate button to perform the calculations and generate the following output.

```
Numeric Results ————————————————————————————————————————————————————————
Hypotheses:  H0: μ = μ0   vs.   H1: μ ≠ μ0
Data Distribution:  Normal
```

				Diff		Effect		
Power	**N**	**μ0**	**μ1**	**μ1 - μ0**	**σ**	**Size**	**Alpha**	**Beta**
0.85339	9	3300.0	2475.0	-825.0	663.0	1.244	0.050	0.14661
0.90307	10	3300.0	2475.0	-825.0	663.0	1.244	0.050	0.09693
0.80426	36	3300.0	2970.0	-330.0	663.0	0.498	0.050	0.19574
0.90409	48	3300.0	2970.0	-330.0	663.0	0.498	0.050	0.09591
0.80105	136	3300.0	3135.0	-165.0	663.0	0.249	0.050	0.19895
0.90070	181	3300.0	3135.0	-165.0	663.0	0.249	0.050	0.09930

This report shows the values of each of the parameters, one scenario per row. Since there were three values of μ1 and two values of power, there are a total of six rows in the report.

We were solving for the sample size, *N*. Notice that the increase in sample size seems to be most directly related to the difference between the two means. The difference in beta values does not seem to be as influential, especially at the smaller sample sizes.

Note that even though we set the power values at 0.8 and 0.9, these are not the power values that were achieved. This happens because *N* can only take on integer values. The program selects the first value of *N* that gives at least the values of alpha and power that were desired.

Example 3 – Finding the Minimum Detectable Difference

This example will consider the situation in which you want to determine how small of a difference between the two means can be detected by the Wilcoxon Signed-Rank test with specified values of the other parameters.

Continuing with the previous example, suppose about 50 SIDS deaths occur in a particular area per year. Using 50 as the sample size, 0.05 as alpha, and 0.80 as power, how large of a difference between the means is detectable?

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Wilcoxon Signed-Rank Tests** procedure window by expanding **Means**, then **One Mean**, then clicking on **Nonparametric**, and then clicking on **Wilcoxon Signed-Rank Tests**. You may then make the appropriate entries as listed below, or open **Example 3** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**µ1 (Search < µ0)**
Alternative Hypothesis	**Two-Sided (H1: µ ≠ µ0)**
Data Distribution	**Normal**
Population Size	**Infinite**
Power	**0.80**
Alpha	**0.05**
N (Sample Size)	**50**
µ0 (Null or Baseline Mean)	**3300**
σ (Standard Deviation)	**663**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results ───
Hypotheses: H0: µ = µ0   vs. H1: µ ≠ µ0
Data Distribution: Normal

Power	N	µ0	µ1	Diff µ1 - µ0	σ	Effect Size	Alpha	Beta
0.80000	50	3300.0	3023.2	-276.8	663.0	0.417	0.050	0.20000

With a sample of 50, a difference of 3023.2 - 3300 = -276.8 would be detectable.

Example 4 – Validation using Chow, Shao, Wang, and Lokhnygina (2018)

Chow, Shao, Wang, and Lokhnygina (2018) presents an example on pages 45 and 46 of a two-sided one-sample *t*-test sample size calculation in which μ0 = 1.5, μ1 = 2.0, σ = 1.0, alpha = 0.05, and power = 0.80. They obtain a sample size of 34.

The Wilcoxon Signed-Rank test power calculations are the same as the one-sample *t*-test except for an adjustment factor for the assumed data distribution. If we assume a uniform data distribution, we should get this same result. If we set the data distribution to Normal, we should get a result of N = 34 × π/3 = 35.605 = 36.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Wilcoxon Signed-Rank Tests** procedure window by expanding **Means**, then **One Mean**, then clicking on **Nonparametric**, and then clicking on **Wilcoxon Signed-Rank Tests**. You may then make the appropriate entries as listed below, or open **Example 4 (a or b)** by going to the **File** menu and choosing **Open Example Template**.

Option	**Value**
Design Tab	
Solve For	**Sample Size**
Alternative Hypothesis	**Two-Sided (H1: μ ≠ μ0)**
Data Distribution	**Uniform**
Population Size	**Infinite**
Power	**0.80**
Alpha	**0.05**
μ0 (Null or Baseline Mean)	**1.5**
μ1 (Actual Mean)	**2**
σ (Standard Deviation)	**1**

Output

Click the Calculate button to perform the calculations and generate the following output.

```
Numeric Results ─────────────────────────────────────────────────────────
Hypotheses:  H0: μ = μ0   vs.   H1: μ ≠ μ0
Data Distribution:  Uniform

                                    Diff          Effect
Power       N        μ0       μ1    μ1 - μ0   σ    Size    Alpha    Beta
0.80778     34       1.5      2.0   0.5       1.0  0.500   0.050    0.19222
```

The sample size of 34 matches Chow, Shao, Wang, and Lokhnygina (2018) exactly. If we now assume a Normal data distribution, the results match our expected outcome.

```
Numeric Results ─────────────────────────────────────────────────────────
Hypotheses:  H0: μ = μ0   vs.   H1: μ ≠ μ0
Data Distribution:  Normal

                                    Diff          Effect
Power       N        μ0       μ1    μ1 - μ0   σ    Size    Alpha    Beta
0.80778     36       1.5      2.0   0.5       1.0  0.500   0.050    0.19222
```

The sample size of 36 matches the expected result.

Example 5 – Validation using Zar (1984)

Zar (1984) pages 111-112 presents an example in which μ0 = 0.0, μ1 = 1.0, σ = 1.25, alpha = 0.05, and N = 12. Zar obtains an approximate power of 0.72 for the one sample *t*-test.

The Wilcoxon Signed-Rank test power calculations are the same as the one-sample *t*-test except for an adjustment factor for the assumed data distribution. If we assume a uniform data distribution, we should get the same power with N = 12. If we assume a Double Exponential data distribution with N = 12 × 2/3 = 8, then the adjusted sample size would be N' = 8/(2/3) = 12 and we should get the same power as Zar (1984).

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Wilcoxon Signed-Rank Tests** procedure window by expanding **Means**, then **One Mean**, then clicking on **Nonparametric**, and then clicking on **Wilcoxon Signed-Rank Tests**. You may then make the appropriate entries as listed below, or open **Example 5 (a or b)** by going to the **File** menu and choosing **Open Example Template**.

Option   **Value**

Design Tab
Solve For .... **Power**
Alternative Hypothesis .... **Two-Sided (H1: μ ≠ μ0)**
Data Distribution .... **Uniform**
Population Size .... **Infinite**
Alpha **0.05**
N (Sample Size) **12**
μ0 (Null or Baseline Mean) **0**
μ1 (Actual Mean) **1**
σ (Standard Deviation) **1.25**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results ───
Hypotheses: H0: μ = μ0   vs. H1: μ ≠ μ0
Data Distribution: Uniform

				Diff		Effect		
Power	**N**	**μ0**	**μ1**	**μ1 - μ0**	**σ**	**Size**	**Alpha**	**Beta**
0.71366	12	0.0	1.0	1.0	1.25	0.800	0.050	0.28634

The difference between the power computed by **PASS** of 0.71366 and the 0.72 computed by Zar is due to Zar's use of an approximation to the noncentral *t* distribution. Notice that the results for the Wilcoxon signed-rank test with the uniform distribution match the one-sample *t*-test results.

If we now assume a double exponential data distribution and set N = 8, the results match our expected outcome.

Numeric Results ───
Hypotheses: H0: μ = μ0   vs. H1: μ ≠ μ0
Data Distribution: Double Exponential

				Diff		Effect		
Power	**N**	**μ0**	**μ1**	**μ1 - μ0**	**σ**	**Size**	**Alpha**	**Beta**
0.71366	8	0.0	1.0	1.0	1.25	0.800	0.050	0.28634

The power of 0.71366 matches the expected result.

Example 6 – Validation using Machin (1997)

Machin, Campbell, Fayers, and Pinol (1997) page 37 presents an example in which µ0 = 0.0, µ1 = 0.2, σ = 1.0, alpha = 0.05, and beta = 0.20. They obtain a sample size of 199.

As in Example 4, if we set the data distribution to Uniform, we should get this same result. If we set the data distribution to double exponential, we should get a result of N = 199 × 2/3 = 132.6667 = 133.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Wilcoxon Signed-Rank Tests** procedure window by expanding **Means**, then **One Mean**, then clicking on **Nonparametric**, and then clicking on **Wilcoxon Signed-Rank Tests**. You may then make the appropriate entries as listed below, or open **Example 6 (a or b)** by going to the **File** menu and choosing **Open Template**.

Option	**Value**
Design Tab	
Solve For	**Sample Size**
Alternative Hypothesis	**Two-Sided (H1: µ ≠ µ0)**
Data Distribution	**Uniform**
Population Size	**Infinite**
Power	**0.80**
Alpha	**0.05**
µ0 (Null or Baseline Mean)	**0**
µ1 (Actual mean)	**0.2**
σ (Standard Deviation)	**1**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results ——
Hypotheses: H0: µ = µ0   vs.   H1: µ ≠ µ0
Data Distribution: Uniform

Power	N	µ0	µ1	Diff µ1 - µ0	σ	Effect Size	Alpha	Beta
0.80169	199	0.0	0.2	0.2	1.0	0.200	0.050	0.19831

The sample size of 199 matches Machin's result. If we now assume a double exponential data distribution, the results match our expected outcome.

Numeric Results ——
Hypotheses: H0: µ = µ0   vs.   H1: µ ≠ µ0
Data Distribution: Double Exponential

Power	N	µ0	µ1	Diff µ1 - µ0	σ	Effect Size	Alpha	Beta
0.80169	133	0.0	0.2	0.2	1.0	0.200	0.050	0.19831

The sample size of 133 matches the expected result.